

A LOWER LIMIT ON $\Omega_m - \Omega_\Lambda$ USING GRAVITATIONAL LENSING IN THE HUBBLE DEEP FIELD

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ABSTRACT

We calculate the expected number of multiply imaged galaxies in the Hubble Deep Field (HDF), using photometric redshift information for galaxies with $m_I < 27$ that were detected in all four HDF passbands. A comparison of these expectations with the observed number of strongly lensed galaxies places a lower limit on the current value of $\Omega_m - \Omega_\Lambda$, where Ω_m is the cosmological mass density of the universe and Ω_Λ is the normalized cosmological constant. Based on current estimates of the HDF luminosity function and associated uncertainties in individual parameters, our 95% confidence lower limit on $\Omega_m - \Omega_\Lambda$ is between -0.44 , if there are no strongly lensed galaxies in the HDF, and -0.73 , if there are two strongly lensed galaxies in the HDF. For a flat universe ($\Omega_m + \Omega_\Lambda = 1$), $\Omega_\Lambda < 0.58$ – 0.79 (95% confidence limit). If the only lensed galaxy in the HDF is the one presently viable candidate, then in a flat universe ($\Omega_m + \Omega_\Lambda = 1$), $\Omega_\Lambda < 0.79$ (95% confidence limit). These lower limits are compatible with estimates based on high-redshift supernovae and with previous limits based on gravitational lensing.

Subject headings: cosmology: observations — gravitational lensing

1. INTRODUCTION

The Hubble Deep Field (HDF) is the deepest optical survey that has been made to date, enabling a detailed study of the galaxy redshift distribution (e.g., Gwyn & Hartwick 1996) and the global star formation history (e.g., Madau et al. 1996; Madau, Pozzetti, & Dickinson 1998). The HDF covers an area of 4.3 arcmin^2 , with a pixel scale of $0''.04$, and has 5σ point-source detection thresholds of 26.7, 28.8, 30.1, and 30.3 mag measured in the Space Telescope magnitude system in F300W, F450W, F606W, and F814W filters, respectively (e.g., Clements & Couch 1996). In this paper, we use AB magnitudes; in this system, the magnitudes of the detection thresholds are, respectively, 28.0, 29.2, 29.9, and 29.5 mag. Complete observational and data reduction details of the HDF are given by Williams et al. (1996).

Spectroscopic redshifts exist for nearly 180 galaxies in the HDF. These redshifts are now complemented by two photometric redshift catalogs, one based on spectral template fitting (Sawicki, Lin, & Yee 1997, hereafter SLY), and the other on empirical color-redshift relations (Wang, Bahcall, & Turner 1998, hereafter WBT). Here we use the magnitudes for HDF sources derived by SLY and the redshifts derived by SLY and WBT. Galaxies in the HDF have redshifts that are estimated to range from 0.1 to 5, with a large portion having redshifts between 2 and 4. Such galaxies have a significant probability of being strongly lensed. The *Hubble Space Telescope* (HST) has proved to be invaluable to gravitational lens discovery programs because of the high-resolution images it produces (e.g., Ratnatunga et al. 1995). The combination of high resolution and deep exposures in multiple colors provides a rich ground for gravitational lens searches, and it was expected that HDF would contain 3–10 lensed galaxies, based on the number of lensed quasars and radio sources in other surveys (Hogg et al. 1996).

Instead, a careful analysis of the HDF (e.g., Zepf et al. 1997) has revealed a surprising dearth of candidates for lensed sources. In fact, the best estimate is either 0 or 1 lensed sources in the entire field, although very faint images with small angular separations may have escaped current

analyses. This lack of lensing has led to suggestions (e.g., Zepf et al. 1997) that the HDF data may be incompatible with the high probability of lensing expected in a universe with a large cosmological constant.

Here we calculate the expected number of detectable, multiply imaged galaxies in the HDF for different cosmological parameters, and we constrain these parameters by comparing the expectations with the observations. In § 2 we discuss our calculation and its inputs, including the inferred redshifts of the galaxies and the luminosity distribution of potential lenses. In § 3 we present our resulting constraints on cosmological parameters, and in particular on $\Omega_m - \Omega_\Lambda$. In § 4 we discuss the potential effect of systematic errors on the expected and observed number of lensed galaxies. Finally, in § 5 we summarize and discuss future prospects for tighter constraints. We follow the conventions that the Hubble constant, H_0 , is $100 h \text{ km s}^{-1} \text{ Mpc}^{-1}$, the present mean density in the universe in units of the closure density is Ω_m , and the present normalized cosmological constant is Ω_Λ . In a flat universe $\Omega_m + \Omega_\Lambda = 1$.

2. EXPECTED NUMBER OF LENSED GALAXIES IN THE HDF

In this section we describe our calculation of the expected number of lensed galaxies in the HDF. In § 2.1 we discuss the formalism for the calculation, and in § 2.2 we consider input quantities and their errors, such as the photometric redshifts and the luminosity distribution of potential lenses.

2.1. Calculating the Number of Lensed Galaxies

In order to calculate the number of lensed galaxies in the HDF, we model the lensing galaxies as singular isothermal spheres (SIS) and use the analytical filled-beam approximation (e.g., Fukugita et al. 1992). At redshifts $z \lesssim 4$, the analytical filled-beam calculations in Fukugita et al. (1992) agree to better than 2% with numerical calculations (e.g., Fig. 1 in Holz, Miller, & Quashnock 1999).

To calculate the probability of observing strong lensing, two additional effects must be included. First, any magnitude-limited sample such as the HDF is subject to so-called magnification bias (e.g., Kochanek 1991), in which

the number of lensed sources in the sample is larger than it would be in an unbiased sample, because lensing brightens into the sample sources that would otherwise not be detected. This is a particularly pronounced effect in quasar lensing surveys (e.g., Maoz & Rix 1993), because the faint end of the quasar luminosity function rises steeply. Second, because identification of a lensed source requires the detection of at least two of its multiple images, if the second brightest of the images is too faint to detect, such sources will not be identified as strongly lensed. For lens search surveys such as the *HST* Snapshot Survey of bright quasars (Bahcall et al. 1992), this effect is small, because the depth of the pointed search is much greater than the depth of the initial search, and hence almost all secondary images will be detected. In the case of the HDF, it is not possible to do a deeper follow-up, and hence for a significant fraction of sources, especially those near the limiting magnitude of the survey, it will be difficult to detect other lensed companions.

If the probability for a source at redshift z to be strongly lensed is $p(z, \Omega_m, \Omega_\Lambda)$ and the number of unlensed sources in the HDF between rest-frame luminosity L and $L + dL$ and between redshifts z and $z + dz$ is $\Phi(L, z)dL dz$, then the number of lensed sources, $d\bar{N}$, in that luminosity and redshift interval expected in the HDF is (see also Maoz et al. 1992)

$$d\bar{N}(L, z) = p(z, \Omega_m, \Omega_\Lambda) dz \times \int \left[\Phi\left(\frac{L}{A}, z\right) \frac{dL}{A} \right] f(A, L, z) q(A) dA, \quad (1)$$

where the integral is over all allowed values of A , the amplification of the brightest lensed image compared to the unlensed brightness $q(A)$ is the probability distribution of amplifications, and $f(A, L, z)$ is the probability of observing the second-brightest image given A , L , and z . Our assumption that the lenses are singular isothermal spheres implies that the minimum amplification is $A_{\min} = 2$ and the probability distribution is $q(A) = 2/(A - 1)^3$. For simplicity, we will assume that $f(A, L, z)$ is a step function (Θ), so that a dimmer image with apparent magnitude brighter than m_{lim} is detected, whereas one dimmer than m_{lim} is not detected. This assumption means that

$$f(A, L, z) = \Theta \left[m_{\text{lim}} - m_i - 2.5 \log_{10} \left(\frac{A}{A - 2} \right) \right], \quad (2)$$

where m_i is the apparent magnitude of the brighter image and $A/(A - 2)$ is the ratio of the brightness of the primary image to the brightness of the secondary image in the SIS approximation.

We assume that the brightness distribution of galaxies at any given redshift is described by a Schechter function in which the comoving density of galaxies at redshift z and with luminosity between L and $L + dL$ is

$$\phi(L, z)dL = \phi^*(z) \left[\frac{L}{L^*(z)} \right]^{\alpha(z)} e^{-L/L^*(z)} dL, \quad (3)$$

where, as before, both L and L^* are measured in the rest frame of the galaxy. Thus, $\Phi(L, z) = \phi(L, z)dV/dz$, where V is the comoving volume in the solid angle of the HDF. We can then write the expected number \bar{N} of lensed galaxies in our selected subsample of the HDF as

$$\begin{aligned} \bar{N} = & \sum_i p(z_i, \Omega_m, \Omega_\Lambda) \int_2^\infty A^{-1-\alpha(z_i)} e^{L_i/L^*(z_i)} e^{-L_i/AL^*(z_i)} \\ & \times \Theta \left[m_{\text{lim}} - m_i - 2.5 \log_{10} \left(\frac{A}{A - 2} \right) \right] \\ & \times \frac{2}{(A - 1)^3} dA \equiv \sum_i \tau(z_i). \end{aligned} \quad (4)$$

Here the sum is over each of the galaxies in our sample, where we have chosen only those galaxies in the HDF with I magnitudes brighter than 27. The index i represents each galaxy; hence, z_i , L_i , and m_i are, respectively, the redshift, the rest-frame luminosity, and the apparent I magnitude of the i th galaxy.

The quantities z_i and m_i can be measured with relatively little error, but the rest-frame luminosities L_i and $L^*(z_i)$ are more difficult to infer because their value depends on uncertain K -corrections. Therefore, we estimate the total average bias by summing the expectation values of $\tau(z_i)$, which are computed by weighting the integral in equation (4) by a normalized distribution of luminosities L_i drawn from the Schechter function appropriate for the redshift z_i of galaxy i , instead of using inferred rest-frame luminosities that are, in any case, very uncertain. We used the tabulated values for Schechter luminosity function in SLY and calculated the magnification bias for individual redshift intervals for which the Schechter function parameters are available in Table 1 of SLY. In principle, the uncertainties in the Schechter function parameters at a given redshift can affect the calculation of the bias, but in practice only the uncertainty in the power-law slope α has a significant effect; in § 3.2 we calculate the result of varying α by the quoted errors for source galaxies with redshifts between 2 and 3, which dominate the expected incidence of lensing.

2.2. Uncertainties in Inputs

2.2.1. Redshifts in Galaxies

We have used two available photometric redshift catalogs for galaxies in the HDF: SLY used spectral fitting techniques to calculate the redshift for all galaxies that appeared in the four HDF passbands with an I -band limiting magnitude of 27. There are 848 such galaxies, 181 of which have spectroscopic redshifts. Recently, WBT computed photometric redshifts for the same sample as SLY, based on empirical relations that were calibrated against spectroscopic redshifts. In Figure 1 we show the redshift distribution of the HDF galaxies according to the SLY (*left panel*) and WBT (*right panel*) catalogs, as a function of the I -band magnitude. These distributions are similar to that found by Gwyn & Hartwick (1996) and contain two peaks, with one at $z \sim 0.6$ and the other at $z \sim 2.3$. Compared to the spectroscopic redshifts, the photometric redshifts of SLY have a larger scatter than the redshifts of WBT. However, as is clear from Figure 1, there is a pronounced (and most probably spurious) lack of photometric redshifts between 1.5 and 2.2 in the WBT catalog, in the same redshift range for which no spectroscopic redshifts are currently available for the HDF. This gap is much less dramatic in the SLY catalog. It is therefore uncertain which catalog is more reliable overall. We therefore estimate cosmological constraints based on both catalogs, but we find (see § 3) that the derived constraints are almost the same for either catalog.

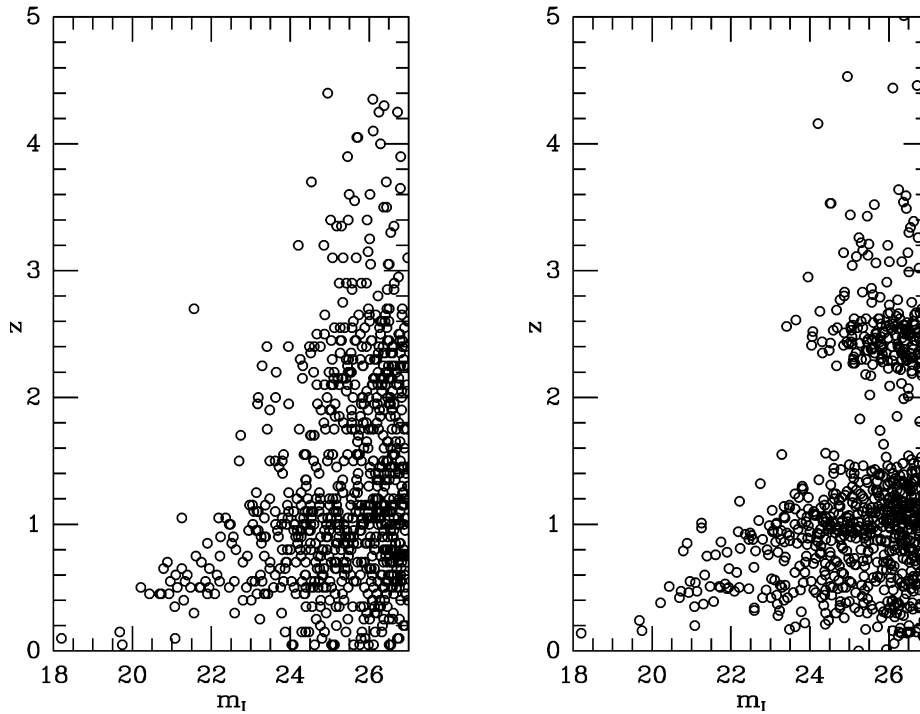


FIG. 1.—Redshift distribution of 848 galaxies with I -band magnitude $\lesssim 27$ in the HDF. The plot shows the photometric redshifts derived by SLY (*left panel*) and WBT (*right panel*) as a function of the I -band magnitude. Both catalogs appear to trace the same redshift distribution, with two peaks ($z \sim 0.6$ and $z \sim 2.3$). However, there is a lack of galaxies in the WBT catalog between $z \sim 1.5$ and $z \sim 2.2$. This is the same range in redshift for which no spectroscopic redshifts are currently available for the HDF.

2.2.2. Properties of Lensing Galaxies

The overall probability of strong lensing depends on the number density and typical mass of lensing galaxies. For singular isothermal spheres, with comoving number density n_0 and velocity dispersion σ , this factor is given by the dimensionless parameter

$$F \equiv 16\pi^3 n_0 R_0^3 \left(\frac{\sigma}{c}\right). \quad (5)$$

where $R_0 \equiv c/H_0$. The parameter F is independent of the Hubble constant, because the observationally inferred number density is itself proportional to h^3 . Note that the probability of lensing, $p(z, \Omega_m, \Omega_\Lambda)$, is directly proportional to F .

At any given redshift, we can estimate F directly from the galaxy luminosity function at that redshift if we know the dependence of the velocity dispersion on the luminosity. A commonly assumed functional form is $L \propto \sigma^\gamma$, so that $\sigma^4 \propto L^{4/\gamma}$. Kochanek (1996) estimates $\gamma = 4.0 \pm 0.5$ and adopts a velocity dispersion for an L^* galaxy in the local universe of $\sigma = 220 \pm 20 \text{ km s}^{-1}$. If the luminosity function at redshift z is given by equation (3), then with the normalization for the velocity dispersion given by Kochanek (1996), we find, by integrating equation (5) over the luminosity function at redshift z , that

$$F = 3.87 \left[\frac{L^*(z)}{L^*(0)} \right] \phi^*(z) \Gamma\left(\alpha + \frac{4}{\gamma} + 1\right), \quad (6)$$

where Γ is the normal gamma function. We assume that the luminosity at which $\sigma = 220 \text{ km s}^{-1}$ corresponds to a B magnitude of $M_B^* = -20.7$. Henceforth, we will also assume $\gamma = 4$.

To estimate F , we concentrate on the redshifts z between 0.5 and 1.0, because foreground lensing galaxies for most of the presently confirmed lensed sources are in this redshift range (e.g., Kochanek 1996). SLY find that in this redshift range, the HDF luminosity function for galaxies is represented by a Schechter function with the following parameters:

$$M_B^* = -19.9 \pm 0.3, \quad \alpha = -1.3 \pm 0.1, \\ \phi^* = 0.042 \pm 0.013 h^3 \text{ Mpc}^{-3}. \quad (7)$$

The best estimate for F is 0.05, if 30% of galaxies in the HDF are ellipticals. If the errors in the parameters were independent of each other, the uncertainty in F would be at least a factor of 2. However, the parameters in the Schechter luminosity function are correlated. For example, in the Century Survey (Geller et al. 1997) the joint error ellipses for α and M_R^* show that, in the R band, $\alpha \approx -1.2 + (M_R^* + 20.7)$. It is therefore plausible that a similar relation holds for the HDF luminosity function parameters. From Table 1 of SLY, we find that, for z between 0.5 and 1.0, $\alpha \approx -1.3 + 0.3(M_B^* + 19.9)$. In addition, the absolute normalization is well determined at the faint end of the luminosity function, where there are many galaxies. If we fix the number density at $0.01L^*$, where galaxies are numerous yet still bright enough to be detected reliably, then a variation in α determines the values of M_B^* and ϕ^* . The 1σ range in α then gives values of F in a tight range, between 0.100 and 0.108. However, the correlation between α and M_B^* is inexact; furthermore, other uncertainties, such as in the velocity dispersion σ , must also be considered. To be conservative and account for systematic uncertainties in the form of the σ - L relation and in the joint errors of the

Schechter parameters, we allow for an overall uncertainty of 30% and take $F = 0.050 \pm 0.015$.

3. CONSTRAINTS ON COSMOLOGICAL PARAMETERS

3.1. Observed Number of Lensed Galaxies in the HDF

The first potential detection of a gravitationally lensed source in the HDF is described in Hogg et al. (1996). However, the candidacy of this object was later questioned by Zepf et al. (1997), who made Keck spectroscopic observations of three lenslike objects in the HDF. These three sources were selected based on the morphological distribution of nearly 750 galaxies, down to a limiting magnitude of 27 in the I band. Discrepancies in the inferred redshifts of the multiple images suggested that two of the three candidates are not lensed sources, leaving one possible gravitational lens in the HDF down to I of 27 (L3.2 in Zepf et al. 1997). It is possible that even this source is not strongly lensed, but currently the data are inconclusive. Zepf et al. (1997) argue that it is unlikely that there is a large population of lensed sources in the HDF that has been missed. This is especially true for sources brighter than $m_I = 27$, 1.5 mag above the detection threshold, m_{lim} , of 28.5 in the I band. However, in order to take into account the possibility that multiply imaged sources with faint secondary images (which still have $m_I < 28.5$) or small angular separations have been missed, we calculate cosmological limits based on the assumed detection of zero, one, and two lensed sources with secondary images brighter than $m_{\text{lim}} = 28.5$, $m_{\text{lim}} = 28.0$, and $m_{\text{lim}} = 27.5$ in the HDF. As we discuss in more detail in the next section, the limits that follow from the detection of at least one lensed source require knowledge of the redshift of the background lensed source. The single candidate of Zepf et al. (1997) has an uncertain redshift, but the redshift is most probably in the range $1.0 \lesssim z \lesssim 2.5$, with a best guess of 1.02.

3.2. Comparing the Expected with the Observed Number of Lensed Galaxies

We have calculated the expected number, \bar{N} , of detectable, multiply imaged galaxies in the HDF, using equation (4) for various combinations of Ω_m , Ω_Λ , m_{lim} , and α , the luminosity function power-law slope for galaxies with redshifts between 2 and 3 (the redshift range expected to produce most of the lensed galaxies in the HDF). We perform each calculation using both the SLY and WBT catalogs. Figure 2 shows the expected number of gravitational lenses in the HDF as a function of Ω_m and Ω_Λ , assuming $m_{\text{lim}} = 28.5$ and $\alpha = -2.1$, and using the SLY photometric redshifts. A universe dominated by Ω_Λ has a higher number of multiply imaged sources than in a universe dominated with a large Ω_m . As shown in Figure 2, \bar{N} is essentially a function of the combined quantity $\Omega_m - \Omega_\Lambda$. This degeneracy in the lensing probability (Carroll, Press, & Turner 1992; Kochanek 1993; Holz et al. 1999) allows us (see below) to constrain $\Omega_m - \Omega_\Lambda$ rather than Ω_m or Ω_Λ individually. In Table 1 we list the expected number of strongly lensed galaxies in the HDF along the $\Omega_m + \Omega_\Lambda = 1$ line as a function of $\Omega_m - \Omega_\Lambda$. These expected numbers, which are listed for the two catalogs separately, have been calculated assuming $m_{\text{lim}} = 28.5$ and $\alpha = -2.1$.

3.3. Likelihood Constraints on $\Omega_m - \Omega_\Lambda$

We constrain the quantity $\Omega_m - \Omega_\Lambda$ by comparing the observed and expected number of lensed galaxies in the

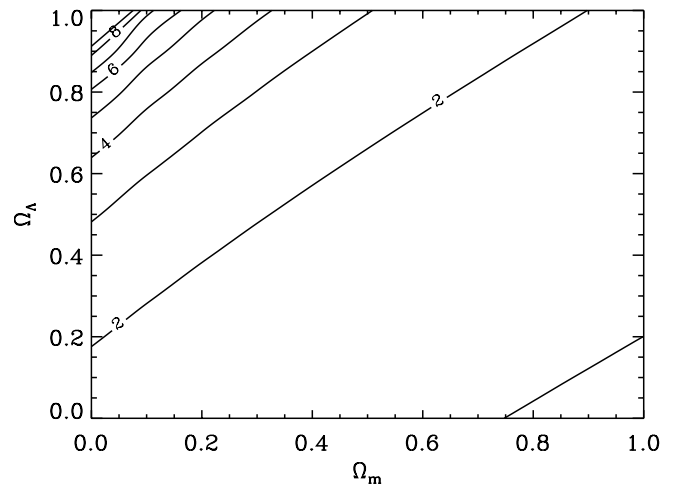


FIG. 2.—Expected number of multiply imaged galaxies, \bar{N} , in the HDF as a function of Ω_m and Ω_Λ . \bar{N} is constant along lines of constant $\Omega_m - \Omega_\Lambda$, allowing for direct constraints on this quantity. Shown here is the expected number based on the SLY catalog and on lens search programs that have been carried out to a limiting magnitude of $m_{\text{lim}} = 28.5$ in HDF I -band images.

HDF. We adopt a Bayesian approach, and take a prior for $\Omega_m - \Omega_\Lambda$ that is uniform between -1 and $+1$. We do this because we do not yet have a precise determination of this quantity (although recent high-redshift supernova measurements favor a value near -0.5 ; e.g., Riess et al. 1998), and we do not wish to consider cosmologies in which either Ω_m or Ω_Λ lies outside the interval $[0, 1]$. We do not constrain Ω_m or Ω_Λ separately; thus, no prior is required for these quantities. Since the prior for $\Omega_m - \Omega_\Lambda$ is uniform, the posterior probability density is simply proportional to the likelihood (see below).

From equation (4), $\bar{N} = \sum_i \tau(z_i)$, where $\tau(z_i)$ is the effective probability that a galaxy at redshift z_i is lensed. Here, $\tau(z_i)$ and \bar{N} depend on $\Omega_m - \Omega_\Lambda$ and are directly proportional to

TABLE 1
EXPECTED NUMBER OF LENSED GALAXIES IN THE HDF

$\Omega_m - \Omega_\Lambda$	SLY	WBT
-1.0.....	14.3	15.8
-0.9.....	8.2	9.0
-0.8.....	5.4	5.8
-0.7.....	4.3	4.6
-0.6.....	3.5	3.7
-0.5.....	3.0	3.2
-0.4.....	2.6	2.8
-0.3.....	2.3	2.4
-0.2.....	2.0	2.2
-0.1.....	1.8	2.0
0.0.....	1.7	1.8
0.1.....	1.6	1.6
0.2.....	1.4	1.5
0.3.....	1.3	1.4
0.4.....	1.2	1.3
0.5.....	1.2	1.2
0.6.....	1.1	1.1
0.7.....	1.0	1.0
0.8.....	1.0	1.0
0.9.....	0.9	0.9
1.0.....	0.9	0.9

the dimensionless parameter F defined in equation (5). Furthermore, we take into account the uncertainty in F by defining $\mathcal{F} \equiv F/0.05$ and taking \mathcal{F} to have a mean of unity and standard deviation $\sigma_{\mathcal{F}} = 0.3$. Thus, we allow for a 30% uncertainty in F (see § 2). The factor \mathcal{F} is then an overall correction to the lensing probability, due to a systematic uncertainty in F .

The likelihood \mathcal{L} , a function of $\Omega_m - \Omega_\Lambda$, is the probability of the data, given $\Omega_m - \Omega_\Lambda$. If there are no lensed galaxies in the HDF, and if \mathcal{F} were known exactly, this probability would be

$$\begin{aligned} \mathcal{L}(0) &= \prod_i e^{-\mathcal{F}\tau(z_i)} = e^{-\mathcal{F}\bar{N}} \\ &= e^{-\bar{N}} e^{-(\mathcal{F}-1)\bar{N}}. \end{aligned} \quad (8)$$

We account for the uncertainty in F by marginalizing the above expression over $\mathcal{F} = 1.0 \pm \sigma_{\mathcal{F}}$, expanding the exponential, and taking expectation values. To second order in $\sigma_{\mathcal{F}}$, we find

$$\langle \mathcal{L}(0) \rangle = e^{-\bar{N}} \left(1 + \sigma_{\mathcal{F}}^2 \frac{\bar{N}^2}{2} \right). \quad (9)$$

We include only contributions from the variance in \mathcal{F} to the expectation value, since we do not know the exact distribution (and higher moments) of \mathcal{F} .

If instead there is one lensed galaxy (at redshift z_j) in the HDF, and if \mathcal{F} were known exactly, the likelihood would be

$$\begin{aligned} \mathcal{L}(1) &= \mathcal{F}\tau(z_j) \prod_i e^{-\mathcal{F}\tau(z_i)} = \tau(z_j)\mathcal{F}e^{-\mathcal{F}\bar{N}} \\ &= \tau(z_j) \left[-\frac{\partial \mathcal{L}(0)}{\partial \bar{N}} \right]. \end{aligned} \quad (10)$$

By again marginalizing over \mathcal{F} and substituting equation (9) into equation (10), we obtain

$$\langle \mathcal{L}(1) \rangle = \tau(z_j) e^{-\bar{N}} \left[1 + \sigma_{\mathcal{F}}^2 \left(\frac{\bar{N}^2}{2} - \bar{N} \right) \right]. \quad (11)$$

In general, if there are n lensed galaxies (at redshifts z_j) in the HDF, then we find

$$\langle \mathcal{L}(n) \rangle = \prod_{j=0}^n \tau(z_j) e^{-\bar{N}} \left\{ 1 + \sigma_{\mathcal{F}}^2 \left[\frac{\bar{N}^2}{2} - n\bar{N} + \frac{n(n-1)}{2} \right] \right\}. \quad (12)$$

In order to constrain $\Omega_m - \Omega_\Lambda$, we calculate the likelihood for cases in which the number n of lensed galaxies present in the HDF is 0, 1, or 2, and for $m_{\text{lim}} = 28.5, 28.0$, and 27.5 . In order to examine the effect of the luminosity function slope α , which has an important effect on magnification bias (eq. [4]), we have also calculated the likelihood by varying α by ± 0.1 (the quoted error in SLY) from the best estimate of -2.1 (for galaxies with redshifts between 2 and 3, which dominate the expected incidence of multiple imaging).

In Table 2, we present the 95% confidence lower limits on $\Omega_m - \Omega_\Lambda$ for various cases. We define a canonical case in which the lens search has been carried out to $m_{\text{lim}} = 28.5$, and has found one lensed galaxy ($n = 1$) in the HDF with a

TABLE 2
95% CONFIDENCE LOWER LIMITS ON $\Omega_m - \Omega_\Lambda$

Parameters	SLY	WBT
$m_{\text{lim}} = 28.5, \alpha = -2.1, n = 1 (z_s = 2.5) \dots\dots$	-0.58	-0.56
$m_{\text{lim}} = 28.5, \alpha = -2.1, n = 0 \dots\dots\dots$	-0.44	-0.42
$m_{\text{lim}} = 28.5, \alpha = -2.1, n = 1 (z_s = 1.0) \dots\dots$	-0.52	-0.49
$m_{\text{lim}} = 28.5, \alpha = -2.1, n = 2 (z_s = 2.5) \dots\dots$	-0.73	-0.70
$m_{\text{lim}} = 28.5, \alpha = -2.0, n = 1 (z_s = 2.5) \dots\dots$	-0.64	-0.62
$m_{\text{lim}} = 28.5, \alpha = -2.2, n = 1 (z_s = 2.5) \dots\dots$	-0.56	-0.54
$m_{\text{lim}} = 28.0, \alpha = -2.1, n = 1 (z_s = 2.5) \dots\dots$	-0.63	-0.60
$m_{\text{lim}} = 27.5, \alpha = -2.1, n = 1 (z_s = 2.5) \dots\dots$	-0.70	-0.67

redshift $z_s = 2.5$. We take $\alpha = -2.1$ in the canonical case. In order to test the effect of a different number of lensed galaxies observed in the HDF, we vary n from this canonical scenario and assume that all lensed galaxies are at a redshift of 2.5 (this gives the weakest lower limit). The cumulative probabilities for the observed number of lenses, as a function of $\Omega_m - \Omega_\Lambda$, are shown in Figure 3, where the plotted curves represent no lensed galaxies in HDF (*solid line*), one lensed galaxy (*short-dashed line*) and two lensed galaxies (*long-dashed line*). If there are no lensed galaxies in the HDF, then at the 95% confidence level $\Omega_m - \Omega_\Lambda > -0.44$, so that in a flat universe $\Omega_\Lambda < 0.72$. If there is one lensed galaxy in the HDF, our constraints depend only slightly on the galaxy redshift. If the galaxy redshift is 1, then $\Omega_m - \Omega_\Lambda > -0.52$, implying $\Omega_\Lambda < 0.76$ in a flat universe. If instead the galaxy redshift is 2.5, then $\Omega_m - \Omega_\Lambda > -0.58$, and hence $\Omega_\Lambda < 0.79$ in a flat universe.

As tabulated in Table 2, the change in I -band lens search magnitude limit from 28.5 to 27.5 has a surprisingly small effect on the limits on $\Omega_m - \Omega_\Lambda$; the effect of the non-detection of secondary images beyond the limiting magnitude is compensated to some extent by the effect of magnification bias. Except in the case where two strongly lensed galaxies

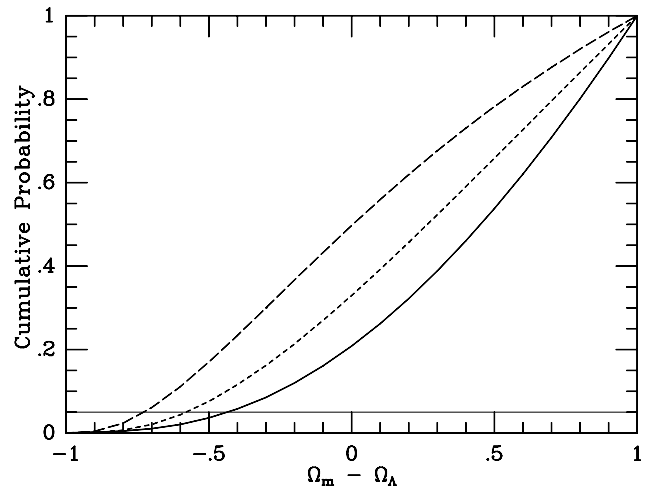


FIG. 3.—Cumulative probability distribution for $\Omega_m - \Omega_\Lambda$ if there is no lensed galaxy (*solid line*), one lensed galaxy (*short-dashed line*), and two lensed galaxies (*long-dashed line*) in the HDF, down to a lens search limiting magnitude of $m_{\text{lim}} = 28.5$ in I -band images (using photometric redshifts from the SLY catalog). The intercepts with the horizontal line show the 95% confidence lower limits on $\Omega_m - \Omega_\Lambda$ (see Table 2).

are present in the HDF, $\Omega_m - \Omega_\Lambda > -0.70$ at the 95% confidence level. This implies that in a flat universe $\Omega_\Lambda < 0.85$, which is consistent with the recent cosmological parameter constraints based on the high-redshift Type Ia supernovae (Riess et al. 1998) and with previous limits on the cosmological constant based on gravitational lensing (e.g., Falco, Kochanek, & Munoz 1998; Kochanek 1996; Chiba & Yoshi 1997).

4. SYSTEMATIC ERRORS

4.1. *Errors Affecting the Expected Number of Lensed Galaxies*

4.1.1. *Errors in the Photometric Redshifts and in F*

Our calculations rely on the accuracy of photometric redshifts, and hence errors in these redshifts produce errors in the estimated number of lensed galaxies. The dispersion of redshifts in, e.g., the WBT catalog with respect to spectroscopically measured ones, range from 0.03 to 0.1 for $z \lesssim 2$ and from 0.14 to 0.36 for $z \gtrsim 2$. The largest effect this could have on the expected number of lensed galaxies would occur if the redshifts were *all* systematically low or high by an amount equal to the quoted error. Even in such an extreme case, the 95% confidence lower limit on $\Omega_m - \Omega_\Lambda$ in our canonical case would only range from -0.64 to -0.50 , using, respectively, redshifts that are all 1σ low or 1σ high compared to the best estimates in the WBT catalog. This is actually a tremendous overestimate of the effect of errors in the photometric redshifts. In reality, a comparison of the photometric redshifts of WBT with available spectroscopic redshifts indicates that the errors are evenly distributed between high and low estimates, so the overall expected number of lensed galaxies is barely affected. The only systematic effect visible in the WBT catalog is a paucity of galaxies in the redshift interval 1.5–2.2. This has a small but visible effect on our $\Omega_m - \Omega_\Lambda$ limits; the results for the WBT catalog are larger than those of the SLY catalog by $\sim 4\%$. This increase is primarily due to the peaked distribution of galaxies between redshifts of 2 and 2.5 in the WBT catalog, while the same galaxies have a much broader distribution in the SLY catalog. In addition, about 10 galaxies in the WBT catalog with $z \sim 2.5$ have $z < 1$ in the SLY catalog. Nonetheless, the two catalogs yield expectations in the number of lensed galaxies that are almost indistinguishable from each other.

Another possible systematic error has to do with the way that the F parameter (eq. [5]) is estimated. One method is to use extensive local surveys of the galaxy luminosity function and velocity dispersions to calculate F , then assume that because most foreground lenses are at a redshift less than unity and galaxy evolution out to that redshift is not believed to be dramatic enough to change F significantly (see Mao & Kochanek 1994), the value of F to use in lensing calculations is the same as it is locally. This is the approach we adopt, and it is supported by the luminosity function in the HDF itself (see § 2.2). Note, however, that if F were to be inferred solely from observations of a particular field, such as the HDF, then because the inferred number density and luminosity of galaxies depends on the assumed cosmology, so will F . The dependence of the inferred F on cosmology also depends on the relation between luminosity and velocity dispersion, but if $\gamma = 4$, the effect is that F is smaller when $\Omega_\Lambda > 0$ than when $\Omega_m = 1$ and $\Omega_\Lambda = 0$, as is often

assumed. If $\Omega_m - \Omega_\Lambda \sim -0.4$, the effect is to decrease F by $\sim 30\%$ for lenses between $z = 0.5$ and $z = 1$. If the field of interest contains a small enough number of potential lenses so that fluctuations are important and F must be derived from that field, this effect may have to be included. In the case of the HDF, however, the number counts are large and the expected level of fluctuations is small, so we assume that F is the same as it is locally.

4.1.2. *Errors in Calculating the Number of Lensed Galaxies*

Another source of systematic error in our study is that we have used the analytical filled-beam expression to estimate the probability of lensing at high redshift. This calculation underestimates the true probability based on numerical techniques by about 2% for the galaxy redshifts of the HDF (Holz et al. 1999). Thus, we have underestimated the expected number of lensed galaxies in the HDF. This is a systematic error in our calculation, and it implies that our lower limits on $\Omega_m - \Omega_\Lambda$ have been underestimated by a similar amount.

We have also assumed that lensing galaxies can be described by isothermal spheres. However, it is likely that galaxies have a nonnegligible core radius. Such a core radius can decrease the expected number of lensed galaxies present in the HDF compared to the number calculated assuming a zero core radius. Kochanek (1996) addressed the issue of a finite core radius by studying lens models described by softened isothermal spheres and showed that a finite core radius increases the velocity dispersion; thus, the decrease in lensing probability due to an added core radius is compensated by the increase in velocity dispersion. In addition, the observed core radii of E and S0 galaxies that dominate lensing are much smaller than their Einstein radii, and hence the effect of a finite core radius is small (Kochanek 1996). We therefore do not expect the effect of finite galaxy core radii to dramatically change the lower limits on $\Omega_m - \Omega_\Lambda$ presented in this paper.

4.2. *Errors Affecting the Observed Number of Lensed Galaxies*

4.2.1. *Reddening Effects*

Given that the light from a lensed galaxy must pass near or through a lensing galaxy, the column depth of dust in the foreground of multiply imaged galaxies may be systematically higher than for the field as a whole. Hence, the extinction of images could be of considerable importance in lens studies. Most lensing galaxies are in a redshift range $z \sim 0.5$ – 1.0 , so it is the properties of galaxies at those redshifts that are most important in evaluating the likely effect of extinction.

By comparing radio-selected and optically selected lens samples, Malhotra, Rhoads, & Turner (1997) suggested that optical lens searches are heavily impaired by extinction due to dust, with a mean magnitude change of 2 ± 1 mag between images. A different conclusion was reached by Falco et al. (1998), who suggested that extinction from dust only produces a mean magnitude change of ~ 0.5 . The conclusion of Falco et al. (1998) that extinction has a minor impact on lensing studies is supported by Kochanek (1996), who presented a reddening model in lensing statistics and determined that extinction in lensing galaxies decreases the probability of observing multiple lensing by only $\sim 10\%$. This calculation concentrated on the $z \sim 0$ elliptical gal-

axes, which are thought to be fairly similar to the galaxies at $z \sim 0.5$ – 1.0 that are doing most of the lensing, although the inferred increase in the massive star formation rate near $z \sim 1$ (e.g., Madau et al. 1998) may imply somewhat higher extinction than at low redshifts.

Given that there is no detailed analysis of the dust distribution associated with HDF galaxies, we have not included the effects of reddening due to dust in the present calculation. However, our variation of the detection threshold magnitude, m_{lim} , in Table 2 is indicative of the possible effects of extinction on our results. We find that even a change as large as 1 mag has a fairly minor effect on our limits.

4.2.2. Source Confusion and Image Separation Effects

The clustering of optical galaxies may confuse lens search programs in two ways: On the one hand, clustering can lead one to infer the existence of multiply lensed galaxies under the assumption that observed galaxies are images of a lensed background object, when in fact the images are unrelated; on the other hand, clustering of galaxies can confuse lensed-image identification by increasing the surface brightness of the surrounding regions near the images. The two gravitationally lensed sources found by Ratnatunga et al. (1995) using *HST* WFPC images have image separations of order of $\sim 1''.2$ – $1''.5$. For presently confirmed lenses, the effective diameter of the foreground lensing galaxies at $z \sim 1$ are of order $\sim 1''.5$, similar to lensed image separations. Thus, galaxy clustering may impair the detection of lensed sources. This effect may in fact be present in the currently confirmed lensed source sample, where almost all of the lensed sources with small-image separations were initially selected in radio searches. However, gravitationally lensed galaxies typically have image shapes that can be differentiated from random clustering of galaxies, and generally have colors different from foreground galaxies. Therefore, a careful examination of the HDF, including both colors and positions, should efficiently reveal lensed sources. The recent discovery of about 10 new small separation lensed sources in the *HST* Medium Deep Survey (K. Ratnatunga 1998, private communication) suggests that efficient lens searches can be and have been made. Therefore, we do not expect that a large number of HDF lenses have been missed because of galaxy clustering or source confusion.

5. SUMMARY AND CONCLUSIONS

By comparing the expected number of lensed galaxies in the HDF to the observed number of lensed galaxies, we have presented limits on the cosmological parameters. We have considered the possibility that present lens search pro-

grams may have missed one to three possible lensed galaxies in the HDF and have given our cosmological parameter constraints for a variety of cases, including the change in limiting magnitude for lens searches.

We find that the expected number of lenses in the HDF is primarily a function of $\Omega_m - \Omega_\Lambda$. A comparison of the expected number of lensed galaxies with the observed number allows us to put a lower limit on the current value of $\Omega_m - \Omega_\Lambda$. Making use of the HDF luminosity function (as determined by SLY), our 95% confidence lower limit on $\Omega_m - \Omega_\Lambda$ ranges between -0.44 and -0.73 (see Table 2). If the only lensed galaxy in the HDF is the one candidate found by Zepf et al. (1997), then $\Omega_m - \Omega_\Lambda > -0.58$. These lower limits are not in conflict with estimates based on high-redshift supernovae (viz., $\Omega_m - \Omega_\Lambda \sim -0.5 \pm 0.4$ [Riess et al. 1998]).

As has been recently noted in the literature, combining $\Omega_m - \Omega_\Lambda$ results from high-redshift supernova measurements with $\Omega_m + \Omega_\Lambda$ results from CMB power spectrum analysis constrains Ω_m and Ω_Λ separately, with much higher accuracy than the individual experiments alone (White 1998; Eisenstein, Hu, & Tegmark 1998). We note that gravitational lensing constraints on $\Omega_m - \Omega_\Lambda$ should also be considered in such an analysis.

We have shown, by comparing our results from two different catalogs, that photometric redshifts can be used to estimate the expected number of lensed galaxies in the HDF with reasonable accuracy. This bodes well for the upcoming Southern Hubble Deep Field redshift catalog that is expected in the near future. The Southern HDF will double the number of high-redshift galaxies and will thus double the expected number of gravitationally lensed galaxies. If, for example, $\Omega_m - \Omega_\Lambda \sim -0.5$, then from Table 1 there could be as many as three lensed galaxies detected in the Southern HDF. The actual number of detected galaxies will lead to strong constraints on $\Omega_m - \Omega_\Lambda$.

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