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Experimental determination of the coefficient of restitution for meter-scale granite spheres

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ABSTRACT

We present results of a series of large-scale experiments to measure the coefficient of restitution for 1-m-diameter rocky bodies in impacts with collision speeds up to ~1.5 m s^{-1}. The experiments were conducted in an outdoor setting, with two 40-ton cranes used to suspend the ~1300-kg granite spheres pendulum-style in mutual contact at the bottoms of their respective paths of motion. The spheres were displaced up to ~1 m from their rest positions and allowed to impact each other in normal-incidence collisions at relative speeds up to ~1.5 m s^{-1}. Video data from 66 normal-incidence impacts suggest a value for the coefficient of restitution of 0.83 ± 0.06 for collisions between ~1-m-scale spheres at speeds of order 1 m s^{-1}. No clear trend of coefficient of restitution with impact speed is discernable in the data.

1. Introduction

Collisions between small solid bodies in the Solar System, and their outcomes, are frequently modeled by granular physics codes (e.g., Richardson et al., 2000; Sánchez et al., 2004; Korycansky and Asphaug, 2006). The "granules" in these codes are typically rocks tens of meters in diameter; in part this is because of the limits of numerical resolution in modeling kilometer-sized bodies, but it is also in part an aspect of reality: we observe that the smallest rocky asteroids are made primarily of meter- to decameter fragments, at least in the case of the one small asteroid seen up close so far, Itokawa, which appears to be a pile of pebble- to house-sized rocks (Fujiwara et al., 2006).

In modeling granular collisions, two considerations must be taken into account. One is that the impact speed is not so high that the notion of elastic rebound is rendered moot. The other is that collisions are not perfectly elastic; the inelasticity is typically expressed as a coefficient of restitution, which is the ratio of the outgoing speed to the incoming speed in a collision. So for instance, laboratory-scale measurements of the coefficient of restitution for low-speed rock-on-rock collisions yield values around 0.8–0.9 (Imre et al., 2008). Is this an adequate description for blocks of material meters to tens of meters in diameter, which comprise the bulk of the mass of small asteroids and planetesimals? Balancing size-dependent failure strength (e.g., Housen and Holsapple, 1999) against the elastic stress accrued during deceleration of an object of a given size to a full stop from a velocity v, we find that at an impact velocity of several m/s the weakest flaws in a rock volume should be activated, releasing elastic energy and lowering the coefficient of restitution. On the other hand, the existence of intact decameter boulders within landslide deposits on Earth indicates that rocks are not, perhaps, so fragile with size, and that an elastic collisional approach, parameterized by a single coefficient of restitution, might apply for a wide variety of regimes of collisional, tidal, or other kinds of evolution.

Among several of these granular codes which rely on assumed values of the coefficient of restitution (e.g., Takeda and Ohtsuki, 2007; Korycansky and Asphaug, 2010), one of the most widely used is the fast N-body code `pkdgrav` (Richardson et al., 2000; Stadel, 2001), which is used to study many problems in planetary science, ranging from the collisional and dynamical evolution of planetary ring systems (e.g., Porco et al., 2008), to problems of planetesimal accretion (Leinhardt et al., 2000, 2009; Leinhardt and Richardson, 2002, 2005; Barnes et al., 2009), to the outcomes of disruptive asteroid collisions (e.g., Michel et al., 2002, 2004; Durda et al., 2004, 2007; also see Jutzi et al., 2010). Within the code is the assumption of a coefficient of restitution characterizing

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the outcomes of low-speed (up to $\sim$10 m s^{-1}) collisions between small bodies (up to $\sim$200 m diameter).[1] Since few experimental data exist from which to extrapolate for the value of the coefficient of restitution appropriate to rocky and icy bodies in this size and speed range, most `pkdgrav` simulations have assumed values of $\sim$0.5.[2] How representative this value is for the size and impact speed range typically used within the various simulations or how variable it may be is simply not well known, and is a significant roadblock to a better understanding of these and other problems in planetary science.

Previous work by other researchers (Bridges et al., 1984; Hatzes et al., 1988; Supulver et al., 1995; Dilley and Crawford, 1996) has already established that the coefficient of restitution varies with size and impact speed for $\sim$1 cm s^{-1} impacts between $\sim$1 and 10-cm diameter water ice spheres, but little or no work has been done to characterize the coefficient of restitution for rocky bodies or for larger ice bodies. Since large rocky bodies tend to have larger and more numerous structural flaws (that fail at lower stress levels and strain rates) than small bodies, collisions between larger bodies might be expected to be more dissipative than impacts at the same speeds between smaller bodies. Without the necessary experimental data, however, we have no reliable basis for setting the coefficient of restitution over the full range of compositions, sizes, and impact speeds presently being modeled.

In this paper we present results of a series of large-scale experiments to measure the coefficient of restitution for 1-m-diameter rocky bodies in impacts with collision speeds up to $\sim$1.5 m s^{-1}. These data, the first gathered at these size scales to our knowledge, offer a significantly greater "lever arm" than has previously existed for reliable scaling of the coefficient of restitution applicable to the compositions, sizes, and impact speeds being widely modeled and will allow parameterizations of its value for variable impact conditions for various planetary science applications.

2. Background and relevance to planetary science

Whenever two bodies undergo a collision, the force that each exerts on the other is an internal force, so that the total linear momentum of the system remains unchanged:

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2', \tag{1}$$

where the subscripts 1 and 2 refer to the two bodies and the primes indicate the velocities after the collision. In terms of the energy balance we can write

$$1/2 m_1 v_1^2 + 1/2 m_2 v_2^2 = 1/2 m_1 v_1'^2 + 1/2 m_2 v_2'^2 + E, \tag{2}$$

where E is the net energy lost as a result of the collision. To compute the velocities after the collision, given the velocities before the collision, we can use Eqs. (1) and (2) together, if we know the value of E. It is conventional in problems of this kind to introduce a parameter ε, the *coefficient of restitution*, defined as the ratio of the relative speed of separation, v', to the relative speed of impact, v:

$$\varepsilon = |v_2' - v_1'|/|v_2 - v_1| = v'/v. \tag{3}$$

In the general case of nonelastic collisions, the energy loss E is related to the coefficient of restitution by

$$E = 1/2 \mu v^2 (1 - \varepsilon^2), \tag{4}$$

where $\mu = m_1 m_2/(m_1 + m_2)$ is the reduced mass. For perfectly elastic collisions, $\varepsilon = 1$; in the case of perfectly inelastic collisions, the two bodies have no relative translational motion after the impact and $\varepsilon = 0$. For real bodies the numerical value of ε lies somewhere between 0 and 1 and depends primarily on the composition and physical properties of the colliding bodies, but is also a function of their relative impact speed and size/mass (see Section 3 below).

Knowledge of the range of values of the coefficient of restitution and its dependence on the size/mass and material properties of the colliding bodies and the impact speed is critical for a number of problems in planetary studies, including planetary ring dynamics, asteroid collision outcomes, and planetesimal accretion and growth.

2.1. Planetary rings

Energy loss during inter-particle collisions in ring systems plays a key role in determining mean free paths, velocity dispersions, viscosity, spreading rates, and ring thickness. As ring particles may have a size distribution covering orders of magnitude in scale (Cuzzi et al., 1984), and since, for example, size-dependent velocity dispersions can lead to particle-size segregation within rings, knowledge of the size dependence of ε is crucial. Recently, Porco et al. (2008) found that the coefficient of restitution may play a key role in understanding the azimuthal brightness asymmetry in Saturn's A ring by controlling the size and compactness of transient aggregates in the ring. Fig. 1 is a simple illustration of how dramatic the choice of ε can be on the equilibrium state of a typical patch of Saturn's rings. As the amount of dissipation increases, wakes and aggregates become more pronounced (these in turn affect the amplitude of any brightness asymmetry).

2.2. Asteroid collisions

Recent numerical simulations of the catastrophic disruption of 100-km scale asteroids have been used to study the formation of asteroid families (e.g., Michel et al., 2002, 2004; Durda et al., 2007) and the formation of asteroid satellites (Durda et al., 2004). In these simulations, fragmentation results of 3D smoothed-particle hydrodynamics (SPH) calculations (e.g., Benz and Asphaug, 1995) are handed off to `pkdgrav` *N*-body calculations to follow the subsequent gravitational reaccumulation and dynamical evolution phases of the interacting fragments. Knowledge of the numerical values of ε covering the range of particle sizes and impact speeds modeled in the `pkdgrav` phase of these simulations would greatly improve the fidelity of these models. In particular, the choice of ε affects the number and size of clumps that reaccumulate after a catastrophic impact (most simulations to date have taken $\varepsilon \approx 0.5$ for expediency).

2.3. Accretion of planetesimals

If planets grow by the pair-wise accretion of planetesimals (the "planetesimal hypothesis"; see Lissauer (1993) for a review), the efficiency of the accretion depends on the details of the planetesimal collision process. Numerical models of planetesimal formation usually assume either "perfect" accretion ($\varepsilon = 0$; e.g., Barnes et al., 2009), or outcomes based on the extrapolation, over many orders of magnitude in scale, of limited laboratory experiments. More recently, Leinhardt et al. (2000, 2009) and Leinhardt and Richardson (2002, 2005) used `pkdgrav` to model early planetesimals as km-size gravitational aggregates colliding at speeds commensurate with conditions in the early Solar System. In these simulations the individual fragments are $\sim$100 m or smaller in size and are

[1] Impacts at >10 m/s likely result in extensive fracturing, a process not modeled by `pkdgrav`, and bodies larger than $\sim$200 m diameter are likely themselves to be extensively fractured, since gravity dominates over material strength in this regime (Asphaug et al., 2002).

[2] The code also allows for a tangential coefficient of restitution that represents a limited form of surface friction, but this quantity is even less well-constrained for realistic materials. We did include some oblique and tangential impacts in the suite of experiments described here, but the data from those impacts will be described elsewhere.

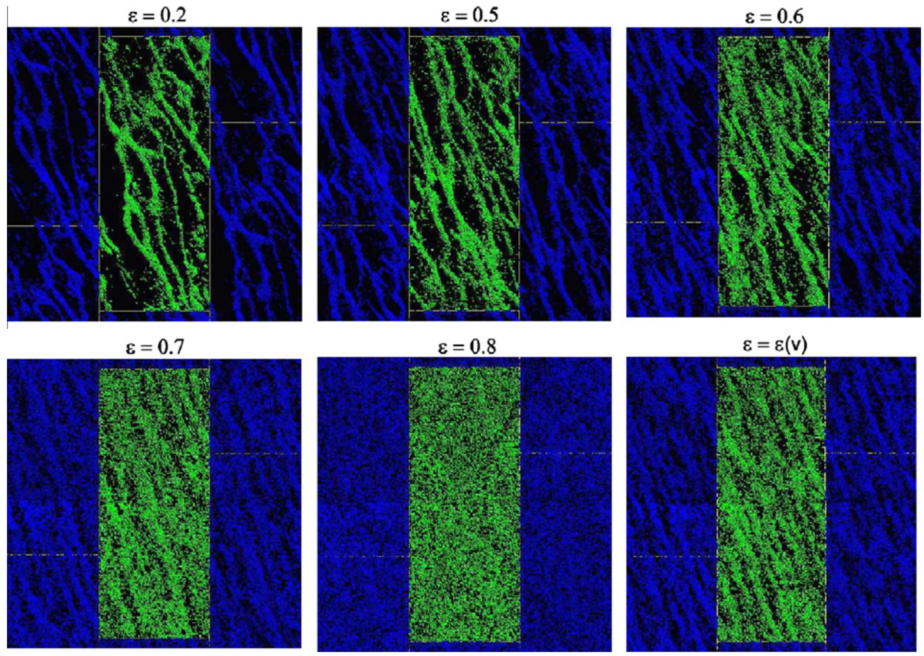

Fig. 1. N-body numerical simulations (using `pkdgrav`) of a patch of Saturn's rings for various choices of the normal coefficient of restitution. The bottom-right panel shows the results using the speed-dependent model of Supulver et al. (1995) with a small amount of transverse motion coupling (tangential coefficient of restitution slightly below unity); the other panels show models with constant ε and no transverse motion coupling. For these illustrative simulations, the patch was centered 100,000 km from Saturn (the approximate location of Saturn's B ring), the dynamical optical depth was set to 0.5 (a little low for the B ring), the runs were carried out for three orbits, the particles had fixed radii of 0.5 m, the patch size was about 100×250 m in dimension (large enough to capture the dominant instability wavelength), the initial velocity dispersions were about 0.1 m s^{-1}, the surface density was 330 kg m^{-2}, and the particle density was 1000 kg m^{-3}. The central patch in each panel is surrounded by "ghost" patches that provide boundary conditions.

assumed to not fracture further at the low (<10 m s^{-1}) impact speeds being modeled. In that case the collision outcome is very sensitive to the adopted value of ε for the individual fragment collisions. Obtaining better values of ε and its functional dependence on size and speed will reduce a major uncertainty in planetesimal accretion modeling.

Relatively few experimental data exist, however, that allow numerical modelers to reliably parameterize the coefficient of restitution characterizing impacts between relatively small planetary bodies of a variety of compositions (i.e., ring particles, accreting planetesimals, small fragments of mutual asteroid impacts, etc.) over the range of impact speeds and body sizes (i.e., speeds up to $\sim$10 m s^{-1} and sizes up to of order 10–100 m in diameter) directly applicable to these and other interesting problems in planetary science.

3. Previous work

Bridges et al. (1984), Hatzes et al. (1988), Supulver et al. (1995), and Dilley and Crawford (1996) have measured the coefficient of restitution for $\sim$1 cm s^{-1} impacts between $\sim$5-cm diameter icy spheres at a range of temperatures (and pressures) applicable to planetary ring studies. Each of these studies was conducted through some variation of pendulum experiments in which moving balls come into contact with a massive stationary block; the coefficient of restitution is measured by the ratio of the speed of the ball after impact to the initial impact speed. The Bridges et al., Hatzes et al., and Supulver et al. experiments employed a compound disk pendulum in order to accurately determine the colliding ice balls' impact velocities at very low speeds, while in the Dilley and Crawford experiments the colliding ice balls were suspended by long, light lines to obtain nearly free collisions.

Bridges et al. (1984) used water ice spheres with a radius of curvature of 2.75 cm at a temperature of $\sim$175 K and at atmospheric pressure to determine that $\varepsilon(v) = 0.32\,v^{-0.24}$ for speeds greater than about 0.05 cm s^{-1}. With an upgraded apparatus, including a new cryostat capable of stable temperatures as low as 85 K and an improved method of accurately determining $\varepsilon(v)$ for speeds as low as 0.005 cm s^{-1}, Hatzes et al. (1988) found that coefficient of restitution data can be well fit by an exponential law of the form $\varepsilon(v) = Ce^{-\gamma v}$ for impact speeds in the range 0.015–2 cm s^{-1} (e.g., Hatzes et al., 1988; Fig. 3). They found that the presence of surface frost can change the speed dependence of ε from an exponential to a power law and that frost or a roughened surface can reduce ε at a given speed by about 10–30%. Hatzes et al. also examined the effects of the geometry of the contact surface by colliding spheres with various radii of curvature molded onto a 2.5-cm radius sphere; this allowed them to increase the effective radius of the colliding sphere, but without the associated increase in actual diameter or mass. Supulver et al. (1995) extended this work to glancing collisions and found that the energy loss for tangential motion is surprisingly low, suggesting that at low impact speeds very little friction is present for smooth ice spheres at temperatures near 100 K.

Dilley and Crawford (1996) focused on determining the size/mass dependence of the coefficient of restitution for frosted ice spheres to test a "viscous dissipation" model introduced by Dilley (1993). The coefficient of restitution is essentially a measure of the fraction of the original impact energy retained after a collision: as the mass of a colliding body increases, its initial energy increases linearly with its mass, but the speed-dependent dissipative forces caused by frost coatings on the icy spheres are independent of mass. Thus, for collisions between low-mass (i.e., low initial energy) bodies, speed-dependent energy losses are dominant. For high-mass (i.e., high initial energy) bodies, the speed-dependent energy losses become less important and the fraction of the initial energy retained should approach unity. To test the Dilley (1993) model and to get around the fact that the inertial mass of the colliding ice ball in disk pendulum experiments is not equal to its actual mass, Dilley and Crawford obtained free pendulum motion by suspending the colliding ice balls by light lines in experiments with simple pendulums. They determined that, indeed, collisions at $\sim$1 cm s^{-1} become quite inelastic for small ice spheres and that the rate at which the coefficient of restitution decreases with decreasing mass roughly agrees with the viscous dissipation model described above.

Higa et al. (1998) investigated the size and impact speed dependence of the coefficient of restitution for water ice spheres in the 0.14–3.6 cm size range over a range of impact speeds from 1 cm s^{-1} to 10 m s^{-1}. They found ε to decrease with decreasing sphere size in the quasi-elastic region and to increase with decreasing sphere size in the inelastic region. Most recently, Heißelmann et al. (2010) utilized a cryogenic parabolic-flight experimental setup to determine ε for 1.5-cm-size ice spheres for collisions with relative speeds between 6 and 22 cm s^{-1}. Their data show a mean value of $\varepsilon = 0.45$, with values ranging from 0.06 to 0.84.

All of the studies summarized above have been focused on water ice impacts because of the interest in their applicability to outer planet ring systems. For asteroid studies, experiments with rock (i.e., non-ice) materials have also been conducted. For example, in addition to the Imre et al. (2008) results mentioned in Section 1, Fugii and Nakamura (2009) examined the compaction and fragmentation in low-speed impacts between 2.5 and 8.3-cm-diameter gypsum spheres, noting that there was not a great deal of difference in the coefficient of restitution between porous and non-porous gypsum. Heißelmann et al. (2010) statistically analyzed the evolution of the kinetic energy of an ensemble of up to 100 cm-scale glass spheres and derived a constant coefficient of restitution of $\varepsilon = 0.64$ for impact speeds of $\sim$3.5 mm s^{-1}. And perhaps most applicable to asteroid studies, the Hayabusa spacecraft obtained an unexpected measurement of the coefficient of restitution of an artificial satellite bouncing on the low-g surface of a rocky asteroid, measuring a value of $\varepsilon \approx 0.84$ for an impact speed of 6.9 cm s^{-1} (Yano et al., 2006).

Although the Dilley and Crawford and Higa et al. experiments focused on determining the size/mass dependence of ε, their experiments still covered less than an order of magnitude in size scale (at cm to dm size scales and mm to cm size scales, respectively), making it difficult to reliably scale the results to significantly larger (i.e., tens of meters) and more massive bodies. This makes the direct application of these experimental data to studies of impacts under other conditions between rocky, asteroidal bodies somewhat problematic. Does the Dilley and Crawford viscous dissipation model, which seems to describe fairly well the behavior of small frost-coated ice spheres, also apply to rough and possibly crumbly surfaces of rocky objects? Might the presence of larger surface structural flaws in larger bodies lead to an opposite effect, so that the coefficient of restitution *decreases* with increasing size in rocky bodies or at sizes larger than $\sim$11 cm for icy bodies?

4. Experiments

To begin to answer some of these questions, we have conducted an extensive series of large-scale experiments to measure the coefficient of restitution for impacts between 1-m-diameter granite spheres with collision speeds up to $\sim$1.5 m s^{-1}. These experiments are intended as the first in a series of similar experiments at a range of size scales, to evaluate the size dependence of the coefficient of restitution of this silicate material.

The two spheres were obtained from a commercial supplier of large natural stone spheres for water sculptures. The "G603" granite was quarried in China (Gary Jackson/Waterfountains.com LLC, personal communication), producing massive blocks that were milled to spheres; the surfaces of the spheres were smoothed but not polished.

Operations for the large-scale experiments described here were conducted at the San Antonio headquarters campus of the Southwest Research Institute (SwRI) during the week of 26–30 May, 2009. We contracted two 40-ton cranes to suspend the spheres for the experiments. Lifting straps were attached to suspend each sphere from its respective crane. The straps were slung under each sphere perpendicular to each other so that the spheres were securely cradled while still allowing significant open rock face for unobstructed rock-on-rock contact during the experiments (Fig. 2).

Before the experiment runs for coefficient of restitution data began, basic characterization/calibration data were obtained. The spheres were weighed using on-board crane sensors at 3000 and

Fig. 2. A 1-m-diameter granite sphere suspended for the large-scale 'pendulum' experiments. Left: placement of support strapping and lifting from forklift. Right: sphere suspended in place for experiment with 1-m scale bar and experiment run number marker shown.

2700 lbs (1361 and 1225 kg, respectively), to within the 100 lb precision allowed by the sensors. One sphere was displaced approximately 1 m from its equilibrium position and allowed to free swing for several cycles to verify that energy losses inherent in the suspension system were small compared with the energy losses in the collisions.

The two cranes were parked back-to-back for the experiment runs, so that the two spheres could be suspended in-line between the cranes and so that attach points on the rear bumpers of the cranes could be used to mount ratcheting 'come alongs' used to displace the spheres (Fig. 3). Three different configurations of the cable/strap suspension for the spheres were used, adjusting the positions of the so-called 'headache ball' (a 90–100 lbs steel sphere) used to stabilize crane loads against winds in order to find a configuration that minimized wobble of the granite spheres after impacts due to 'double pendulum' effects.

We began the experiment series with sphere displacement at a minimum (displacements of only ∼10 cm) in order to assess both the safety of the strap/sling supports and to insure that the spheres would not be overly damaged or even destroyed during the first few runs. For these early runs the spheres were simply displaced by hand, by pulling on rope loops that had been fed through the strap supports. For the first set of runs, we displaced only one sphere and let it impact the other, un-displaced, sphere. In this manner we slowly increased the displacement of the single sphere until displacements of over a meter were achieved, yielding impact speeds of order ∼1 m/s. For higher impact speeds we needed greater displacements. In part, this was achieved by displacing both spheres from their rest positions and allowing them to impact at the original, un-displaced point of contact. Also, we found it physically too difficult to pull and hold the spheres for these higher impact speeds and devised a mechanical mechanism to accomplish the task. Ratcheting cable pullers ('come alongs') were attached to the bumpers of the cranes and heavy-duty (175 lb rating) cable ties were looped to the rope loop holding the sphere. The cable ties were then cut on cue to release the sphere. This arrangement had the advantages of being able to pull and maintain greater displacement force than could be done by hand (thus achieving higher relative impact speeds) and of establishing much more stabilized initial conditions before sphere release.

5. Analysis of data and results

We obtained a total of 108 science data runs, of which 90 were head-on collisions, intended to measure the normal coefficient of restitution. All runs were imaged with high-definition video to record the raw science data: the ratio of initial sphere displacement and speed before impact to displacement and speed after impact. To ensure that prominent measurement points would always be resolved on each sphere we placed several white, ∼1-in. squares of self-adhesive Velcro patches on each sphere (Fig. 2, right).

Video data from 66 runs were reduced and analyzed to derive the coefficient of restitution as a function of impact speed. Wind-induced wobble, camera malfunctions, and bad lighting forced us to discard data from 24 runs.

A typical experiment run consists of ∼150 frames of video data. In each frame the horizontal position (pixel index) of the center of each sphere is measured, as a function of time (frame number). In principle, the velocity of each sphere before and after impact can be obtained from the position of the spheres in the few frames before and after the impact event. Since we are able to track the position of the spheres to within one pixel, and the frame rate is a very stable 20.97 frames per second, this would give us the coefficient of restitution with a relative accuracy of $\sim 2/v$, where v is the impact speed in pixels per frame. Unfortunately, this turned out to provide insufficient accuracy as most of our runs were at impact speeds corresponding to 3–4 pixels per frame. Our cameras had more than sufficient definition, but in retrospect a higher frame rate would have been useful.

To overcome the low-frame-rate problem, we needed to obtain the impact and separation velocities at better than one pixel-per-frame accuracy. This can be done by taking advantage of the pixel data from the entirety of the run, not just the few frames surrounding the impact event. Each sphere executes simple pendulum motion, with different amplitude before and after collision. The relative speeds of the spheres both before and after impact can be determined from the fit amplitudes and angular frequencies of the two pendula. The uncertainty of these velocity values depends on the goodness-of-fit statistics, and can be better than for the individual velocities obtained by comparing two sequential frames. This works very well for runs in the medium- to

Fig. 3. Granite spheres in equilibrium position. Left: load-stabilizing 'headache balls' (see text) situated low near the spheres. Right: headache balls moved higher up to minimize sphere wobble after impacts. In this un-displaced, equilibrium position the two spheres were just barely in contact with each other, with no appreciable normal contact force.

high-impact-speed range of our experiment. Coefficients of restitution obtained from low-impact-speed runs still have quite high uncertainties.

Fig. 4 shows the resulting derived coefficient of restitution as a function of impact speed for the 66 experiment runs analyzed. A clear speed-dependent trend is not apparent. In fact, the data appears to be scattered enough to suggest that ε is independent of impact speed in the range of speeds tested. If this is the case, the coefficient of restitution derived from experimental data should be scattered because of measurement errors, and the distribution of values should be approximately normal. A Q–Q plot of the sample data versus 66 values drawn from a standard normal distribution (Fig. 5) supports this conjecture. (In the language of statistical analysis – our sample data cannot be used to reject the null hypothesis.)

If no clear trend exists in the range of impact speeds investigated, a good representative value of ε can at least be extracted from the data. If the scatter in ε values is indeed due to random measurement error, then the sample is best represented by a normal distribution with a mean of 0.83 and standard deviation 0.06 (weighted by the inverse of individual uncertainties). Thus, we suggest using $\varepsilon = 0.83 \pm 0.06$ for the value of the coefficient of restitution for a collision between ~1-m-scale spheres at speeds of order 1 m s^{-1}. Note that this value is significantly higher than the value of 0.5 commonly used in numerical simulations, in the absence of data to suggest otherwise. It is also comfortably in the same range of values for ~1-cm-scale granite spheres as measured by Imre et al. (2008).

6. Discussion

As discussed in the Introduction, our motivation for these large-scale experiments was to extend the size scale of experimental data in order to more reliably extrapolate model results upwards toward real-world asteroid sizes. Our range of impact speeds, however, was limited to speeds up to about 1.5 m s^{-1} due to logistical mechanical constraints – forces involved were just too great to ratchet our spheres back any further to get higher speeds. Thus, we never approached near the impact speed regime necessary to see material compaction effects similar to those that Fugii and Nakamura (2009), for instance, saw in their gypsum experiments;

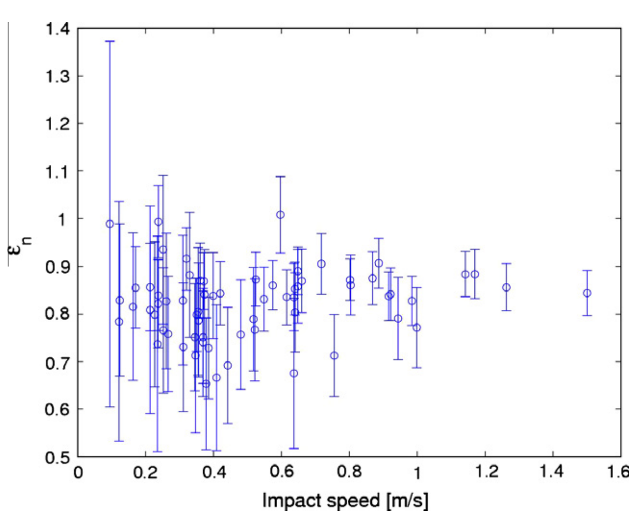

Fig. 4. Coefficient of restitution as a function of impact speed for 1-m-diameter granite spheres. Each point on the graph was obtained from a fit of a pixel number versus frame number curve, with ~170 points, to a 2-part sinusoidal, with different amplitudes before and after impact. The error bars are derived from the confidence intervals of the fitted parameters (amplitude and angular frequency).

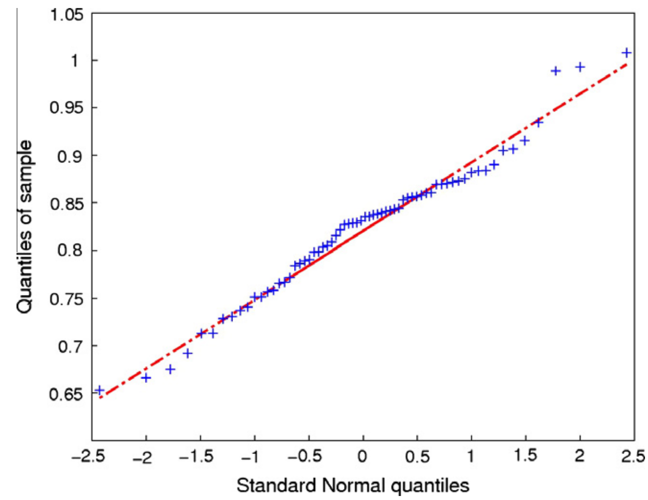

Fig. 5. Q–Q plot of sample data versus values drawn from a standard normal distribution. If the range of measured coefficient of restitution is due to normally distributed measurement errors, the sample data should show as approximately linear on the quantile versus quantile plot.

we certainly never got near any speed great enough to see large scale cracking. A priori we suspected, based on blind extrapolations of Weibull-type collisional physics approaches, that at the ~2 m s^{-1} impact speeds we planned to achieve we might approach an impact regime where we would see some change (reduction?) in coefficient of restitution due to small-scale surface cracking or compaction, but the Fugii and Nakamura results suggest otherwise – they did not see much change in ε all through the impact speed range right up to fragmentation.

The granite material from which our spheres were produced obviously survived intact through the quarrying and milling process. They thus represent a natural selection toward a likely stronger and large-flaw-free end-member sample of this particular granite. This fact should be considered when comparing our measured value for ε to other rocky materials and when using these data in contexts where values representing more porous and weaker materials might be more appropriate.

Also, irregular body shapes might cause the *effective* coefficient of restitution for collisions between real fragments in actual planetary contexts to approach the lower, ~0.5 value assumed in many of our `pkdgrav` numerical applications. Protuberances will tend to get tamped down on impact, lowering the effective coefficient of restitution (due to internal material reconfiguration), and off-axis impacts into irregular surfaces generate more angular momentum transfer than a typical tangential coefficient of restitution would provide. In future experiments we hope to use nearly spherical but still very rough ~10-cm and ~3-cm-scale granite fragments to measure their effective coefficient of restitution for normal-incidence collisions, checking for moderately increased dissipation/rotational coupling with increased surface irregularity. These data can then be compared with the values for spherical bodies of the same material to provide appropriate, realistically constrained inputs for `pkdgrav` or other numerical codes for which a proper choice of ε appears crucial.

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Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at doi:10.1016/j.icarus.2010.09.003.

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