Setting frequency parameters in the WASP database

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Calculations for multiple mixers are based on a formalism that uses sideband information and LO frequencies:

$$sb := \mathrm{sign}(f_{sig} - f_{LO}) \quad \text{and} \quad f_{IF} := (f_{sig} - f_{LO}) \cdot sb$$

Here sb = +1 for USB ($f_{LO} < f_{sig}$), -1 for LSB ($f_{LO} > f_{sig}$). With this scheme, LOs can be removed from the calculation by setting their frequencies to zero and the corresponding sb to 1. Upconversion can be accomodated by multiplying the LO frequency by -1 and setting sb to 1. This sign convention is somewhat backwards but keeps the LO frequencies positive for the more common case of downconversion. Note that CSO uses sb = 0 for LSB, sb = 1 for USB.
For a concrete example in the worksheet, use

$$f_{rest} := 230 \cdot GHz \quad f_{offset} := 0 \cdot GHz \quad v_{source} := 0 \cdot \frac{km}{s} \quad v_{offset} := 0 \cdot \frac{km}{s} \quad v_{radial} := 20 \cdot \frac{km}{s}$$

and the characteristics of Chip Sumner's IF converter box:

$$chipLO := \begin{pmatrix} 13.75 \\ 17.25 \\ 8.75 \\ 12.25 \end{pmatrix} \cdot GHz \quad chipIFctr := \begin{pmatrix} 7.75 \\ 11.25 \\ 14.75 \\ 18.25 \end{pmatrix} \cdot GHz \quad IFctr := \frac{5.75 \cdot GHz + 20.25 \cdot GHz}{2}$$

$$IFctr = 13.000\,GHz$$

$$chipsb := \mathrm{sign}(chipIFctr - chipLO) \quad chipsb = \begin{pmatrix} -1 \\ -1 \\ 1 \\ 1 \end{pmatrix}$$

All of the IF center frequencies are larger than the WASP 6 GHz center frequency, so all mixers are downconverters and the LO frequencies stay positive.

so pick an index m for the converter channel (indices run from 1 in this sheet), then

$$m := 4 \quad f_{LO_2} := chipLO_m \quad f_{LO_2} = 12.250\,GHz \quad sb_2 := chipsb_m \quad sb_2 = 1$$

First the general expressions

The sky frequency takes the line rest frequency, source velocity, various offsets, and the radial velocity between the telescope and the reference frame

$$f_{sky} := (f_{rest} + f_{offset}) \cdot \left[1 - \frac{(v_{source} + v_{offset} + v_{radial})}{c}\right] \qquad f_{sky} = 229.985\,GHz$$

Calculate the LO frequency from f_{sky} and the IF frequency.
sb = +1 for USB ($f_{LO} < f_{sig}$), -1 for LSB ($f_{LO} > f_{sig}$)

$$f_{IF_1} := IFctr \quad sb_1 := 1 \quad f_{IF_1} = 13.000\,GHz$$

$f_{LO_1} := f_{sky} - sb_1 \cdot f_{IF_1}$ $\qquad f_{LO_1} = 216.985\,GHz$

Set LO manually for further tests: $\quad ff_{LO_1} := 216.985 \cdot GHz$

LO and sideband vectors: $\quad f_{LO} = \begin{pmatrix} 216.985 \\ 12.250 \end{pmatrix} GHz \qquad sb = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Work out line and image frequencies in the reference frame: f_{line} and f_{image}

$$f_{line} := \left(f_{rest} + f_{offset}\right) \cdot \left[1 - \frac{\left(v_{source} + v_{offset}\right)}{c}\right] \qquad f_{line} = 230.000\,GHz$$

$$f_{image} := f_{line} - 2 \cdot sb_1 \cdot f_{IF_1} \qquad f_{image} = 204.000\,GHz$$

$\dfrac{f_{line} + f_{image}}{2} = 217.000\,GHz$ as it should be

Calculate frequency of spectrometer reference channel in reference frame (see Appendix).
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$f_{ref} := 4.5 \cdot GHz \qquad N := \text{length}(sb) \qquad N = 2$

$$f_{in}\left(f_{IFN}\right) := f_{IFN} \cdot \prod_{i=1}^{N} sb_i + \sum_{j=2}^{N} \left(f_{LO_j} \cdot \prod_{i=1}^{j-1} sb_i \right) + f_{LO_1}$$

$$f_{ref\_chan} := f_{in}\left(f_{ref}\right) \qquad f_{ref\_chan} = 233.735\,GHz$$

Correspondence between delta frequency in spectrometer IF and in reference frame is:

$$scale\_sense := \prod_{i=1}^{N} sb_i \qquad scale\_sense = 1$$

Line comes out at:

$$f_{IFN}\left(f_{in}\right) := f_{in} \cdot \prod_{i=1}^{N} sb_i - \sum_{j=1}^{N} \left(f_{LO_j} \cdot \prod_{i=j}^{N} sb_i \right) \qquad f_{sky} = 229.985\,GHz$$

$f_{IFN}\left(f_{sky}\right) = 0.750\,GHz$ in channel $\quad m = 4 \quad chipIFctr_m = 18.250\,GHz$

WASP band is 4.25 to 7.75 GHz

SDFITS and WASP database keywords concerning frequency scale

CRPIX	Reference pixel, set to 1.
CRVAL	Reference value in frame frequency: f_{ref_chan} - f_{radial}.
CDELT	Increment in frame frequency, CWRES*scale_sense
FREQRES	Channel spacing, equal to step size in cwsiginvert
OBSFREQ	Observed frequency in reference pixel. We don't understand how this differs from CRVAL, if it does, yet.
IMAGFREQ	Corresponding image frequency at reference pixel. We don't understand this keyword, either, so we'll skip it for the CSO and use our own IMREF.
RESTFREQ	Line rest frequency, f_{rest} above (e-notebook entry)
RVSYS	Radial velocity between source and telescope. This is the sum of all velocity offsets including the radial velocity correction.
VFRAME	Radial velocity of telescope to frame. We usually call this the doppler correction.

New keywords:

CWFREQ#	Center frequency of lowest-frequency spectrometer channel. This defines frequency of the reference channel, channel 1 (note shift from zero-based counting here). Obtain from cwsigsinvert.
CWRES#	Channel-to-channel spacing in spectrometer. Obtain from cwsigsinvert, possibly with binning.
LOn-#	Vector containing LO frequencies for mixer n and beaddr #. Length = number of mixers. Index from 1 to 9. Default value is zero.
SBn-#	Vector containing sideband info (+1 for USB, -1 for LSB conversions) for mixer n and beaddr #. Length = number of mixers. Index from 1 to 9. Default value is 1.
IMREF	Image frequency in rest frame for reference channel for beaddr #. Corresponds to value in CRVAL1, which is probably the same as OBSFREQ. We define our own IMREF just to keep things tidy since the exact definition of IMAGEFREQ isn't clear.
VSOURCE	Nominal source velocity
VOFFSET	Velocity offset (e-notebook entry)
FOFFSET	Frequency offset (e-notebook entry; may also be important for LO setting if the PLL has a frequency offset)

Correspondence to CSO scoreboard file

velocity	Source velocity, v_{source} or VSOUR above
frequency	Line rest frequency, f_{rest} or RESTFREQ above
v_offset	Velocity offset, v_{offset} above
f_offset	Frequency offset, f_{offset} above
if_frequency	IF frequency that corresponds to requested velocity and frequencies, including offsets. This is used as a constraint on the LO frequency setting.
lock_freq multiplier	Multiplied together these define the first LO frequency, perhaps with an offset for a phase-lock loop. The product is LO1-#.
sideband	Indicates lower or upper sideband mixing, CSO uses sb = 0 for LSB, sb = 1 for USB, not our convention. This is SB1-# once converted with sb = 2*sideband-1.
v_type	Reference frame type: 0 for radial, 1 for LSR, 2 for heliocentric, 3 for relativistic.

To CLASS FITS

Oddballs that don't follow convention

CRVAL1	Set to zero
VELO-RAD VELO-LSR VELO-HEL VELO-RED	*Velocity* of reference channel in appropriate coordinate system: radial, local standard of rest, heliocentric, or redshift. Sum of velocity and velocity_offset. Frame is selected by v_type in scoreboard file.
DELTAV	Velocity spacing of channels, signed
IMAGEFREQ	Image *frequency* corresponding to VELO-x above.

Appendix

Functions for computing the frequency in IF number N, f_{IFN}, from an input signal f_{in}, and vice versa. Requires at least two LOs, but the second and subsequent LOs may have frequency = 0 and sb = 1.

$$f_{LO} := \begin{pmatrix} 225 \\ 8 \\ 0 \end{pmatrix} \cdot GHz \qquad ctrs := \begin{pmatrix} 230 \\ 6 \\ 7 \end{pmatrix} \cdot GHz \qquad sb := sign(ctrs - f_{LO}) \qquad f_{sky} := ctrs_1$$

$$f_{LO} = \begin{pmatrix} 225.000 \\ 8.000 \\ 0.000 \end{pmatrix} GHz \qquad sb = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

$N := length(sb) \qquad N = 3$ N is the number of LOs

$$f_{IFN}(f_{in}) := f_{in} \cdot \prod_{i=1}^{N} sb_i - \sum_{j=1}^{N} \left(f_{LO_j} \cdot \prod_{i=j}^{N} sb_i \right) \qquad fl := f_{IFN}(f_{sky})$$

$$fl = 3.000\,GHz$$

$$f_{in}(f_{IFN}) := f_{IFN} \cdot \prod_{i=1}^{N} sb_i + \sum_{j=2}^{N} \left(f_{LO_j} \cdot \prod_{i=1}^{j-1} sb_i \right) + f_{LO_1} \qquad fh := f_{in}(fl)$$

$$fh = 230.000\,GHz$$

$$\prod_{i=1}^{N} sb_i = -1 \qquad f_{sky} - fh = 0.000\,Hz$$

$$v_{radial} := c \cdot \left[1 - \frac{f_{sky}}{(f_{rest} + f_{offset})} \right] - (v_{source} + v_{offset}) \qquad f_{IF_1} := (f_{sky} - f_{LO_1}) \cdot sb_1$$

$$v_{radial} := c \cdot \left[1 - \frac{(f_{LO_1} + f_{IF_1} \cdot sb_1)}{(f_{rest} + f_{offset})} \right] - (v_{source} + v_{offset}) \qquad f_{sky} := f_{LO_1} + f_{IF_1} \cdot sb_1$$

$$(v_{source} + v_{offset}) := c \cdot \left[1 - \frac{(f_{LO_1} + f_{IF_1} \cdot sb_1)}{(f_{rest} + f_{offset})} \right] - v_{radial}$$

Global variables for worksheet $\quad ORIGIN \equiv 1 \qquad c \equiv 299792458 \cdot \frac{m}{sec}$

Formulas for CSO scoreboard and WASP database keywords

The companion document is John Ward and Kate Isaak's write_fits.cc routine for the CSO.

$f_{rest} := 230 \cdot GHz$ $\quad f_{offset} := 0 \cdot MHz$ $\quad v_{source} := 10000 \cdot \frac{km}{s}$ $\quad v_{offset} := 0 \cdot \frac{km}{s}$ $\quad v_{radial} := -100 \cdot \frac{km}{s}$

$f_{LO_2} := 0 \cdot GHz$ $\quad sb_2 := 1$ $\quad f_{LO_3} := 0 \cdot GHz$ $\quad sb_3 := 1$

$sb\_if\_frequency := 6 \cdot GHz$ $\quad sb\_sideband := 1$

$sb\_multiplier := 2$ $\quad sb\_vtype := 1$

$sb\_velocity := v_{source}$ $\quad sb\_velocity = 1 \times 10^4 \frac{km}{s}$

$sb\_v\_offset := v_{offset}$ $\quad sb\_v\_offset = 0 \frac{km}{s}$

$sb\_frequency := f_{rest}$ $\quad sb\_frequency = 230.000\,GHz$

$sb\_f\_offset := f_{offset}$ $\quad sb\_f\_offset = 0.000\,GHz$

$sb_1 := sb\_sideband \cdot 2 - 1$ sideband multiplier entry for first LO in multiplier array SBn-#

$$f_{sky} := (sb\_frequency + sb\_f\_offset) \cdot \left[1 - \frac{\left(sb\_velocity + sb\_v\_offset + v_{radial}\right)}{c}\right]$$

$f_{LO_1} := f_{sky} - sb\_if\_frequency \cdot sb_1$ $\quad f_{LO_1} = 216.405\,GHz$

$f_{sky\_LO} := f_{LO_1} + sb\_if\_frequency \cdot sb_1$

$sb\_lock\_freq := \frac{f_{LO_1}}{sb\_multiplier}$ $\quad sb\_lock\_freq = 108.202\,GHz$

$f_{LO_1} := sb\_lock\_freq \cdot sb\_multiplier$ $\quad f_{LO_1} = 216.405\,GHz$ first LO entry in LO array LOn-#

$$f_{line} := (sb\_frequency + sb\_f\_offset) \cdot \left[1 - \frac{(sb\_velocity + sb\_v\_offset)}{c}\right]$$

$f_{image} := f_{line} - 2 \cdot sb\_if\_frequency \cdot sb_1$

$f_{sky} = 222.405\,GHz$ $\quad f_{line} = 222.328\,GHz$ $\quad f_{LO_1} = 216.405\,GHz$ $\quad f_{image} = 210.328\,GHz$

$f_{sky\_LO} = 222.405\,GHz$ $\quad f_{sky} - f_{sky\_LO} = 0.000\,Hz$ $\quad sb_1 = 1$

$f_{sky} - f_{line} = 76.720\,MHz$ $\quad -\frac{v_{radial}}{c} \cdot \left(f_{rest} + f_{offset}\right) = 76.720\,MHz$ radio definition

$RESTFREQ := sb\_frequency$ $\quad RESTFREQ = 230.000\,GHz$

$VOFFSET := sb\_v\_offset$ $\quad VOFFSET = 0\,\frac{km}{s}$

$FOFFSET := sb\_f\_offset$ $\quad FOFFSET = 0.000\,MHz$

$VSOURCE := sb\_velocity$ $\quad VSOURCE = 10000\,\frac{km}{s}$

DELTAV

$\boxed{CWRES := 10 \cdot MHz}$ (spacing between pixels from cwsigsinvert)

$$CDELT := CWRES \cdot \prod_{i=1}^{last(sb)} sb_i \qquad CDELT = 10.000\,MHz$$ for SDFITS

$$DELTAV := -\frac{CDELT}{RESTFREQ + FOFFSET} \cdot c \qquad DELTAV = -13.034\,\frac{km}{s}$$ for CLASS

CRVAL frequency of spectrometer channel 1 on sky in observing frame (e.g. LSR)

$$f_{radial} := f_{LO_1} + sb\_if\_frequency \cdot sb_1 - (RESTFREQ + FOFFSET) \cdot \left[1 - \frac{(VSOURCE + VOFFSET)}{c}\right]$$

$f_{radial} = 76.720\,MHz$ frequency shift from doppler effect

$v_{radial} = -100\,\frac{km}{s}$

$$VFRAME := -\frac{f_{radial}}{RESTFREQ + FOFFSET} \cdot c \qquad VFRAME = -100\,\frac{km}{s}$$

$\boxed{CWFREQ := 4000 \cdot MHz}$ (spectrometer's first channel center frequency, from cwsigsinvert)

$$N := length(sb) \qquad f_{in}(f_{IFN}) := f_{IFN} \cdot \prod_{i=1}^{N} sb_i + \sum_{j=2}^{N} \left(f_{LO_j} \cdot \prod_{i=1}^{j-1} sb_i \right) + f_{LO_1}$$

$CRVAL := f_{in}(CWFREQ) - f_{radial}$ $\quad CRVAL = 220.328\,GHz$

$f_{line} = 222.328\,GHz$

$$VELOx := \frac{(RESTFREQ + FOFFSET) - CRVAL}{RESTFREQ + FOFFSET} \cdot c \qquad VELOx = 12606.891\,\frac{km}{s}$$

$OBSFREQ := CRVAL$

IMREF

$IMREF := CRVAL - 2 \cdot sb_1 \cdot CWFREQ$ $\quad f_{image} = 210.328\,GHz$

$IMAGEFREQ := IMREF$ $\quad IMREF = 212.328\,GHz$