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THE INTRINSIC POLARIZATION OF MIRA VARIABLES

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It is shown that light emerging from the atmospheres of Mira variables may exhibit a degree of polarization much higher than the classical value for a pure scattering atmosphere. It is suggested that temperature variations over the stellar surface could lead to a large observable polarization.

Since the discovery that some late-type stars show time dependent polarization (Serkowski 1966), a large number of Mira and semi-regular variables have been found to exhibit this effect (Zappala 1967, Kruszewski *et al.* 1968; Dyck 1968). While the details of the wavelength dependence vary greatly from star to star, the polarization is larger in the blue than in the yellow, and the largest values have been found in the ultraviolet where polarizations of 8 per cent and greater have been measured. Some of the variables with the largest polarizations, such as V CVn, V CrB, and R Gem, show a quite regular pattern, with the maximum polarization occuring at minimum light.

Two models have been advanced to explain this polarization. Donn *et al.* (1966) suggested that graphite platelets aligned by a stellar magnetic field produce polarization in a manner analogous to the current theories of interstellar polarization. Kruszewski *et al.* (1968) considered Rayleigh scattering in an extended, asymmetric envelope. Rayleigh scattering is an attractive mechanism because it leads naturally to a steep increase of polarization with frequency, an effect observed in many of these stars. Both suggestions encounter difficulties, however. It might not be possible to form the required number of graphite particles in an M type variable. Kruszewski *et al.* (1968) point out the difficulty of producing graphite in an atmosphere without enough free carbon to form detectable amounts of C_2. On the other hand, the maximum polarization obtainable from the highly asymmetric envelope considered by Kruszewski *et al.* is 5.46 per cent, and we would expect a smaller value from a less artificial shape.

It is the purpose of this letter to suggest that a rather high degree of polarization may arise in the photospheric layers of Mira variables, so that irregularities in the distribution of brightness over the stellar surface or some distortion of the shape of the star could lead to a considerable overall polarization. Two conditions favor polarization of the radiation emerging from the photosphere. First, Rayleigh scattering by H_2 is a dominant source of opacity at shorter wavelengths in M type stars (Tsuji 1966, Gingerich *et al.* 1966). In addition, since the maximum of the radiated energy is in the infrared, the gradient of the Planck function may be very great at visual and ultraviolet wavelengths. This means that, if the optical depth at these frequencies is not much greater than the mean optical depth, most of the radiation is directed outwards from the deep layers, and this favors strong polarization of the scattered light.

The transfer equations for the case where there is both scattering and absorption have been given by Code (1950) in terms of the two perpendicular intensities I_l and I_r. We prefer to use the Stokes parameters I and Q. From the formal solutions of the transfer equations, it follows that the Stokes parameters of the emergent radiation can be written as

$$I(\mu) = \int_0^\infty \{s(\tau) + (\tfrac{1}{3} - \mu^2)p(\tau)\}e^{-\tau/\mu}\,\frac{d\tau}{\mu} \tag{1}$$

and

$$Q(\mu) = \int_0^\infty \{(1 - \mu^2)p(\tau)\}e^{-\tau/\mu}\,\frac{d\tau}{\mu}, \tag{2}$$

where the quantities in brackets are 'source functions' written in terms of two auxiliary functions $s(\tau)$ and $p(\tau)$, which can be shown to satisfy two integral equations analogous to the Schwarzschild–Milne equation (see Chandrasekhar 1960. §11.2, for a similar case of scattering according to Rayleigh's phase function, but without polarization). These equations are

$$s(\tau) = (1 - \lambda)\left\{\frac{1}{2}\int_0^\infty s(x)E_1(|\tau - x|)dx + \right.$$

$$\int_0^\infty p(x)\left[\frac{1}{6}E_1(|\tau-x|)-\frac{1}{2}E_3(|\tau-x|)\right]dx+\lambda B(\tau) \quad (3)$$

and

$$p(\tau) = \frac{3}{8}(1-\lambda)\left\{\int_0^\infty s(x)\left[\frac{1}{2}E_1(|\tau-x|)-\frac{3}{2}E_3(|\tau-x|)\right]dx + \int_0^\infty p(x)\left[\frac{5}{3}E_1(|\tau-x|)-4E_3(|\tau-x|) + 3E_5(|\tau-x|)\right]dx\right\}. \quad (4)$$

Here $\lambda=\kappa/(\kappa+\sigma)$ is the ratio of absorption to total extinction, $B(\tau)$ is the Planck function, and we have dropped the subscripts indicating frequency dependence. If B and λ are known functions of τ, and we introduce a function $s^*=s-B$ which will vanish in the limit $\tau\to\infty$ (as does $p(\tau)$), then we can replace the integrals over s^* and p by Gaussian quadratures. The resulting system of linear algebraic equations can be solved as Gebbie (1967) did for the isotropic scattering case. Essentially the same method can be applied to the problem of constant flux in a pure scattering atmosphere ($\lambda\equiv 0$) and this was done to check the accuracy of the calculations; the $I(\mu)$ and $Q(\mu)$ obtained agree with the exact solution of Chandrasekhar (1960) to at least 3 significant figures.

Our knowledge of the atmospheric structure of late giants is still rather primitive. The most appropriate calculation available at present is a model by Auman (1969) with an effective temperature of 2000° and $\log g = -1.0$. If we consider radiation in the ultraviolet at 3600 Å, where large polarizations have been observed, this model shows that Rayleigh scattering by atomic and molecular hydrogen is the main source of opacity, and that absorption by H^- becomes appreciable only below $\tau\cong 2$ where H_2 dissociates; H^- absorption provides only 6 per cent of the opacity at $\tau=3$. (Here τ is the optical depth at a standard frequency of 11 700 Å. The monochromatic optical depth τ_ν (3600 Å) is about $\frac{1}{2}\tau$ for all $\tau\gtrsim 2$ and $\tau_\nu<\tau$ for $\tau<6$.)

In addition to H^-, we would expect a major contribution to the absorption from the many overlaping atomic and molecular lines in this region of the spectrum. Quantitative estimates of the ratio of this absorption to the scattering are not available, however, so we can only say that the amount of absorption will be more nearly comparable to the scattering than would be predicted from the model atmosphere.

We have obtained two solutions which illustrate the high degree of polarization which could occur in Mira variables. We have assumed a gray temperature distribution, $T^4(\tau)=\frac{3}{4}T_e^4(\tau+q(\tau))$, with $T_e=2000°$, and have further taken $\tau_\nu=\tau$. (Actually, in Auman's model, the temperature drops much more sharply at the surface because of the extreme non-gray nature of the atmosphere. This, along with $\tau_\nu<\tau$, would enhance the polarization effects discussed here.) Now at 3600 Å the exponent in the Planck function is $h\nu/kT=\alpha\, T_e/T$, where $\alpha\cong 20$, so that $B_\nu(\tau)$ is a very steep function of τ. Table I gives two solutions for these conditions, one with a constant ratio of absorption to total extinction of $\lambda=1/6$, and the other with the scattering increasing toward the surface according to the formula $\lambda=\tau/(1+\tau)$. The classical solution for a pure scattering atmosphere ($\lambda\equiv 0$) is given for comparison.

It is seen that, when the gradient of B_ν is so large, the addition of some absorption to a scattering atmosphere greatly increases the polarization. Thus for the $\lambda(\tau)=\tau/(1+\tau)$ solution at $\mu=0.5$ the polarization is more than 5 times as large as for pure scattering. If we reduce the absorption towards the limit $\kappa=0$ at all τ, then the polarization $Q(\mu)/I(\mu)$ must tend towards the pure scattering solution regardless of $B_\nu(\tau_\nu)$. On the other hand, if $\sigma\to 0$ for all τ, the polarization must vanish. It is interesting that, in these examples the polarization attains values for some intermediate functions $0<\lambda(\tau)<1$ which are much greater than the pure scattering limit. (The examples of $\lambda(\tau)$ presented here do not, of course, represent the *maximum* polarization possible for this $B_\nu(\tau_\nu)$—either for a specific μ or in any mean sense.) Furthermore, although we know that Rayleigh scattering is proportional to ν^4, the variation with frequency of $B_\nu(\tau)$ and $\lambda(\tau_\nu)$ is not so straightforward, especially if the absorption component is mainly due to atomic and molecular lines. The resulting frequency dependence of the polarization could be quite complex. The apparent difference between the polarization of carbon stars and M stars (Kruszewski *et al.* 1968) would be understandable in terms of different absorption bands and lines. Also, $B_\nu(\tau)$ will become steeper at higher frequencies, and under favorable circumstances this might combine with the ν^4 dependence of the scattering to yield the very sharp increases of polarization in the ultraviolet observed for some stars.

It thus appears likely that radiation emerging obliquely from the atmospheres of Mira variables

TABLE I

Polarization and Limb Darkening of the Emergent Radiation

	$Q(\mu)/I(\mu)$ (polarization)			$I(\mu)/F$		
μ	$\lambda = 1/6$	$\lambda = \tau/(1+\tau)$	$\lambda = 0$	$\lambda = 1/6$	$\lambda = \tau/(1+\tau)$	$\lambda = 0$
0.00	0.2290	0.2833	0.1171	0.309	0.354	0.414
0.05	0.2057	0.2624	0.0898	0.350	0.378	0.475
0.10	0.1910	0.2516	0.0745	0.385	0.391	0.524
0.15	0.1785	0.2421	0.0631	0.419	0.402	0.570
0.20	0.1669	0.2316	0.0541	0.455	0.415	0.614
0.25	0.1556	0.2192	0.0467	0.492	0.430	0.658
0.30	0.1444	0.2045	0.0404	0.532	0.451	0.700
0.40	0.1217	0.1691	0.0303	0.621	0.509	0.784
0.50	0.0989	0.1294	0.0225	0.724	0.602	0.866
0.60	0.0765	0.0912	0.0163	0.847	0.738	0.948
0.70	0.0551	0.0585	0.01112	0.991	0.927	1.029
0.80	0.0351	0.0329	0.00682	1.161	1.178	1.109
0.90	0.0167	0.0137	0.00316	1.360	1.501	1.189
1.00	0.00	0.000	0.000	1.592	1.907	1.269

and other M type giants and supergiants will be strongly polarized. To explain the observed polarization of Mira variables on this basis, however, requires a sufficient asymmetry in the star so that the integrated polarization does not cancel too effectively.

The mechanism behind the light variations of Mira variables is not too well understood. It is possible that these stars pulsate in a fashion that is not entirely spherically symmetric. This is at least possible in the case of R Gem which is known to have a magnetic field of 400 gauss (Babcock 1960) as well as a large variable polarization (Dyck 1968). A magnetic field is only measurable if it is strongly organized over the stellar surface, and since we measure only one component, the actual field strength must exceed the measured value. The energy in a 400 gauss field, $H^2/8\pi = 6 \times 10^3$, is one or two orders of magnitude larger than the photospheric gas pressure of these stars.

We can consider both distortions of the surface of the star and variations of the temperature over the surface. We should not overlook the fact that modest variations in the surface temperature can result in great differences in flux in the visual and ultraviolet regions of the spectrum. As a crude estimate of this effect, suppose the temperature varies over the surface as

$$T(\theta) = T_{\text{pole}}(1 - \epsilon \sin^2\theta), \tag{5}$$

where θ is the co-latitude. We have calculated the net polarization seen by an observer in the equatorial plane under the following conditions: (1) $T_{\text{pole}} = 2000°$, (2) the polarization and limb darkening at any point is like that given by the $\lambda = \tau/(1+\tau)$ solution of Table I, (3) the flux varies as $B_\nu(T(\theta))$. The results are presented in Table II.

TABLE II

Polarization due to a Temperature Gradient over the Stellar Surface

ϵ	Polarization (per cent)
0.05	1.10
0.10	2.87
0.15	5.42
0.20	8.50
0.25	11.43

Even a rather small variation could produce observable effects: $\epsilon = 0.05$ corresponds to $\pm 50°$. Larger variations in surface temperature, perhaps accompanied by changes in the shape of the star, might explain the large values observed for some objects.

With a knowledge of the mean absorption due to atomic and molecular lines we could undertake a detailed calculation of the wavelength dependence of the polarization. On the observational side, it would be valuable to know how the polarization varies (or fails to vary) across absorption bands in the spectrum. If the polarization originates at some distance from the star in an envelope of graphite particles we would not expect any correlation of the amount of polarization and the absorption features. The model presented here, on the contrary, would imply some change of polarization across absorption bands, although even to say whether this would be an increase or decrease would depend on the details of $\lambda(\tau_\nu)$ and $B_\nu(\tau_\nu)$.

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