ASTR498E High Energy Astrophysics (Spring 2006)

Homework 5

Due date; Tuesday 16th May 2006

1. **Relativistic Jets :** The best evidence that AGN jets contain material moving at relativistic velocities comes from observations of "superluminal motion".

   (a) Suppose that a blob is traveling down a jet at a velocity v, with an angle θ between the blob's velocity vector and the observer's line of sight to the system. Show that the apparent velocity of the blob on the plane of the sky is given by,

   $$V_{\rm app} = \frac{v}{1 - v\cos\theta/c}; \tag{1}$$

   (b) Show that, for a given velocity v, the apparent velocity $V_{\rm app}$ is maximized when $\cos\theta = v/c$ and, at that maximum, has a value $V_{\rm app} = \Gamma v$ where $\Gamma = (1 - v^2/c^2)^{-1/2}$ is the standard Lorentz factor.

   (c) In the M87 jet, we see blobs with apparent motions of $V_{\rm app} \approx 6c$. Calculate an approximate value for the velocity of these blobs.

   (d) Observations of some systems reveal blob motions in both the approaching jet and the receding jet (i.e. the counter-jet). Blobs in the receding jet are always seen to be moving slower than those in the approaching jet, and never attain apparent superluminal velocities. If we assume that both jets have the same true velocity and are oriented back-to-back, show that one can solve uniquely for both the jet speed v and jet angle θ if you know the apparent speeds of blobs in both jets. [Hint — you might find it useful to consider drawing lines on the (v,θ)-plane.]

2. **Dark Energy and the Big Rip :** In class, we discussed how the equation of state of Dark Energy can be parameterized as $p = w\rho_{\rm de}$, where $\rho_{\rm de}$ is the energy density of the dark energy. In a Universe which has both matter and dark energy, we can rewrite the equation of state as $p = w(\rho_{\rm tot} - \rho_{\rm matter})$. Hence, we can describe the Universe with an "effective" equation of state $p = w_{\rm eff}\rho_{\rm tot}$ where $w_{\rm eff} = w(1 - \rho_{\rm matter}/\rho_{\rm tot})$.

   (a) An analysis of one of Friedmann's equation shows that the total energy density of the Universe depends on the scale factor ("size") of the Universe R according to $\rho_{\rm tot} \propto R^{-3(1+w_{\rm eff})}$. Briefly discuss the behaviour of this expression in the two special cases of pure normal matter ($w_{\rm eff} = 0$) and pure vacuum energy $w_{\rm eff} = -1$.

   (b) Use the other Friedmann equation,

   $$\frac{1}{R}\frac{d^2R}{dt^2} = -\frac{4\pi G}{3}(1 + 3w_{\rm eff})\rho_{\rm tot}, \tag{2}$$

   to show that the resulting time-dependence of the scale factor is $R \propto t^{-2/3(1+w_{\rm eff})}$.

(c) Show that acceleration only occurs when $w_{\text{eff}} < -1/3$.

(d) At the current time, we know that $w_{\text{eff}} > -1$. However, suppose that the equation of state of dark energy itself has $w < -1$. As the Universe expands and dark energy becomes more dominate, the value of w_{eff} will move closer to w. Explain why something dramatic will happen at the point where $w_{\text{eff}} = -1$. What will be the nature of this dramatic event? [This event is often referred to as the *Big Rip*].