Simulating Collisions in the Solar System

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0.1 Introduction & Techniques

In this review I will outline the importance of discrete particle collisions in the context of planetesimal dynamics and show how they can be simulated by direct methods. Direct methods are those in which the evolution of each particle is tracked explicitly. Although more computationally expensive than statistical techniques, direct methods have the advantage of making fewer approximating assumptions about the nature of interactions (such as the collective gravity in a system). The techniques will be illustrated in §0.2 using several examples from a broad range of applications.

0.1.1 Collisions in the Solar System

Under the "Planetesimal Hypothesis" of Solar System formation, planets grow by the pairwise accretion of planetesimals (see [8] for a review). Detecting and resolving particle collisions accurately is therefore of critical importance. In the case of planet formation, direct methods that incorporate collision mechanics provide bonus information, such as the exact planet genealogy (an indication of radial mixing) and the spin evolution (since the net spin is the product of individual impacts). The question of planet formation will be addressed in more detail in §0.2.5.

Other physical processes in the Solar System in which collisions play a key role include the production and dynamics of dust. Grain-grain collisions can lead to fractal growth (§0.2.3) and interesting collective behaviour (§0.2.2), even when gravity is unimportant, while collisions between larger bodies can replenish the dust population. Roughly 15% of nearby stars surveyed by IRAS show infrared excess that could be indicative of surrounding dust clouds [3]. In systems such as β Pic it is believed that this dust must be replenished by collisions between larger parent bodies, possibly the functional equivalent of our own Kuiper Belt Objects, i.e. debris left over from the formation of the Solar System. Asteroid collisions in our own Solar System are thought to be responsible for the Zodiacal dust cloud.

Collisions are important in more fundamental ways as well. For one, collisions provide a mechanism for transport of angular momentum in dense media. The momentum is transported via sound waves and gives rise to a nonlocal viscosity that may regulate behaviour in systems such as Saturn's dense particle rings (§0.2.1). In general, collisions provide a mechanism for damping excitations induced by gravitational scattering and shear in a Keplerian disk. By removing heat, the effective volume of the system decreases, causing the collision frequency to increase. In sufficiently cold systems this can lead to runaway growth of particles, a key ingredient of planet formation.

Finally, on a more technical note, simulations involving physical collisions are inherently discretized and do not require softening. This is an advantage so long as no hard binary formation is expected (otherwise regularization techniques might be required). The particle equations of motion therefore incorporate Newton's law of gravity exactly.

0.1.2 Collision Detection

To detect collisions in direct simulations, a code must make use of either *hindsight* or *foresight*. To use foresight, each collision must be predicted in advance. This is only feasible for linear trajectories, such as in the absence of gravity, or when using a low-order integrator like "leap-frog". In this case, the time until surface contact of particles of size R_1 and R_2, relative position $\mathbf{r}$, and relative velocity $\mathbf{v}$ is given by:

$$\delta t = -\frac{(\mathbf{r}\cdot\mathbf{v})}{v^2}\left\{1 - \sqrt{1 - \frac{\left[r^2 - (R_1 + R_2)^2\right] v^2}{(\mathbf{r}\cdot\mathbf{v})^2}}\right\}. \tag{1}$$

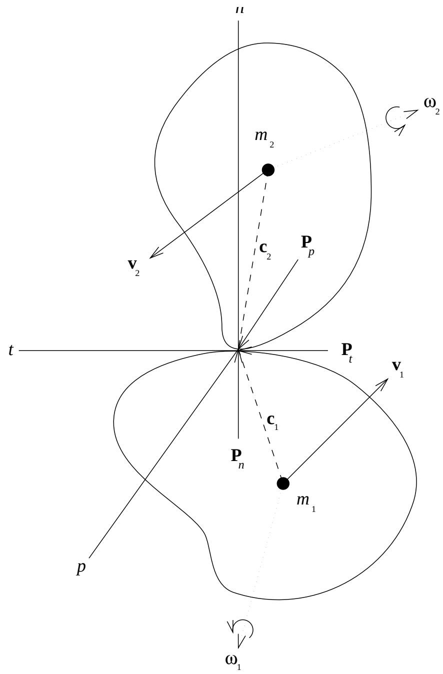
A necessary condition for collision is that $\mathbf{r}\cdot\mathbf{v} < 0$ (i.e. $\delta t > 0$) and that the argument of the square root be non-negative (otherwise there is no intersection). Note that to evaluate the collisions in the correct sequence, all potential colliders must be considered at each step.

Hindsight is necessary if the particle trajectories between updates are complicated, such as when using a high-order integrator. In this case collisions are detected *after* they actually happen. The goal is to minimize the interpenetration distance by choosing a time-step that is sensitive to close encounters, requiring an integration scheme that can handle changes in time-step size or that uses individual particle steps. A versatile expression is given by [1, 10]:

$$\delta t = \eta \left(\frac{a\ddot{a} + \dot{a}^2}{\dot{a}\dddot{a} + \ddot{a}^2}\right)^{1/2}, \tag{2}$$

where a is the particle acceleration and η is a dimensionless parameter $\ll 1$ chosen such that overlaps are $\lesssim$1% of the sum of the particle radii. Once a collision has been detected, the particle positions must be corrected so that the particles just touch before evaluating the collision outcome. For small overlaps it is safe to simply move the particles out along their line of centres [11]. For this hindsight method, only the nearest neighbour need be checked for collision, information that usually comes for free during gravity calculations.

Note that the collision methods described here are only valid for *instantaneous*, *isolated*, *single point contact* collisions: surface deformation effects are ignored, only one collision can take place at a time, and non-spherical colliding bodies must have only one contact point. This is sufficient for most cases, though the isolation condition is sometimes violated in the most dense simulations (e.g. rubble-piles, §0.2.4), resulting in small but manageable errors.



Physics

$$
\begin{aligned}
m_i \Delta \mathbf{v}_i &= (-1)^{i-1}\mathbf{P},\ i=1,2 \\
\Delta \mathbf{h}_i &= \mathbf{c}_i \times (-1)^{i-1}\mathbf{P},\ i=1,2 \\
u_n' &= -\epsilon_n u_n \\
P_t &= \mu_t P_n \\
P_p &= \mu_p P_n
\end{aligned}
$$

Definitions

$$
\begin{aligned}
\mathbf{h}_i &= \mathbf{I}_i \boldsymbol{\omega}_i,\ i=1,2 \\
\mathbf{u} &= \mathbf{v} + \boldsymbol{\sigma} \\
\mathbf{v} &= \mathbf{v}_2 - \mathbf{v}_1 \\
\boldsymbol{\sigma} &= \boldsymbol{\sigma}_2 - \boldsymbol{\sigma}_1 \\
\boldsymbol{\sigma}_i &= \boldsymbol{\omega}_i \times \mathbf{c}_i,\ i=1,2
\end{aligned}
$$

Figure 1: Definitions and equations for the collision problem.

0.1.3 Collision Resolution

There are many possible outcomes following a collision, depending on the nature of the colliding bodies. Here I will concentrate only on *bouncing*. Other possibilities are briefly described in §0.2.3 and §0.2.5. Fig. 1 illustrates the important physical parameters for the collision of two arbitrarily shaped bodies and lists the governing equations along with the necessary definitions. The $\mathbf{I}_i$'s refer to the inertia tensors of the colliding bodies. The quantity ϵ_n is the normal coefficient of restitution; μ_t and μ_p parameterize surface friction. The physics consists of statements of linear and angular momentum conservation, along with a kinematic equation and two kinetic equations. The latter three equations are *empirical* in origin—for complicated body shapes and surface friction they do *not* guarantee a physically meaningful outcome [5]. Remarkably, fully self-consistent bouncing models based on simple restitution parameters have yet to be devised. Fortunately, self-consistent solutions are derivable for certain special cases: non-central impact without surface friction [12]; central impact between spheres with surface friction [11]; and the simple case of central impact without surface friction, in which only linear momentum conservation and the kinematic equation play a role, i.e.:

$$\Delta \mathbf{v}_1 = (1+\epsilon_n)\frac{m_2}{m_1+m_2}\mathbf{u}_n,\ \Delta \mathbf{v}_2 = -\frac{m_1}{m_2}\Delta \mathbf{v}_1. \tag{3}$$

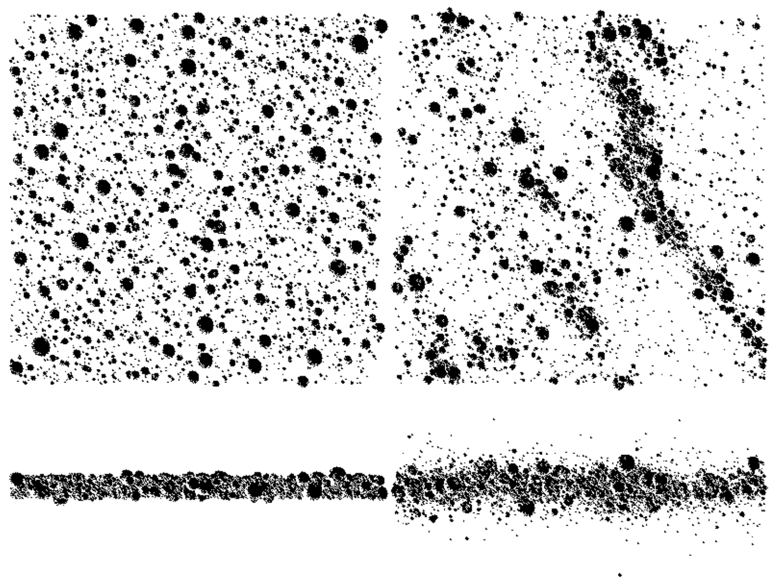


Figure 2: Local simulation of Saturn's B ring, showing the initial conditions (left) and the state after 3 orbits (right). Note the clumping that develops in the top view.

0.2 Applications

0.2.1 Planetary Rings

The brightness of Saturn's rings and the size distribution of the particles suggest that collisional processes are still at work. Due to the thinness of the rings and the observed density of material, collisions are thought to be frequent ($\gtrsim 1$ collision/particle/orbit in the dense B ring). Most of the features in planetary rings are attributed to resonances and shepherding by moons. However, the densest rings may be subject to gravitational instabilities that cause particles to clump together loosely for a while and then dissipate in the tidal field after just a few orbits. [13] showed that Saturn's A and B rings may be susceptible to formation of wakes similar to those found by [7] in stellar disks. The difference is that physical collisions play an important role in the rings case, reducing random motions and causing clumps to form. Both [13] and [11] have performed simulations illustrating these phenomena (Fig. 2). The instability wavelengths are too small to have been detected by Voyager, but the Cassini spacecraft, with an estimated 30 m resolution, should be able to see them.

0.2.2 Dynamics of Granular Media

As a quick example of the role of collisions in the dynamics of granular media, consider the following experiment: start with three identical non-gravitating balls in 1D with velocities such that the outer two balls converge on the inner ball. The outcome of the first two collisions can be written as a matrix equation. If the coefficient of restitution ϵ satisfies $0 < \epsilon < 7 - 4\sqrt{3}$, there is at least one real eigenvalue of the matrix between 0

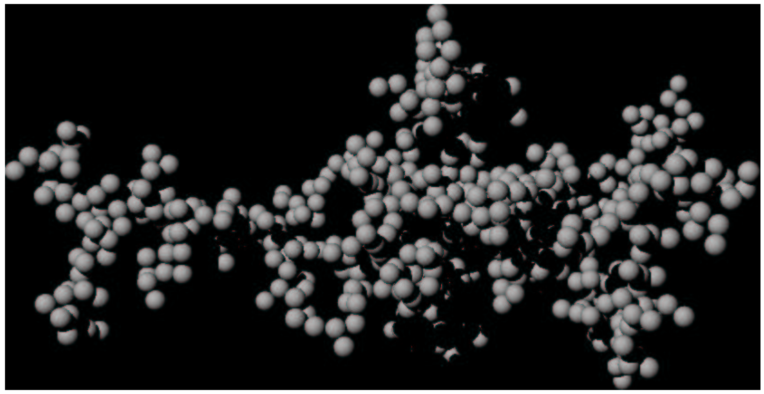


Figure 3: An example of a fractal aggregate.

and 1. This means the system will "collapse", suffering an infinite number of collisions in a finite amount of time. This remarkable result has been confirmed by numerical simulation and has been found to work in 2D as well [9]. In unpublished work, I've shown that the phenomenon also occurs in 3D. Now consider the case where ϵ slightly exceeds its minimum critical value. In that case, a fourth particle must be introduced to ensure collapse. This works in general: for larger values of ϵ, more particles are required to ensure collapse. Hence in a large granular system, collapse could occur even among relatively elastic particles. This might help explain some collective phenomena observed in granular systems in which patterns are seen to develop over large distances.

0.2.3 Fractal Aggregate Dynamics

If particles are constrained to stick at the point of contact, fractal aggregates such as the one shown in Fig. 3 form. This is an example of ballistic accretion, and for identical particles leads to a fluffy aggregate of fractal dimension ~ 2. Using the equations in §0.1.3 along with Euler's torque-free rigid body equations of motion, it is possible to observe such clusters bounce off each other [12]. Fractals may play a role in the early stage of planet formation: their fluffy nature would allow dust aggregates to couple more easily with the nebular gas and be transported greater distances. They also grow more quickly than shapes with higher fractal dimension. Elsewhere in this volume, Blum shows how to grow these fractals in the laboratory.

0.2.4 Tidal Distortion and Disruption of "Rubble-Piles"

With careful collision treatment it is possible to simulate "rubble-piles", that is, loose collections of material bound together only by self-gravity. [2] showed that if tidal disruption caused Comet D/Shoemaker-Levy 9 (SL9) to fragment into ~ 21 pieces during its penultimate encounter with Jupiter in 1992, then the progenitor had to be virtually strengthless: at periapsis, tidal forces stretched the rubble-pile into a needle-like shape; as the rubble-pile receded from the planet, gravitational instabilities along its length caused the particles to accrete into discrete clumps, making a "string of pearls". The comet could not simply have been in 21 pieces to begin with, as the pieces would have paired off after the disruption, making too few observed fragments.

The discovery of SL9 has sparked a series of investigations into the dynamics of rubble-piles. [14] showed that SL9-like disruptions of cometary rubble-piles can explain the crater chain population observed on the Galilean satellites. [4] can account for the observed crater chains on the Moon by invoking tidal disruption of Earth-crossing asteroids. They also show that formation of crater chains on the Earth via disruption of asteroids by the Moon is very unlikely. A comprehensive investigation by the same authors of rubble-pile distortion and disruption is currently in preparation for *Icarus*. It is found that less catastrophic tidal encounters result in milder stripping of material from the rubble-pile, which can lead to binary asteroid formation. Other phenomena that may be explained by rubble-pile distortion and disruption include the unusual shapes of some asteroids, the range of observed asteroid spin periods, the population of doublet craters on the terrestrial planets, and possibly even the origin of the Martian satellites.

0.2.5 Formation of the Solar System

Direct numerical simulations involving particle collisions can improve our understanding of the Mid to Late stages of planet formation. This period is characterized by runaway growth of a few large bodies in a swarm of smaller, faster moving planetesimals, followed by isolation of the largest protoplanets until slow mutual perturbations nudge them into crossing orbits, ultimately leading to giant impacts. There are a number of outstanding questions that such simulations can help address, including: the extent of radial mixing, the origin of planetary spin, the clearing of the asteroid belt, and the role of giants in terrestrial planet formation. A key ingredient in such simulations is the prescription used to determine collision outcomes. Typically the outcome is based on the relative impact energy [6], with the following outcomes in increasing order of energy: agglomeration, bouncing, cratering (mass transfer), and fragmentation. Of course, no one really knows in detail what happens when, say, two large asteroids collide. We must rely on extrapolation from laboratory experiments. However, carefully crafted simulations can be modified to including new findings as they become available.

I am currently working with Tom Quinn and George Lake at the University of Washington to modify a powerful spatially and temporally adaptive cosmology code, making it suitable for simulations of Solar System formation. The code uses domain decomposition to balance work between multiple processors, and a k-D tree to reduce the cost of computing long-range forces. New components that must be incorporated include collision detection and resolution, and sensitive hierarchical time-steps. Using a Cray T3E we plan to simulate 10^6 planetesimals for 10^6 yr or more, which would revolutionize the state of the art in direct simulation of Solar System formation.

0.3 Summary

I have shown some of the important roles collisions play in planetesimal dynamics. Collisions can be incorporated realistically into numerical simulations, allowing direct modeling of a variety of interesting problems. Applications include planetary rings, granular physics, fractal aggregates, tidal disruption, Solar System formation, and more!

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