ASTR320 Theoretical Astrophysics (Spring 2007)

Homework 7

(Due Thursday 19th April 2007)

1. **The static universe :** In class, you were introduced to the "Friedmann equation". For the case of a universe that has only matter and a cosmological constant (i.e., radiation is negligible), this equation is

$$\frac{1}{R}\frac{d^2R}{dt^2} = -\frac{4}{3}\pi G\rho + \frac{\Lambda}{3}. \tag{1}$$

   In this question, you will use this to explore the so-called "static universe".

   (a) Show that there is a special density ρ_s for which the Universe is *static*, i.e., neither expanding nor contracting with time.

   (b) Suppose that we have a static universe. Show that this static universe is unstable, i.e., that a small deviation away from the special density leads to a universe that either expands at an accelerating rate forever or collapses down to a big crunch. Please note you do not need to actually solve the differential equation — you simply need to put forward convincing arguments as to why the solution will have this behaviour.

2. **Fusion reactions in the early universe :** The temperature of the Universe at the time that radiation ceased to dominate its energy budget ($300,000$ yr after the big bang) was about $25,000K$. Before this, the universe expanded in a manner according to the radiation dominated case discussed in class. At sufficiently early times, the Universe was hot enough for the hydrogen to undergo nuclear fusion and rapidly produce helium. Given that the required temperature is $T \sim 10^8 K$, estimate the age of the Universe when hydrogen fusion occured.