ASTR320 Theoretical Astrophysics (Spring 2007)

Homework 4

(Due Thursday 8th March 2007)

1. **The Roche radius :** In class, we analyzed the restricted three-body problem and determined that if the binary system has a large mass ratio (i.e., $M_1 \gg M_2$), the L1 and L2 points are a distance

$$d = a(M_2/3M_1)^{1/3} \tag{1}$$

from the center of the small object. Use this fact to answer the following questions (you will need to look up some information to answer these questions... please list all sources of information):

   (a) Consider the Sun-Earth system. How far from the Earth (in kilometers) is the L2 point? Explain why this is an attractive location to place some science satellites.

   (b) Suppose an asteroid gets captured into a circular orbit around the Earth. If the asteroid gets close enough to the Earth, the Earth's tidal forces will rip the asteroid to pieces. By considering the location of the L1/L2 points in the Earth-asteroid binary system, show that the critical radius around the Earth separating disruption from non-disruption is a function of only the mass of the Earth and the *average density* of the asteroid. What is this critical radius if the asteroid has an average density of $\rho = 2 \times 10^3\,\mathrm{kg\,m^{-3}}$?

   (c) If the orbit of a star in a galaxy takes it too close to the central supermassive black hole, it can be ripped to pieces by the tidal forces of the black hole. Basing your answer on part (b) above, explain why a Red Giant star is more easily torn to pieces than a main sequence star.

2. **Gravitational potential energy of distributed systems :** Here, we derive the results that were quoted in class for the gravitational potential energy of distributed (spherically symmetric) collections of mass.

   (a) Suppose that the density at a distance r from the center of the distribution is $\rho(r)$. By imagining that you are "peeling" away concentric shells of the distribution, carefully explain why the gravitational potential energy of the distribution is

$$W = -(4\pi)^2 G \int_{r=0}^{\infty} r\rho(r) \left(\int_{x=0}^{r} x^2 \rho(x)\, dx \right) dr. \tag{2}$$

   (b) Suppose that the density is constant ($\rho(r) = \rho_0$) within some sphere of radius R_0 and is zero outside of this radius. Evaluate this integral to show that

$$W = -\frac{3GM^2}{5R_0}, \tag{3}$$

   where M is the total mass.

(c) Now suppose that $\rho(r) \propto r^{-\alpha}$ out to a radius $R = R_1$ and is zero outside of that radius. Show that

$$W = -\frac{3GM}{5R_1}\left(\frac{1-\alpha/3}{1-2\alpha/5}\right). \tag{4}$$

[You may assume that $alpha < 3$.]