

MODELING KICKS FROM THE MERGER OF GENERIC BLACK HOLE BINARIES

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ABSTRACT

Recent numerical relativistic results demonstrate that the merger of comparable-mass spinning black holes has a maximum “recoil kick” of up to $\sim 4000 \text{ km s}^{-1}$. However, the scaling of these recoil velocities with mass ratio is poorly understood. We present new runs showing that the maximum possible kick perpendicular to the orbital plane does not scale as $\sim \eta^2$ (where η is the symmetric mass ratio), as previously proposed, but is more consistent with $\sim \eta^3$, at least for systems with low orbital precession. We discuss the effect of this dependence on galactic ejection scenarios and retention of intermediate-mass black holes in globular clusters.

Subject headings: black hole physics — galaxies: nuclei — gravitational waves — relativity

1. INTRODUCTION

Recently, numerical exploration of the radiative recoil “kick” of merging black holes has progressed considerably. In particular, efforts in this regard have led to suggested phenomenological formulae for the kick, largely based on post-Newtonian (PN) predictions such as that given by Kidder (1995) which have proved surprisingly successful. For example, Gonzalez et al. (2007) found that in cases of unequal masses ($q \equiv m_1/m_2 < 1$) and no spin, a simple modification of the PN formula originally found by Fitchett (1983) fits the numerical data quite well. For cases of spins perpendicular to the orbital plane (i.e., parallel with the orbital angular momentum), a formula proposed by Baker et al. (2007) is also consistent with numerical data. This formula is loosely based on PN calculations, with spins perpendicular to the orbital plane producing kicks in the orbital plane. For spins with components in the orbital plane, Campanelli et al. (2007) have proposed a formula, again derived from PN calculations, that agrees well with numerical results for equal masses.

This last type of kick, which is perpendicular to the orbital plane, is of particular interest because its computed magnitude can be very large (up to thousands of kilometers per second). In the current literature (specifically Campanelli et al. 2007), the mass ratio dependence is drawn from the leading-order PN approximation. It is unclear whether this approximation is sufficient to predict the strong-field dynamics that presumably determines the kick. Indeed, hints of a deviation from this form are evident for mass ratio $q = 1/2$ in the runs of Lousto & Zlochower (2008a). Therefore, although the angular dependence of the proposed formula is consistent with symmetry arguments, which are independent of the strong-field dynamics (Boyle et al. 2008; Boyle & Kesden 2007), the mass ratio dependence of this formula is currently not well justified.

Characterization of the dominant kick for unequal masses is especially important because, although the largest possible kicks would eject the remnant from any galaxy, for astrophysical applications the distribution of kick speeds matters most. For example, Bonning et al. (2007) find no evidence for quasars ejected from their hosts (although Komossa et al. 2008

may have seen a 2650 km s^{-1} kick). If quasar activity is commonly induced by major galaxy mergers that lead to coalescence of supermassive black holes, the implications of this therefore depend in part on how frequently one expects a merger to allow ejection. Even for very large recoils, more than half of galaxies would still retain black holes at their cores (Schnittman 2007). However, the kick speed distribution has a major impact on the hierarchical growth of massive black holes at redshifts $z > 5$ (e.g., Volonteri 2007).

Here we investigate how the out-of-plane kick depends on the mass ratio, and find that, for mass ratios in the range $q = 1$ to $q = 1/3$ and spins $S_i \leq 0.2m_i^2$, the kick drops off more rapidly with decreasing mass ratio than proposed by Campanelli et al. (2007). Specifically, we find that a large body of numerical data on kicks are well represented by

$$\begin{aligned} \mathbf{V}_{\text{recoil}} &= v_m \hat{e}_1 + v_{\perp} (\cos \xi \hat{e}_1 + \sin \xi \hat{e}_2) + v_{\parallel} \hat{e}_2, \\ v_m &= A \eta^2 \sqrt{1 - 4\eta} (1 + B\eta), \\ v_{\perp} &= H \frac{\eta^2}{(1 + q)} (\alpha_2^{\parallel} - q\alpha_1^{\parallel}), \\ v_{\parallel} &= \frac{K\eta^3}{(1 + q)} [q\alpha_1^{\perp} \cos(\phi_1 - \Phi_1) - \alpha_2^{\perp} \cos(\phi_2 - \Phi_2)], \end{aligned} \quad (4)$$

where v_m is the mass asymmetry contribution, and v_{\perp} and v_{\parallel} are the spin contributions that yield kicks perpendicular and parallel to the orbital angular momentum. The quantity $\eta \equiv q/(1 + q)^2$ is the symmetric mass ratio; α_i^{\parallel} is the projection of the dimensionless spin vector $\alpha_i = S_i/m_i^2$ of black hole i along the orbital angular momentum, while α_i^{\perp} and ϕ_i are the magnitude and angle with respect to some reference angle in the orbital plane of its projection, α_i^{\perp} , into the orbital plane; and Φ_1 and Φ_2 are constants for a given mass ratio. Here $A = 1.35 \times 10^4 \text{ km s}^{-1}$, $B = -1.48$, $H = 7540 \pm 160 \text{ km s}^{-1}$, $\xi = 215^\circ \pm 5^\circ$, and $K = (2.4 \pm 0.4) \times 10^5 \text{ km s}^{-1}$. This formula, similar in form to that of Campanelli et al. (2007), synthesizes results from Gonzalez et al. (2007) for equation (2) and from Baker et al. (2007) for equations (1) and (3).³ For ξ

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³ Note that in Baker et al. (2007) we used a simpler form for the zero-spin contribution, equivalent to eq. (2) with $B = 0$.

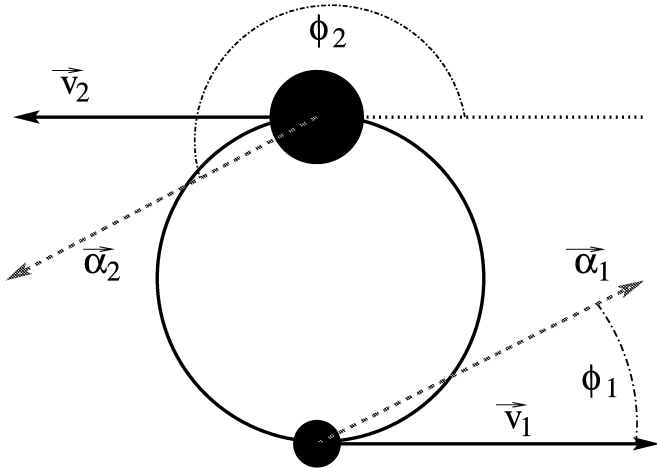


FIG. 1.—Configuration of black holes for all new simulations. The two holes' spins $\alpha_{(1,2)}$ lie initially in the orbital plane, at angles ϕ_1 and ϕ_2 to v_1 , the smaller hole's initial velocity.

and H we have fit available numerical data from Herrmann et al. (2007), Koppitz et al. (2007), and Baker et al. (2007). The qualitatively new part, the factor of η^3 in equation (4), replaces the factor of η^2 originally proposed by Campanelli et al. (2007) and is motivated by new numerical evolutions presented here.

2. INITIAL DATA AND METHODOLOGY

We simulated the inspiral and merger of a range of spinning black hole binaries, with mass ratios in the range $1/1.1 \geq q \geq 1/3$. The initial configuration of momenta and spins is illustrated in Figure 1. The parameters used in the numerical evolutions are presented in columns (1)–(3) of Table 2. For these evolutions, the smaller hole (m_1) has a dimensionless spin $|\alpha_1| = 0.2$, while the larger hole's spin is $|\alpha_2| = q^2|\alpha_1|$. Both spins initially lie in the orbital plane, at angles ϕ_1 and ϕ_2 to the initial velocity of hole 1 (see Fig. 1).

To perform our simulations, we employed the Hahndol evolution code, as described by Baker et al. (2007) and J. G. Baker et al. (2008, in preparation). We chose initial coordinate separations of $7.0M$ for the $q \geq 1/2$ cases and $8.0M$ for the $q = 1/3$ cases (where M is the total mass of the system) to yield between one and four orbits prior to merger; informed by PN theory (Damour et al. 2000), we chose the corresponding momenta to minimize initial eccentricity. The finest grid spacing in all the runs presented here was $h_f = 3M/160$. We also performed a single high-resolution simulation of $h_f = M/64$ for the $q = 1/2$ case and found the kicks and all other relevant quantities agreed with the corresponding $h_f = 3M/160$ simulation to within $\sim 1\%$; additionally, we performed a set of lower resolution $h_f = 3M/128$ simulations in the $q = 1/3$ case, demonstrating consistency of the amplitude of v_{\parallel} with the $h_f = 3M/160$ simulations to within 6% .

3. RESULTS AND DISCUSSION

The recoil kicks resulting from the new simulations are in columns (4) and (5) of Table 2. Note that the in-plane kick agrees well with what would have been expected given no spin (v_m). This indicates negligible orbital precession, which we also verified from the trajectories of the black hole centers.

To conceive of plausible candidates for the mass scaling of v_{\parallel} , we begin with the spin expansion and symmetry arguments

TABLE 1
MAXIMUM PERCENT ERROR RESULTING FROM
VARIOUS MODELS OF THE KICK, AS
DISTINGUISHED BY OVERALL MASS RATIO
DEPENDENCE (SEE EQUATION [6])

q (1)	$K\eta^2$ (2)	$K(a_2\eta^2 + a_4\eta^4)$ (3)	$K\eta^3$ (4)
1/1.1	0.22	0.23	0.20
1/1.3	0.75	0.80	0.78
1/1.5	1.26	1.31	1.28
1/2	15.57	2.46	1.41
1/3	39.58	10.98	9.11

of Boyle et al. (2008) and Boyle & Kesden (2007). For the spin configurations considered here,

$$v_{\parallel} = D(q)\alpha_1^{\perp} \cos[\phi_1 - \Phi(q)] - D(1/q)\alpha_2^{\perp} \cos[\phi_2 - \Phi(1/q)], \quad (5)$$

where D and Φ are some functions of mass ratio q , and we note that Φ must also depend on the initial separation. Further restricting ourselves to forms related to the factor of $S_1/m_1 - S_2/m_2$ appearing in PN calculations of the kick, which informed Campanelli et al. (2007) and Lousto & Zlochower (2008a) and has been numerically well verified in the equal-mass case, we substitute $D(q) = qC(\eta)/(1+q)$ to obtain

$$v_{\parallel} = \frac{C(\eta)}{(1+q)} [q\alpha_1^{\perp} \cos(\phi_1 - \Phi_1) - \alpha_2^{\perp} \cos(\phi_2 - \Phi_2)], \quad (6)$$

where $\Phi_1 \equiv \Phi(q)$ and $\Phi_2 \equiv \Phi(1/q)$. Equation (4) arises from the choice $C(\eta) = K\eta^3$, with $K = 2.4 \times 10^5$.

There are several other possibilities for the form of $C(\eta)$ in the literature. Campanelli et al. (2007) assume $C(\eta) = 6.0 \times 10^4 \eta^2$. In this case, $(\phi_1 - \Phi_1)$ and $(\phi_2 - \Phi_2)$ are related to $(\Theta - \Theta_0)$ from Campanelli et al. (2007) as well as the angle between the spin vectors implicit in $|\alpha_2^{\perp} - q\alpha_1^{\perp}|$.

Another possibility arises from the known relation between v_{\parallel} and the difference between the energy radiated in the $(l, m) = (2, 2)$ and $(2, -2)$ harmonics of the radiation (Schnittman et al. 2008; Brüggmann et al. 2007). With no spin, these quantities are equal. With spin, we expect that $v_{\parallel} \sim \dot{E}_{22(\text{peak})} F$, where $\dot{E}_{22(\text{peak})}$ is the peak power radiated in the $(2, 2)$ harmonic, and F represents the spin-dependent asymmetry between $\dot{E}_{22(\text{peak})}$ and $\dot{E}_{2-2(\text{peak})}$, i.e., $F \sim 1 - \dot{E}_{2-2(\text{peak})}/\dot{E}_{22(\text{peak})}$. For black holes with no spin, we have found that $\dot{E}_{22(\text{peak})} = a_2\eta^2 + a_4\eta^4$, where $a_2 = 0.0044$ and $a_4 = 0.0543$, gives a good fit to the numerical data (Baker et al. 2008). We do not expect spins orthogonal to the orbital angular momentum to change the scaling of the radiated energy significantly. If we further assume that the asymmetry factor F is independent of η , which finds some support in PN analysis since to leading order $\dot{P}_{\parallel}/\dot{E}$ is independent of η , then we hypothesize that $C(\eta) \propto (a_2\eta^2 + a_4\eta^4)$.

In Table 1 we summarize the agreement of various kick formulas with the numerical data. For each formula, which has the form of equation (6), we found the best Φ_1 and Φ_2 , per mass ratio, according to a least-squares fit to the data given in Table 2. For each mass ratio, the resulting percent error is given for each model, maximized across initial angle. Referring to equation (6), the column headings $K\eta^2$, $K(a_2\eta^2 + a_4\eta^4)$, and $K\eta^3$ of Table 1 represent choices for $C(\eta)$ that were tested, where in each case K has been chosen so as to reproduce the

TABLE 2
INITIAL CONFIGURATION AND FINAL KICK
FOR EACH SIMULATION

q (1)	ϕ_1 (deg) (2)	ϕ_2 (deg) (3)	v_m (km s ⁻¹) (4)	v_{\parallel} (km s ⁻¹) (5)
1/1.1	0	180	24	-542
	315	135	24	-657
	270	90	25	-384
1/1.3	0	180	67	-386
	315	135	67	-525
	270	90	69	-348
1/1.5	60	240	92 (94)	-381
	0	180	95 (94)	-135
	315	135	91 (94)	168
	270	90	90 (94)	364
1/2	0	180	137 (140)	-37
	315	135	136 (140)	111
	270	90	136 (140)	193
	315	90	...	75
	0	90	...	-55
1/3	0	180	166	49
	315	135	166	48
	270	90	163	17
	0	0	162	114

NOTES.—Here $\phi_{1(2)}$ is the angle made by the spin vector of hole 1(2) with the velocity vector of hole 1, as shown in Fig. 1. Numerical results for the kick components v_m (where available) and v_{\parallel} are shown. Kicks for equivalent spinless runs are in parentheses.

value of the formula of Campanelli et al. (2007) in the equal-mass case.

Now we consider the agreement of our data with the $C(\eta) = K\eta^2$ scaling of Campanelli et al. (2007) (Table 1, col. [2]). The error of the best fit grows significantly with mass ratio, hence the mass ratio dependence of this formula is inaccurate. One might suppose that precession of the spins into the orbital plane could account for this. However, the v_m column in Table 2 shows that the in-plane kicks are close to those measured without spins (given in parentheses); hence this does not explain the discrepancy in v_{\parallel} from the η^2 scaling. We have experimented with other values of K to resolve the discrepancy. For example, the maximum error of the η^2 model can be reduced to less than 10% for the $q = 1/3$ case, but not without increasing the maximum error of other mass ratios closer to unity to significantly greater than 10%.

Since original submission of this Letter, new data presented by Lousto & Zlochower (2008b) seem to support η^2 scaling, although the cases analyzed are complicated by considerable orbital precession. For example, their in-plane kicks are apparently at odds with previous formulae. It is possible that strongly precessing orbits require different fitting formulae, but this has yet to be settled.

The choice $C(\eta) = K(a_2\eta^2 + a_4\eta^4)$, motivated above, fits the data much more successfully (Table 1, col. [3]). Other scalings can be motivated through post-Newtonian-based analysis (Schnittman et al. 2008). However, a better empirical model was found to be $C(\eta) = K\eta^3$ (Table 1, col. [4]). For now we consider this our best fit, and leave open the interesting question of how to accurately relate this prefactor directly to E_{22} .

Our results affect the distribution of kick speeds, given various assumptions about the spin parameters, spin orientations, and mass ratios involved in coalescences. This has particular application to the retention of the products of mergers of massive black holes in the current universe (e.g., Bonning et al.

2007) and electromagnetic signatures of kicks (e.g., Shields et al. 2007; Lippai et al. 2008), as well as coalescences in the early structure formation phase of redshift $z \sim 5-30$ (Merritt et al. 2004; Boylan-Kolchin et al. 2004; Haiman 2004; Madau & Quataert 2004; Yoo & Miralda-Escudé 2004; Volonteri & Perna 2005; Libeskind et al. 2006; Micic et al. 2006; Volonteri 2007), and for current-day mergers of intermediate-mass black holes (IMBHs), which might exist in dense stellar clusters (Taniguchi et al. 2000; Miller & Hamilton 2002a, 2002b; Mouri & Taniguchi 2002a, 2002b; Miller & Colbert 2004; Gültekin et al. 2004, 2006; O’Leary et al. 2006, 2007). Note that $q = 1$ to $q = 1/3$ is in the range of ratios expected for major mergers of galaxies, and as Sesana et al. (2004) show, this range is expected to account for most massive black hole mergers in the early $z > 10$ phase of black hole assembly.

Our new formula implies an important revision in our understanding of how easily IMBHs with $M \sim 10^2-10^3 M_{\odot}$ are retained in globular clusters. A rich cluster has an escape speed $v_{\text{esc}} \approx 50 \text{ km s}^{-1}$ (Webbink 1985). Gültekin et al. (2006) showed that the Newtonian kicks involved in binary-single interactions are insufficient to reach this speed if the IMBH is at least $\sim 15-20$ times more massive than the objects with which it interacts. Using the Campanelli et al. (2007) formula, however, the maximum kick from gravitational radiation is $v_{\text{max}} = 6 \times 10^4 \text{ km s}^{-1}\eta^2$, implying that even IMBHs 30–35 times more massive than the black holes with which they merge could get ejected. Holley-Bockelmann et al. (2007) focusing on cases in which stars lose little mass through their evolution and thus can leave behind stellar-mass black holes with masses $>60-100 M_{\odot}$, use this to argue that most IMBHs of even 1000 M_{\odot} will be ejected from globulars. If instead stellar-mass black holes have masses $\sim 10 M_{\odot}$, a mass of at least 400 M_{\odot} would still be required to guarantee retention.

In contrast, our new formula suggests a maximum kick of $v_{\text{max}} = 2.4 \times 10^5 \text{ km s}^{-1}\eta^3$. Thus if $\eta < 0.06$, $v_{\text{max}} < 50 \text{ km s}^{-1}$. Therefore, an IMBH interacting with 10 M_{\odot} black holes will stay in a rich globular if its initial mass is $M > 170 M_{\odot}$, comparable to what is necessary for retention against Newtonian three-body kicks.

Our results also have implications for whether merged supermassive black holes stay in their host galaxies. The figure of merit is the fraction of kicks that exceed typical escape speeds from galactic centers (ranging from roughly 500 km s^{-1} for a small spiral to 2000 km s^{-1} for a giant elliptical), given assumptions about the distribution of spins and orbital orientations. The calculation of record for this purpose is that by Schnittman & Buonanno (2007) who used a kick formula based on effective one-body analysis and is different from that of Campanelli et al. (2007); this formula underestimates the highest kicks. Table 3 compares the fraction of kicks above 500 and 1000 km s^{-1} using the Schnittman & Buonanno (2007) formula (an underestimate), the Campanelli et al. (2007) formula (an overestimate), and our results. It is clear that the Schnittman & Buonanno (2007) results were conservative: the fraction of large kicks is significantly higher than their estimate for comparable-mass mergers with plausible spins.

Barring mechanisms to retain supermassive black holes after major mergers, one would expect tens of percent of merged galaxies to have no central black hole, in strong contradiction with observations (see Ferrarese & Ford 2005). Low spin magnitudes would lower kicks, but this is contrary to spin inferences from Fe K α lines; see Iwasawa (1996), Fabian et al. (2002), Reynolds & Nowak (2003), and Brenneman & Reynolds (2006). Alignment of spins is another possibility; since

TABLE 3
FRACTION OF KICK SPEEDS ABOVE A GIVEN THRESHOLD

Mass Ratio and Spin (1)	Speed Threshold (2)	SB (3)	CLZM (4)	This Work (5)
$1/10 \leq q \leq 1, a_1 = a_2 = 0.9$	$v > 500 \text{ km s}^{-1}$	0.12 (+0.06, -0.05)	0.364 ± 0.0048	0.2283 ± 0.0014
	$v > 1000 \text{ km s}^{-1}$	0.027 (+0.021, -0.014)	0.127 ± 0.0034	0.085 ± 0.0008
$1/4 \leq q \leq 1, a_1 = a_2 = 0.9$	$v > 500 \text{ km s}^{-1}$	0.31 (+0.13, -0.12)	0.699 ± 0.0045	0.618 ± 0.0014
	$v > 1000 \text{ km s}^{-1}$	0.079 (+0.062, -0.042)	0.364 ± 0.0046	0.2547 ± 0.0013
$1/4 \leq q \leq 1, 0 \leq a_1, a_2 \leq 1$	$v > 500 \text{ km s}^{-1}$...	0.428 ± 0.0045	0.3484 ± 0.0015
	$v > 1000 \text{ km s}^{-1}$...	0.142 ± 0.0034	0.0974 ± 0.0009

NOTES.—Fraction of kick speeds above a given threshold, compared with the results of Schnittman & Buonanno (2007) (SB) and Campanelli et al. (2007) (CLZM). In all cases we assume an isotropic distribution of spin orientations.

pure gravity does not do this (Schnittman 2004; Bogdanovic et al. 2007), external torques such as those from nuclear gas would be required (Bogdanovic et al. 2007).

In conclusion, we have performed a systematic study of the mass ratio dependence of the out-of-plane kicks produced by the merger of spinning black holes. Our work shows that the Campanelli et al. (2007) candidate kick formula overestimates the out-of-plane kick systematically. However, we find that an additional factor of 4η agrees with our numerical results to within 10% (and typically $\sim 1\%$) for mass ratios between 1 and $1/3$. This has considerable implications for black hole retention in early dark matter halos, galaxies, and globular clusters.

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