Monthly Notices of the ROYAL ASTRONOMICAL SOCIETY
MNRAS **497,** 3076–3082 (2020)
Advance Access publication 2020 July 22
doi:10.1093/mnras/staa2074

Beaming as an explanation of the repetition/width relation in FRBs

L. Connor [1,2]★ M. C. Miller[3] and D. W. Gardenier [1,2]

[1] *Anton Pannekoek Institute, University of Amsterdam, Postbus 94249, NL-1090 GE Amsterdam, the Netherlands*
[2] *ASTRON, the Netherlands Institute for Radio Astronomy, Oude Hoogeveensedijk 4, NL-7991 PD Dwingeloo, the Netherlands*
[3] *Department of Astronomy and Joint Space-Science Institute, University of Maryland, College Park, MD 20742-2421, USA*

Accepted 2020 July 7. Received 2020 July 7; in original form 2020 March 30

ABSTRACT

It is currently not known if repeating fast radio bursts (FRBs) are fundamentally different from those that have not been seen to repeat. One striking difference between repeaters and apparent non-repeaters in the Canadian Hydrogen Intensity Mapping Experiment sample is that the once-off events are typically shorter in duration than sources that have been detected two or more times. We offer a simple explanation for this discrepancy based on a selection effect due to beamed emission, in which highly beamed FRBs are less easily observed to repeat, but are abundant enough to detect often as once-off events. The explanation predicts that there is a continuous distribution of burst duration – not a static bimodal one – with a correlation between repetition rate and width. Pulse width and opening angle may be related by relativistic effects in shocks, where short-duration bursts have small solid angles due to a large common Lorentz factor. Alternatively, the relationship could be a geometric effect where narrow beams sweep past the observer more quickly, as with pulsars. Our model has implications for the FRB emission mechanism and energy scale, volumetric event rates, and the application of FRBs to cosmology.

Key words: methods: statistical – fast radio bursts.

1 INTRODUCTION

Fast radio bursts (FRBs) are short-duration (µs–ms) extragalactic radio transients whose origins remain a mystery (Cordes & Chatterjee 2019; Petroff, Hessels & Lorimer 2019). To date, approximately 800 FRBs have been detected, of which ∼120 have been published (Petroff et al. 2016) and ∼700 will be published by the Canadian Hydrogen Intensity Mapping Experiment (CHIME) in a forthcoming catalogue (Fonseca et al. 2020). The majority of these have not been seen to repeat. In this paper, we interchangeably refer to such sources as 'once-off events' and 'apparent non-repeaters', as we cannot know that they will not repeat in the future. There are 20 FRBs that have been found to repeat, all but two of which were discovered by CHIME (Spitler et al. 2016; CHIME/FRB Collaboration 2019b, c; Kumar et al. 2019; Fonseca et al. 2020). One of the repeaters, FRB 180916.J0158+65, was found by CHIME to exhibit a 16.35-d periodicity in its repetition activity (The CHIME/FRB Collaboration 2020). There is now also a claim that the first source found to repeat, FRB 121102, does so with a tentative ∼160-d periodicity in its activity level (Rajwade et al. 2020). No FRB periodicity has been detected on time-scales between 10^{-3} and 10^{3} s that might be associated with a neutron star rotation period.

It remains an open and important question as to whether repeating FRBs and apparent non-repeaters form two physically distinct classes. There are a number of ways a repeating FRB might be observed as a once-off event, including low repeat rate, clustered repetition (Connor, Pen & Oppermann 2016; Scholz et al. 2016; Oppermann, Yu & Pen 2018), or an unfavourable luminosity function and telescope sensitivity (Connor & Petroff 2018; Caleb et al. 2019; Kumar et al. 2019). For example, CHIME has only once detected the repeater FRB 121102, so in the absence of previous observations that source would appear to be a non-repeater (Josephy et al. 2019).

One curious distinction has emerged between repeaters and non-repeaters in the pulse width distribution found by CHIME. They have found that repeaters typically emit longer duration pulses, and once-off FRBs are narrower (CHIME/FRB Collaboration 2019c; Fonseca et al. 2020). This has been shown for the first 12 published apparent non-repeaters (CHIME/FRB Collaboration 2019a) and 18 repeating FRBs discovered by CHIME. It was also suggested by Scholz et al. (2016), who noticed that the non-repeating FRBs detected at Parkes were shorter in duration than FRB 121102. Robust comparison across different surveys is difficult, but the CHIME FRBs were all found on the same telescope by the same pipeline, and it is difficult to explain the width/repetition relation with an instrumental selection effect.

We provide an explanation for the broad duration of repeating FRBs relative to once-off events using beamed emission. If FRBs are beamed with a wide distribution of opening angles, and there is a positive correlation between opening angle and pulse width, then broad FRBs will have a higher observed repeat rate even if the intrinsic repetition statistics are the same.

In this paper, we first describe our model, including a simple Monte Carlo simulation demonstrating the proposed selection effect due to beaming. We then speculate on the origin of the required relationship between beaming and pulse width, followed by the implications for FRB emission, rates, energetics, and future searches for repeating sources.

2 MODEL

In our picture, most or all FRBs repeat. The distribution of intrinsic repetition rates is currently unknown, and our model remains agnostic

★ E-mail: liam.dean.connor@gmail.com

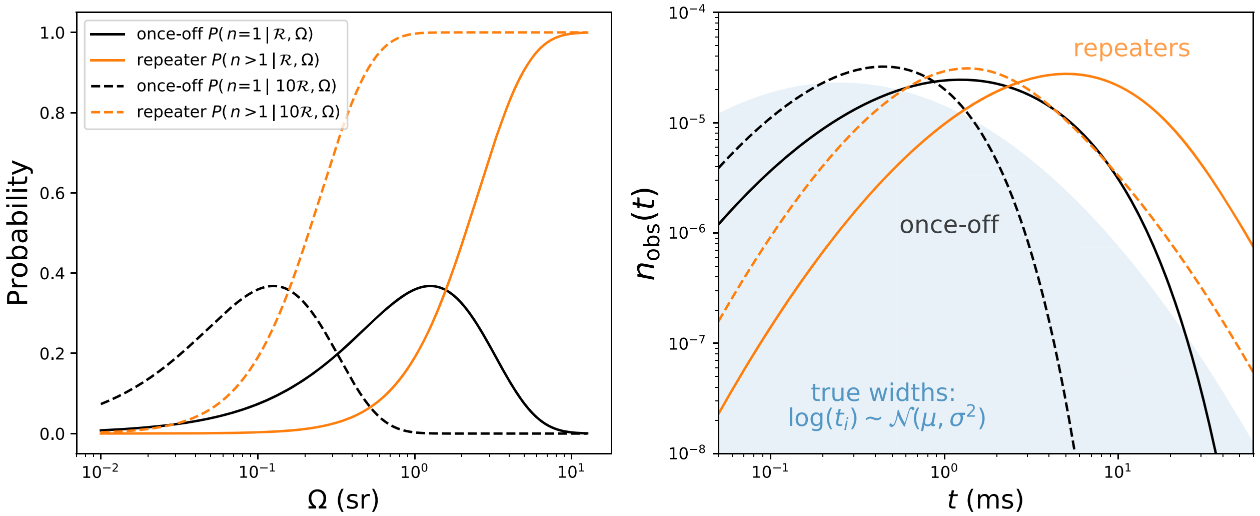

Figure 1. The left-hand panel shows the probability of a beamed FRB emitting in the direction of an observer once (black) and more than once (orange) as a function of opening angle Ω. We use two intrinsic repeat rates, $\mathcal{R}$ (solid) and $10\,\mathcal{R}$ (dashed), corresponding to an expected value of 5 and 50 bursts per $T_{\rm obs}$, respectively. The right-hand panel shows the observable pulse width distributions of repeaters and apparent non-repeaters, having converted beaming solid angle Ω to pulse width, t, via equation (6). There are more broad-duration repeating FRBs and narrower single detections.

to its shape. Instead, the key distinction between observed repeaters and FRBs that have only been detected once is their beaming angle, not their intrinsic repeat frequency (which would be zero in the case where non-repeaters are a distinct class of cataclysmic FRBs). The CHIME data show that in roughly 1 yr of observing, which corresponds to ∼60 h on each source, there are more once-off FRBs than repeaters, and the repeaters are longer in duration. To explain these two facts, our model requires the following two main assumptions:

(i) There exists a positive correlation between beaming angle and pulse duration.

(ii) The intrinsic beaming angle distribution of FRBs is such that there are more highly beamed sources than ones with large opening angles, for the region of widths to which we are sensitive.

If sources with beaming solid angle, Ω, emit repeat bursts in random directions, then on average the probability of detecting a burst from a given source is $\frac{\Omega}{4\pi}$. In principle, the FRBs need not emit uniformly over the full sphere: So long as the directions of its repeat bursts are spread over a solid angle that is larger than Ω, the effect still holds. Assuming Poissonian repetition where the j-th source has a repetition rate $\mathcal{R}_j$ and opening angle Ω_j, the expected number of bursts in observing time $T_{\rm obs}$ is

$$N^j_{\rm exp} = \frac{\Omega_j}{4\pi}\mathcal{R}_j T_{\rm obs}. \quad (1)$$

The probability of a source being detected exactly once is

$$P(n\!=\!1\,|\,\Omega_j, \mathcal{R}_j) = e^{-\frac{\Omega_j}{4\pi}\mathcal{R}_j T_{\rm obs}}\frac{\Omega_j}{4\pi}\mathcal{R}_j T_{\rm obs} \quad (2)$$

and the probability of its repeating twice or more towards the observer is

$$P(n\!\geq\!2\,|\,\Omega_j, \mathcal{R}_j) = 1 - P(n\!=\!0\,|\,\Omega_j, \mathcal{R}_j) - P(n\!=\!1\,|\,\Omega_j, \mathcal{R}_j) \quad (3)$$

$$= 1 - e^{-\frac{\Omega_j}{4\pi}\mathcal{R}_j T_{\rm obs}}\left(1 + \frac{\Omega_j}{4\pi}\mathcal{R}_j T_{\rm obs}\right). \quad (4)$$

To get the beaming angle and pulse width distribution of *detected* events, two more steps are required. First, $P(n=1)$ and $P(n\geq 2)$ must be multiplied by the intrinsic distribution of beaming angles, $n_i(\Omega)$. This quantity is the differential intrinsic beaming angle distribution that gives the number of bursts with opening angle in each bin dΩ and is defined as

$$n_i(\Omega) \equiv \frac{{\rm d}n_i}{{\rm d}\Omega}. \quad (5)$$

Next, we must ask which of those events will actually be detected by a radio telescope, after deleterious smearing effects due to finite time and frequency sampling. In Fig. 1, we plot in the left-hand panel the probability that a source is pointed towards the observer once (black) and more than once (orange), for two different repetition rates, as a function of beaming angle. The right-hand panel shows these probability curves multiplied by the intrinsic Ω distribution, assuming a lognormal distribution in beaming angle with a mean of 0.04 sr, corresponding to a burst-duration mean of 200 µs. To get this pulse width distribution, we have assumed a simple mapping between burst duration t and Ω such that

$$t = 5\,{\rm ms} \times \frac{\Omega}{1\ {\rm sr}}. \quad (6)$$

We speculate on the origin of such a relationship in Section 3.1. Note that the right-hand panel of Fig. 1 corresponds to the observed width distribution for repeaters and apparent non-repeaters in the absence of instrumental smearing and propagation effects such as scattering or plasma lensing. The orange curves illustrate that beaming angle provides a selection effect such that repeating FRBs will have statistically larger widths than once-off events. Conversely, a wide event that has only been detected once is more likely to repeat in the future than a narrow apparent non-repeater.

2.1 Monte Carlo simulation

In order to estimate a realistic pulse width distribution of detected FRBs, we must include non-linear instrumental effects such as temporal smearing. To do this, we have built a simple Monte Carlo simulation. This also enables us to add jitter to the pulse width/beaming relationship and test different input distributions for Ω, $\mathcal{R}$, brightness, and dispersion measure (DM). Though we have assumed for Fig. 1 that the underlying repeat rate need not vary between sources, in reality there will be an intrinsic distribution $n_i(\mathcal{R})$ that may include some true non-repeaters, i.e. weight at $\mathcal{R} = 0$.

We start by simulating 100 000 FRBs, all with the same intrinsic repeat rate but with a broad distribution of beaming angles, as shown in Fig. 2. We then simulate 1000 repeat bursts for each source, using Poissonian statistics and with an emission direction that is drawn randomly from a uniform distribution on the sphere. Each FRB is observed for 20–30 h, drawn from a uniform distribution. We then check which if any of their repeat bursts would be detectable with CHIME.

If a burst from a given FRB was emitted within its $T_{\rm obs}$ and with favourable pointing, it was deemed 'observable' (left-hand panel of Fig. 2), which is to say that the observer line of sight fell within that FRB's top-hat beam during the pre-defined time window. Within that subset of observable bursts, we take the linear relationship between Ω and t used in equation (6). We then apply instrumental smearing effects, assuming a CHIME-like back end. We assume for simplicity that all FRBs have DM = 1000 pc cm^{-3}. A pulse with a duration $t_{\rm i}$ as it arrives at our telescopes will be smeared to

$$t_{\rm obs} = \sqrt{t_{\rm i}^2 + t_{\rm s}^2 + t_{\rm DM}^2}, \qquad (7)$$

due to the finite time and frequency sampling of radio telescopes. Here, $t_{\rm s}$ is the instrument's sampling time and $t_{\rm DM}$ is the time-scale associated with intra-channel dispersion smearing, given by

$$t_{\rm DM} = 8.3 \times 10^{-3}\,{\rm DM}\,\frac{\Delta\nu_{\rm MHz}}{\nu_{\rm GHz}^3}\quad {\rm ms}. \qquad (8)$$

We use the values for CHIME's back end because they have the largest sample of repeaters, and that is where the repetition/width relation is most pronounced. Its current sampling time is $t_{\rm s}$ = 0.983 ms and its frequency channel width in MHz is $\Delta\nu_{\rm MHz} = 0.024$. Using DM = 1000 pc cm^{-3} and a central frequency in GHz of $\nu_{\rm GHz}$ = 0.6, the minimum detection width for an FRB in our simulation is $t_{\rm obs} = \sqrt{t_{\rm s}^2 + t_{\rm DM}^2} \approx 1.35$ ms. That is why the right-hand panel of Fig. 2 has no detections below that value, despite the large number of submillisecond simulated events.

Instrumental smearing also decreases the pulse's signal-to-noise ratio (S/N) and lowers its chances of being detected. To account for this in our simulation, we apply the corresponding reduction in S/N. A pulse whose brightness corresponds to some S/N, $s_{\rm i}$, in the absence of smearing, will be detected with an S/N

$$s_{\rm obs} = s_{\rm i}\left(\frac{t_{\rm i}^2}{t_{\rm i}^2 + t_{\rm s}^2 + t_{\rm DM}^2}\right)^{1/2}. \qquad (9)$$

We assume for now that there is no correlation between width and brightness, and draw each value $s_{\rm i}$ from a Euclidean distribution in brightness with $n(s_{\rm i}) \propto s_{\rm i}^{-5/2}$. In Section 3.1, we discuss how this assumption may not hold if the relationship between Ω and t is due to relativistic beaming, which would cause narrow bursts beamed in our direction to be brighter. If the pulse's resulting $s_{\rm obs}$, as computed in equation (9), is above an S/N threshold, $s_{\rm min}$, we label it a 'detection'. In the right-hand panel of Fig. 2, we show the subset of simulated bursts that were observable and still above $s_{\rm min}$ after smearing. We have ignored a potential selection effect by using a single DM value. If once-off FRBs are in fact brighter, then they will be visible at greater distances and may have higher DMs, which can lead to detection biases. However, based on the current CHIME sample of repeaters and single detections, whatever differences might emerge between their DMs will be relatively small and their distributions will overlap significantly.

It is clear that detected repeaters are wider in duration than the apparent non-repeaters. The resulting distributions are similar to the observed widths in CHIME, in that apparent non-repeaters are more abundant and cluster around the smearing width and are narrower in duration.

For the sake of isolating the beaming selection effect proposed in this paper, we have used a simplified model and must include the following caveats. FRB repetition is often clustered and not described by a homogeneous Poisson process (Scholz et al. 2016; Oppermann et al. 2018; Gourdji et al. 2019). This increases the variance on the number of detected bursts in an observing window, even if the mean remains the same, $N_{\rm exp} = \frac{\Omega_j}{4\pi}\mathcal{R}_j T_{\rm obs}$. We have assumed a delta function distribution in DM with $n_i({\rm DM}) = \delta({\rm DM} - 1000)$ when that is known to not be the case. We have also used a perfect mapping between Ω and pulse duration $t_{\rm i}$. Still, we have experimented with adding noise to the $\Omega(t_{\rm i})$ function such that there is jitter in the mapping, and find that so long as there is a correlation between beaming angle and duration, there is a difference between the detectable widths of repeaters versus apparent non-repeaters. To tie this to observations, we have also tried adding noise to the $t_{\rm i}/\Omega$ relation for an individual repeating FRB, not just from source to source. We have taken the fractional root-mean-square error (RMSE) on the pulse widths of FRB 180916.J0158+65 as the spread for our simulation, because it is the CHIME repeater with the greatest number of detections. For FRB 180916.J0158+65, this value is ∼0.6. With this level of noise, we find that repeaters are noticeably broader than single detections, even from just the first 19 repeating sources. It is possible that our estimated RMSE is biased low, because very narrow and very broad FRBs are more likely to be missed. We also tested different distributions in Ω and DM and recover the same effect as long as the two main assumptions in Section 2 are met. Finally, the pulse widths reported by CHIME are fitted widths rather than the maximum-S/N boxcar that was used to discover the FRB (CHIME/FRB Collaboration 2019a, c; Fonseca et al. 2020). They are able to do this effectively because of their large fractional bandwidth, which can disentangle intrachannel dispersion smearing, scattering, and intrinsic duration, which have different frequency dependences. This is why some of CHIME's published FRB widths are shorter than the sampling time of the instrument. We have opted instead to use the final smearing width, because our simulation did not produce dynamic spectra of individual bursts to fit, which is why the red points in Fig. 3 go to shorter time-scales than the orange points. None the less, the width/repetition effect is expected with both methods.

3 DISCUSSION

3.1 Predictions

In our model, repeating FRBs and those that have only been detected once are not two fundamentally different source classes. Therefore, we do not expect a bimodal distribution of FRB widths in which repeaters have a characteristic duration and non-repeaters have a different one. This is in contrast to GRBs, which can be divided into two classes along burst duration, where short-hard bursts are typically less than 2 s and long-soft bursts are longer than 2 s (Kouveliotou et al. 1993; see the review in Berger 2014).

Unlike with GRBs, our model suggests that frequent repeaters are simply the long-duration tail of the non-repeater distribution. The repeater width distribution will move towards shorter durations as the exposure to each source increases, because that leads to a

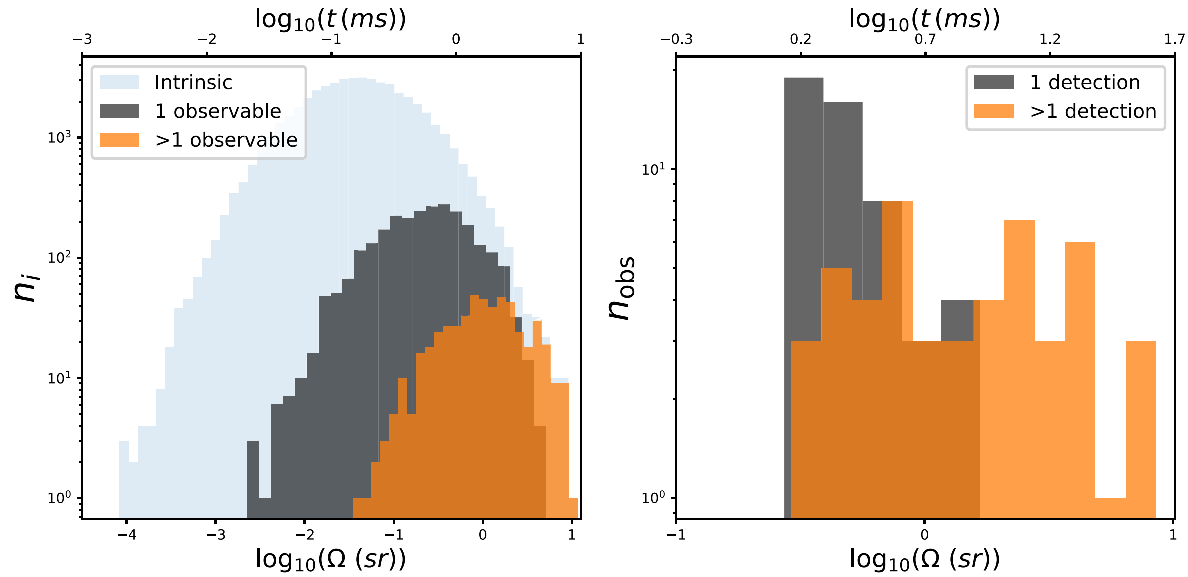

Figure 2. The blue histogram in the left-hand panel shows the beaming angles of 100 000 simulated FRBs, all with the same intrinsic repeat rate. The black and orange counts are FRBs that were 'observable' once and more than once, respectively, thanks to their favourable viewing angle and their being within the temporal observing window. The right-hand panel shows the detected distributions of pulse widths, after accounting for instrumental smearing with a CHIME-like back end. The detected repeaters are statistically significantly wider than the more-numerous single-detection FRBs in this realization.

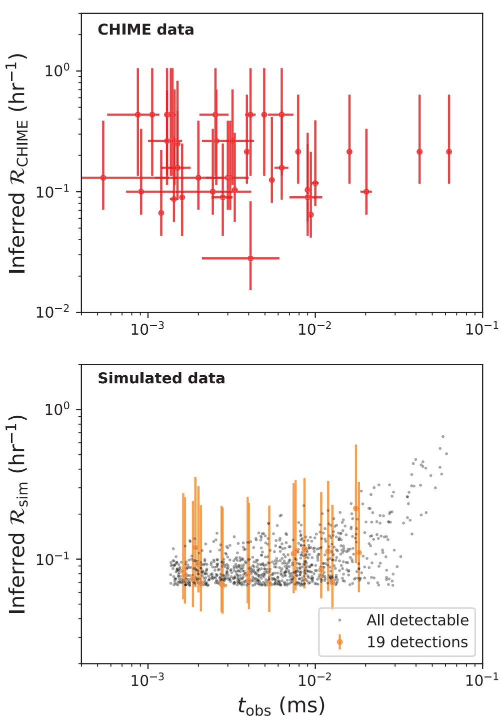

Figure 3. The inferred repetition rate versus pulse width of CHIME repeaters (top panel) and simulated events (bottom panel). In both cases, the first 19 repeater detections do not result in a significant correlation. The Poissonian error bars we use are lower limits for the true error bars, which are presently unknowable due to the biases and selection effects described in the text.

greater probability of seeing a previously once-off source repeat. If, instead, repeaters and non-repeaters have distinct and *static* width distributions, there are several summary statistics that could be used. For example, the bimodality coefficient, β, is defined between 0 and 1, where $\beta = 5/9$ corresponds to a uniform distribution and greater values can imply multimodality. It can be used as a test statistic, assuming that the underlying distribution is generated by a mixture of two normal distributions[1] (whether in t or $\log t$). It is computed as

$$\beta = \frac{g^2 + 1}{k + \frac{3(N-1)^2}{(N-2)(N-3)}}, \qquad (10)$$

where N is the number of samples in the data set, k is excess kurtosis, and g is the sample skewness. Another method called Kernel Mean Matching has been used in astronomical data sets such as globular cluster metallicity and GRB duration (Ashman, Bird & Zepf 1994). In the case of FRBs, the observed width distribution can depend strongly on the time/frequency resolution of the survey back end (Connor 2019). Before a test for bimodality can be done, burst-duration selection effects must be understood and a large-enough sample of temporally resolved FRBs must be obtained. CHIME will be able to do this if their selection function can be measured, because they can save raw voltage data and can fit for pulse widths below their instrumental smearing time-scale. If the presence of two peaks were found, and they do not change with increased exposure, this would be good evidence against the claim that all FRBs are repeaters with a continuum of repetition frequencies.

We also expect a positive correlation between FRB duration and repetition rate. The strength of this correlation will depend on the mapping between beaming angle and intrinsic pulse width, as well as observational selection effects. Again, this is because our population is not divided into repeaters and non-repeaters. That is, even within the set of detected repeaters, we expect those that repeat more frequently to have wider beams. From our simulation, we find that such a correlation persists so long as the Ω/t relationship is not made too noisy and a large-enough collection of repeaters have been observed for more than an average repeat period. This is shown in the

[1] https://en.wikipedia.org/wiki/Multimodal_distribution

bottom panel of Fig. 3, where there is a discernible relationship for the large number of black points, but none for the first 19 detections in our simulation.

We looked for such a relation in the CHIME repeater data, but found no significant correlation between mean repetition rate and pulse width (top panel of Fig. 3). The correlation was tested using a Pearson product-moment correlation, but was not found to be constraining either in the linear or in a logarithmic space. At this point, that is unsurprising for the following reasons. With just 19 repeaters observed for a median ∼23 h, many sources have only repeated two or three times. This combined with the uncertainty of CHIME's exposure to each source (from unknown beam shapes, day-to-day sensitivity, etc.) leads to large uncertainties in their repeat rate, which we take as $\mathcal{R}^j_{\rm CHIME} = N_j/T^j_{\rm obs}$. Furthermore, the inferred repeat rate of sources that have been detected just a few times is highly biased. Suppose there were 1000 FRBs all with the same intrinsic Poissonian repeat rate, and they were observed for a duration less than their common repeat period. The first dozen repeater detections would necessarily have a much higher inferred repeat rate than their underlying repetition frequency. Therefore, we should expect the CHIME repeaters that have been detected fewer than 5 times to repeat less frequently in the future than they have thus far. Finally, as discussed in Section 2, temporally clustered, non-Poissonian repetition increases the error bars on the number of bursts detected in an observing window, furthering uncertainty in repeat rate. The Poissonian error bars we use for the red points in Fig. 3 are therefore lower limits on the true uncertainty.

In Fig. 3, the orange points show the first 19 FRBs detected in our Monte Carlo simulation. As with CHIME repeaters, many have not been detected more than a couple of times and there is no strong correlation with pulse width. In order to establish our proposed correlation, more repeating sources are needed and each source must be observed for longer to decrease uncertainty on their repeat period. The black points in the bottom panel of Fig. 3 give an idea of the broader trend and the number of repeaters required to measure this correlation. In our simulation (and similarly with CHIME), the repeaters have only been observed for 20–30 h, so there is a floor in the repeat rate at 2 per 30 h. That floor can be seen in the plot's black points. Until those sources are observed for longer, they do not offer much information on the repeat rate/width correlation, but the broader, more repetitive sources do. A better estimate of their repeat rates could be obtained by tracking telescopes and with coming years of CHIME exposure.

One consequence of any model in which repeaters and once-off FRBs are drawn from the same population, such as ours, is that the pulses ought to have similar individual burst structure. The first repeating source to be discovered, FRB 121102, often emits FRBs with a characteristic 'march down' in their dynamic spectra, such that adjacent subpulses peak at subsequently lower frequencies (Hessels et al. 2019). This has also been detected in some, but not all, repeating FRBs discovered by CHIME (CHIME/FRB Collaboration 2019b, c; Fonseca et al. 2020). We might then expect that with sufficient time/frequency resolution and S/N, some narrow apparent non-repeaters would have similar structure in their dynamic spectra, but on shorter time-scales. This is possible now that many FRB back ends allow for the preservation of raw voltage data that can be coherently dedispersed (Farah et al. 2018; Bannister et al. 2019; CHIME/FRB Collaboration 2019c; Ravi et al. 2019). We emphasize that while this should be true for our model, it will also be the case for any scenario in which repeaters and apparent non-repeaters come from similar sources in similar environments, not only under the beaming selection effect we have put forth.

Depending on the physical origin of a beaming/pulse width correlation, there will be more model-specific predictions, some of which we touch on in the next section. Here we have described only the most generic consequences of a situation in which broadly beamed FRBs are more easily detected as repeaters.

3.2 Origin of the Ω/*t* relationship

Beamed emission occurs in astrophysical sources when collimated beams of particles are moving at speeds close to c. If those particles are moving with a Lorentz factor, Γ, radiation is seen by the observer within an angle $\alpha \sim \Gamma^{-1}$. If those relativistic particles form a pencil beam, so too will the radiation and

$$\Omega \sim \alpha^2 \propto \Gamma^{-2}. \qquad (11)$$

If the particles are confined to a thin sheet, the contraction is effectively only in one dimension (Katz 2017) and we get

$$\Omega \sim \alpha \propto \Gamma^{-1}. \qquad (12)$$

The purpose of this paper is to propose a simple model for the origin of the longer durations of repeating FRBs relative to apparent non-repeaters rather than offer a unique emission mechanism or progenitor. Our model requires that FRBs are differentially beamed and that their opening angle scales with pulse width. It also requires that repeat bursts from the same source point in different directions over a larger area than its own beaming solid angle Ω. While we do not attempt to explain this phenomenon at the emission level with high certainty, below we provide examples of how the Ω/*t* relationship could emerge in the context of previously proposed FRB models.

3.2.1 *Relativistic temporal modulation*

In a subset of FRB models, relativistic plasma is sporadically expelled from magnetars, which then collides with surrounding material or the magnetar's wind, producing radio emission $\sim 10^{11}$–10^{15} cm away from the star – well outside of its magnetosphere (Lyubarsky 2014; Beloborodov 2017, 2020; Margalit & Metzger 2018; Metzger, Margalit & Sironi 2019). This is preferentially expected to occur in young, hyper-active systems, which are thought to generate magnetic flares more frequently than the older magnetars we observe in our own Galaxy. Such models provide a natural connection between burst duration and observability, due to relativistic effects.

In these emission models, the ultra-relativistic shock has Lorentz factor, $\Gamma_{\rm sh}$. The resulting electromagnetic radiation is compressed in time by the Doppler effect and undergoes relativistic beaming, in the observer's frame. From Beloborodov 2020, the burst duration in the observer's frame is

$$t_{\rm i}(r) \approx \frac{R_{\rm GHz}}{c\,\Gamma^2_{\rm sh}}, \qquad (13)$$

where $R_{\rm GHz}$ is the distance from the star at which the observer-frame emission peaks at GHz frequencies. The beamed solid angle is

$$\Omega \approx \frac{\pi}{\Gamma^2_{\rm sh}}, \qquad (14)$$

and therefore a natural connection between beaming solid angle and pulse width emerges

$$t_{\rm i} \approx \frac{R_{\rm GHz}}{c\,\pi}\,\Omega. \qquad (15)$$

In this scenario, both $t_{\rm i}$ and Ω scale with $\Gamma_{\rm sh}$, which may vary from burst to burst and from source to source.

The variation in Lorentz factors between sources must be larger than the variation within a source between bursts, so that frequently observed repeating FRBs are broader in duration (smaller Γ_{sh}) on average. There is already some evidence of this being the case if FRB data are interpreted within the shocked gas framework (see fig. 3 of Margalit, Metzger & Sironi 2020). Within the flaring magnetar model, there is an alternative explanation for the proclivity of wider FRBs to repeat often that is more intrinsic to the source. Burst width increases with the density of material around the neutron star, with which the relativistic flare must collide, and therefore width and repeat rate are correlate positively (Margalit et al. 2020).

3.2.2 Rotation period and opening angle

Another explanation for the connection between opening angle and pulse duration comes from models in which beamed emission is sweeping past the observer. It is not known if the beamed radio emission from pulsars forms a circularly symmetric pencil beam (Rankin 1990) or if it is a fan beam that is narrow in the direction transverse to the observer but broad in the orthogonal direction (Michel 1987; Wang et al. 2014; Oswald, Karastergiou & Johnston 2019). Even in the fan beam model, the emission does not span 180 deg in the latitudinal direction, so highly beamed emission is still more difficult to observe. Empirically, Rankin (1990) found that pulsar widths correlate with the neutron star rotation period in seconds P_{sec} as

$$t_i \approx 6.6 \times 10^{-3}\ \mathrm{s}\ \sqrt{P_{sec}}. \qquad (16)$$

The duty cycle, W, which is proportional to the radio beam's opening angle transverse to the observer, scales as $\frac{1}{\sqrt{P}}$. Therefore, old, non-recycled pulsars with large periods have smaller opening angles. It must be noted, however, that there may be selection biases related to periodicity searching and pulsar duty cycle. None the less, it has been proposed that one reason old pulsars become unobservable is not just that they become too faint as they approach the death line, but that their narrow beams are less likely to point in the direction of the observer as their magnetic and spin axes become more aligned (Johnston & Karastergiou 2017).

That scenario is similar to the selection effect we have described in this paper except that in the case of pulsars, older sources have spun down so much that their pulse durations are wider, despite having narrower opening angles. If FRBs are produced by a rotating, pulsar-like object (Cordes & Wasserman 2016; Kumar, Lu & Bhattacharya 2017), then we require wide opening angles to correspond to long-duration bursts, so W and P should not anticorrelate in the same way as Galactic pulsars. Within this framework, our model would also require that each repeat burst is emitted sporadically at a range of pulse phases, since periodicity has not been seen in repeating FRBs on short time-scales (10^{-3}–10^{3} s). One tension with this explanation comes from the fact that the polarization position angle (PA) is known to be flat across the pulse (Michilli et al. 2018; CHIME/FRB Collaboration 2019c), even though a PA swing is expected in the standard rotating vector model. If the emitting region is close to the rotational equator and the beaming angle is much less than a radian, then the PA may be relatively constant, but the magnetic field geometry may offer clues about a possible connection between opening angle and duration.

In Katz (2017), it was suggested that while FRBs are often thought to radiate nearly isotropically and repeat with low duty cycle, it may be that they are almost always emitting, but the emission is beamed and only occasionally so in the direction of the observer. In this 'wandering narrow beam' model, the burst duration depends on both opening angle and the angular speed at which the beam drifts past the line of sight. As long as the distribution of angular speeds does not dominate the observed pulse width, such a scenario would lead to frequent repeaters being of longer duration than once-off FRBs, due to our proposed selection effect.

Relativistic beaming is also expected in models that invoke orbiting planets or asteroids around neutron stars (Mottez & Zarka 2015; Mottez, Voisin & Zarka 2020). It was proposed that FRBs could be generated in the Alfvén wings of orbiting bodies interacting with highly relativistic pulsar winds, analogous to the electrodynamics of the Jupiter–IO system. In their model, the radio flux is concentrated within $\Omega \propto \Gamma_W^{-2}$, where Γ_W is the Lorentz factor of the pulsar wind. The duration of this pulse in the observer frame is determined by the angular size of the beamed emission as it sweeps by the observer, so $t_i \propto \Gamma_W^{-1}$ (Mottez et al. 2020).

3.3 Implications

FRBs can have peak flux densities as large as the brightest single pulses from Galactic pulsars, despite coming from roughly a million times farther away. The 10–15 order-of-magnitude gap in pseudo-luminosity can be explained by beaming, which is central to our model. However, while beamed emission alleviates burst energetics, it exacerbates the already-high volumetric rates of FRBs (Nicholl et al. 2017; Ravi 2019). Therefore, FRB emission models that invoke significant beaming must offer a way of producing $\frac{4\pi}{\Omega}$ times more bursts. We note, however, two useful quantities that are beaming invariant: FRB brightness temperature and the total power emitted by the source population. The former implies that the necessity of coherent emission is not relaxed by beaming, and the latter has implications for the global energy requirements of FRB emitters in the Universe.

In the case of relativistic beaming, it may be possible to detect short-duration, high-Γ FRBs to greater distances, assuming that the rest-frame burst luminosity is the same as broader FRBs. Conversely, frequently repeating FRBs would be closer. We note that the repeater FRB 180916.J0158+65 is at $z \approx 0.034$, just 150 Mpc away (Marcote et al. 2020), and has an average pulse width that is wider than typical CHIME non-repeaters (CHIME/FRB Collaboration 2019c). A repeating FRB detected within ∼30 Mpc would be an ideal system to search for electromagnetic radiation beyond the radio and to precisely study the host environment. Depending on the ability of DM to predict z, a large sample of repeating FRBs may have a lower average DM than once-off events in the CHIME data after accounting for selection effects like dispersion smearing.

If the FRB width distribution continues to show the duration/repetition trend, broad once-off FRBs ought to be followed up to search for repeat bursts, independent of our explanation for the phenomenon's origin. CHIME is a transit instrument and cannot point, but Northern hemisphere telescopes like Apertif (van Leeuwen et al., in preparation) and the Karl G. Jansky Very Large Array (Law et al. 2018) could follow up wide single-detection FRBs found by CHIME, assuming that width is not dominated by scattering or instrumental smearing. For example, FRB 121102 has only been detected once by CHIME and the one detection was very broad, at 34 ms (Josephy et al. 2019). If that source were not already known to be a repeater, FRB 121102 would have been an obvious candidate to follow up based only on the width criteria. As CHIME observed between 400 and 800 MHz, higher frequency coverage could offer interesting insights into frequency dependence of pulse width and repetition activity. There appears to be a width/frequency relation in

FRB 121102, with a shorter time-scale at higher frequencies (Gajjar et al. 2018). That correlation appears to be unrelated to instrumental smearing and may be a useful probe of the FRB emission mechanism and beaming.

The apparent proclivity of once-off FRBs to be shorter in duration and more abundant than longer duration, more repetitive sources has implications for FRB applications. Many proposed methods of using FRBs as probes of cosmology (Madhavacheril et al. 2019; Weltman & Walters 2019), the intergalactic medium (McQuinn 2014; Vedantham & Phinney 2019), or fundamental physics (Muñoz et al. 2016; Eichler 2017) require larger numbers of sightlines. This can be hard to attain if the FRBs to which you are sensitive are frequent repeaters, because the number of distinct sources is inversely proportional to repetition rate for a fixed all-sky FRB rate (10^3 FRBs per sky per day could be produced by just 40 sources with hourly repetition, for example). Therefore, if there exists a population of narrow FRBs that are currently being missed due to instrumental smearing (Connor 2019), surveys that hope to detect a large number of distant sources for FRB applications must mitigate these effects in their telescope back end to go after submillisecond events. Even if the detection rate of a given telescope were not increased by changing its time/frequency resolution, the number of sightlines could increase substantially.

ACKNOWLEDGEMENTS

We thank the essential services members of the Netherlands for allowing us to carry out this research from our homes during the COVID-19 outbreak. The research of LC and DWG was supported by the European Research Council under the European Union's Seventh Framework Programme (FP/2007-2013)/ERC Grant Agreement No. 617199 ('ALERT'). MCM thanks the Radboud Excellence Initiative for supporting his stay at Radboud University. We thank our referee for constructive feedback that has improved our paper. We thank Joeri van Leeuwen, Jason Hessels, and Lorenzo Sironi for helpful conversations. We also thank Ben Margalit, Shu-Qing Zhong, and Jonathan Katz for helpful comments on the first draft of this manuscript.

4 DATA AVAILABILITY

All of the analysis presented in this paper ought to be reproducible using the text in our manuscript and the PYTHON code in our publicly available Jupyter notebook.[2]

REFERENCES

Ashman K. M., Bird C. M., Zepf S. E., 1994, AJ, 108, 2348
Bannister K. W. et al., 2019, Science, 365, 565
Beloborodov A. M., 2017, ApJ, 843, L26
Beloborodov A. M., 2020, ApJ, 896, 142
Berger E., 2014, ARA&A, 52, 43
Caleb M., Stappers B. W., Rajwade K., Flynn C., 2019, MNRAS, 484, 5500
CHIME/FRB Collaboration, 2019a, Nature, 566, 230
CHIME/FRB Collaboration, 2019b, Nature, 566, 235
CHIME/FRB Collaboration, 2019c, ApJ, 885, L24
Connor L., 2019, MNRAS, 487, 5753
Connor L., Petroff E., 2018, ApJ, 861, L1
Connor L., Pen U.-L., Oppermann N., 2016, MNRAS, 458, L89
Cordes J. M., Chatterjee S., 2019, ARA&A, 57, 417
Cordes J. M., Wasserman I., 2016, MNRAS, 457, 232
Eichler D., 2017, ApJ, 850, 159
Farah W. et al., 2018, MNRAS, 478, 1209
Fonseca E. et al., 2020, ApJ, 891, L6
Gajjar V. et al., 2018, ApJ, 863, 2
Gourdji K., Michilli D., Spitler L. G., Hessels J. W. T., Seymour A., Cordes J. M., Chatterjee S., 2019, ApJ, 877, L19
Hessels J. W. T. et al., 2019, ApJ, 876, L23
Johnston S., Karastergiou A., 2017, MNRAS, 467, 3493
Josephy A. et al., 2019, ApJ, 882, L18
Katz J. I., 2017, MNRAS, 467, L96
Kouveliotou C., Meegan C. A., Fishman G. J., Bhat N. P., Briggs M. S., Koshut T. M., Paciesas W. S., Pendleton G. N., 1993, ApJ, 413, L101
Kumar P. et al., 2019, ApJ, 887, L30
Kumar P., Lu W., Bhattacharya M., 2017, MNRAS, 468, 2726
Law C. J. et al., 2018, ApJS, 236, 8
Lyubarsky Y., 2014, MNRAS, 442, L9
Madhavacheril M. S., Battaglia N., Smith K. M., Sievers J. L., 2019, Phys. Rev. D, 100, 103532
Marcote B. et al., 2020, Nature, 577, 190
Margalit B., Metzger B. D., 2018, ApJ, 868, L4
Margalit B., Metzger B. D., Sironi L., 2020, MNRAS, 494, 4627
McQuinn M., 2014, ApJ, 780, L33
Metzger B. D., Margalit B., Sironi L., 2019, MNRAS, 485, 4091
Michel F. C., 1987, ApJ, 322, 822
Michilli D. et al., 2018, Nature, 553, 182
Mottez F., Zarka P., 2015, in SF2A-2015: Proc. Annu. Meeting French Soc. Astron. Astrophys. p. 237
Mottez F., Voisin G., Zarka P., 2020, preprint (arXiv:2002.12834)
Muñoz J. B., Kovetz E. D., Dai L., Kamionkowski M., 2016, Phys. Rev. Lett., 117, 091301
Nicholl M., Williams P. K. G., Berger E., Villar V. A., Alexander K. D., Eftekhari T., Metzger B. D., 2017, ApJ, 843, 84
Oppermann N., Yu H.-R., Pen U.-L., 2018, MNRAS, 475, 5109
Oswald L., Karastergiou A., Johnston S., 2019, MNRAS, 489, 310
Petroff E. et al., 2016, PASA, 33, e045
Petroff E., Hessels J. W. T., Lorimer D. R., 2019, A&AR, 27, 4
Rajwade K. M. et al., 2020, MNRAS, 495, 3551
Rankin J. M., 1990, ApJ, 352, 247
Ravi V. et al., 2019, Nature, 572, 352
Ravi V., 2019, Nat. Astron., 3, 928
Scholz P. et al., 2016, ApJ, 833, 177
Spitler L. G. et al., 2016, Nature, 531, 202
The CHIME/FRB Collaboration, 2020, Nature, 582, 351
Vedantham H. K., Phinney E. S., 2019, MNRAS, 483, 971
Wang H. G. et al., 2014, ApJ, 789, 73
Weltman A., Walters A., 2019, preprint (arXiv:1905.07132)

[2]https://github.com/liamconnor/frb-beaming

This paper has been typeset from a TEX/LATEX file prepared by the author.