Merger rates in primordial black hole clusters without initial binaries

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ABSTRACT

Primordial black holes formed through the collapse of cosmological density fluctuations have been hypothesized as contributors to the dark matter content of the Universe. At the same time, their mergers could contribute to the recently observed population of gravitational-wave sources. We investigate the scenario in which primordial black holes form binaries at late times in the Universe. Specifically, we re-examine the mergers of primordial black holes in small clusters of $\sim$30 objects in the absence of initial binaries. Binaries form dynamically through Newtonian gravitational interactions. These binaries act as heat sources for the cluster, increasing the cluster's velocity dispersion, which inhibits direct mergers through gravitational-wave two-body captures. Meanwhile, three-body encounters of tight binaries are too rare to tighten binaries sufficiently to allow them to merge through gravitational-wave emission. We conclude that in the absence of initial binaries, merger rates of primordial black holes in the considered scenario are at least an order of magnitude lower than previously suggested, which makes gravitational-wave detections of such sources improbable.

Key words: gravitational waves – cosmology: dark matter – black hole mergers.

1 INTRODUCTION

The concept of black holes formed directly from the gravitational collapse of the cosmological density fluctuations in the early Universe, known as primordial black holes (PBHs), dates back to the 1970s (Hawking 1971). If the density fluctuations collapsed before $\sim$1 s after the big bang, the baryons that produced them would be locked up and thus there would not be a conflict with light element nucleosynthesis. Then, PBHs would behave as non-baryonic cold dark matter (DM) throughout the subsequent evolution of the Universe. The large variety of mechanisms that could have produced the density fluctuations that seed PBHs yield possible mass functions that extend from the Planck mass to the mass of a galaxy cluster (see Carr, Kühnel & Sandstad 2016; Carr et al. 2020, for reviews). In principle, all of the DM in the Universe could be PBHs. The abundance of PBHs in different mass regimes have been strongly contested in recent years by a number of astrophysical and cosmological experiments, leaving only three mass windows in which PBHs could still provide an important contribution to the DM: asteroid mass PBHs (10^{16}–10^{17} g), sublunar mass PBHs (10^{20}–10^{26} g), and stellar mass PBHs (20–100 $M_\odot$; Carr et al. 2020).

Recently, detections of gravitational waves (GWs) from merging black holes of 10–40 $M_\odot$ by Advanced Laser Interferometer Gravitational-wave Observatory (LIGO) and Virgo have returned attention to stellar mass PBHs (Abbott et al. 2016; Abbott et al. 2019). Shortly after the first LIGO detection (GW150914), several authors suggested that it might have been a merger of PBHs. There are a number of ways to produce PBH binaries. These can be divided roughly into two channels: initial binaries produced in the early Universe (e.g. Sasaki et al. 2016; Ali-Haïmoud, Kovetz & Kamionkowski 2017) and binaries formed much later through dynamical interactions (e.g. Bird et al. 2016; Clesse & García-Bellido 2017). In the initial binaries channel, PBHs can form in binaries as a result of chance proximity, as first described in Nakamura et al. (1997). In the second channel, single PBHs that formed in the early Universe form binaries much later due to dynamical interactions of PBHs in DM haloes. We note that constraints on PBH abundance from GW observations are based

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mostly on the early Universe scenario because later production of PBH binaries is highly subdominant (e.g. Ali-Haïmoud et al. 2017). However, because the early Universe channel has a number of caveats and uncertainties, in this paper we revisit the dynamical interaction channel.

Let us assume that $30\,M_\odot$ PBHs form clusters, and that PBH clusters have the large-scale structure distribution of DM haloes (e.g. the Press & Schechter 1974 mass function). For each PBH cluster, there is a finite probability that two initially unbound PBHs pass close enough to gravitationally capture each other and form a bound binary system through the emission of a burst of GWs at periapsis. Following this idea, Bird et al. (2016) derived a PBH capture rate between 10^{-4} and $1400\,\mathrm{Gpc}^{-3}\,\mathrm{yr}^{-1}$ depending on assumptions about how PBHs cluster in DM haloes. This broad range encompasses the best estimate based on all the events of the first and the second advanced detector observing runs, $53.2^{+58.2}_{-28.8}\,\mathrm{Gpc}^{-3}\,\mathrm{yr}^{-1}$ (Abbott et al. 2019). In particular, Bird et al. (2016) find that the major contribution to the total merger rate of PBH binaries comes from clusters of $\sim 10^3\,M_\odot$, while smaller DM clusters have too few PBHs and evaporate promptly due to weak gravitational interactions. Gravitational two-body captures are more efficient in low-mass PBH clusters because PBHs in low-mass clusters move more slowly and because small clusters are more concentrated. While the first statement is based on dynamical considerations, the second statement is a consequence of the hierarchical formation of DM haloes in the ΛCDM cosmological model: because low-mass haloes assemble earlier, when the mean density of the Universe is higher, they have higher concentrations than high-mass haloes (e.g. Navarro, Frenk & White 1997; Wechsler et al. 2002).

Ali-Haïmoud et al. (2017, see also earlier work of Afshordi, McDonald & Spergel 2003) pointed out that due to the discrete nature of PBHs, in clusters composed of $\lesssim 1000$ PBHs, Poisson perturbations dominate over standard adiabatic perturbations (considered in Bird et al. 2016). This implies that low-mass PBH clusters form much earlier, and therefore they are also denser. As a consequence, using scalings appropriate for large-N dynamical systems, Ali-Haïmoud et al. (2017) conclude that clusters with $\lesssim 3 \times 10^4\,M_\odot$ also evaporate by the present time and do not contribute to the merger rate of PBHs.

In low-mass PBH clusters, binaries can also form through non-dissipative three-body interactions, in which one PBH removes enough kinetic energy to leave the other two in a bound state. Once formed, binaries serve as a source of energy in a cluster (Heggie & Hut 1993). By interacting with single PBHs (and other binaries, if present) they heat the cluster, which then expands and partially evaporates (i.e. loses objects). This enhances the rate of evaporation relative to that obtained by only considering weak gravitational interactions, as in Bird et al. (2016). Moreover, cluster expansion causes the density of the cluster and the merger rate through two-body capture to decrease (see equation 5).

In this work, we perform a suite of N-body simulations to quantify the effect of dynamical interactions on the PBH merger rate. We focus on low-mass, low-N clusters for which analytical scalings (see Ali-Haïmoud et al. 2017 and Section 2 below) may break down. Specifically, we investigate what binary formation process provides the major contribution to the PBH merger rate in few $\times\,10^2$–$10^3\,M_\odot$ clusters. We find that in this mass regime the first hard binary is formed within a few hundred Myr. After the formation of the first binary the cluster expands by a factor of 20 in a time comparable to the age of the Universe. Consequently, the rate of two-body captures at the present time drops by an order of magnitude.

Spatial clustering of PBHs has been discussed in light of current LIGO observations (e.g. Raidal, Vaskonen & Veermäe 2017; Ballesteros, Serpico & Taoso 2018; Bringmann et al. 2019; Inman & Ali-Haïmoud 2019). In this paper, we investigate the dynamics for the specific initial PBH distributions proposed by Bird et al. (2016). We refer the reader to Bringmann et al. (2019) for a discussion of plausible PBH distributions.

This paper is organized as follows. In Section 2, we present theoretical considerations using as an example a $450\,M_\odot$ cluster composed of $15 \times 30\,M_\odot$ PBHs. In Sections 3 and 4, we describe the set-up for N-body simulations and present our results. In Section 5, we discuss the implications of these results for the merger rate of PBHs, and present our conclusions.

2 THEORETICAL CONSIDERATIONS

In this section, we consider an example of an N-body non-relativistic interacting system composed of $N = 15$ PBHs, which we call a *cluster*. For simplicity, we assume that the cluster is composed of a single-mass species of $m = 30\,M_\odot$, such that the total mass of the cluster is $M = Nm = 450\,M_\odot$. This example falls in the mass range that provides the major contribution to the PBH merger rate in Bird et al. (2016).[1] Finally, we assume the cluster to be spherically symmetric with radius $R = 2 \times 10^5$ au (i.e. 1 pc).

The number density of PBHs in the cluster is

$$n \sim \frac{N}{R^3} = 15\,\mathrm{pc}^{-3} \approx 2 \times 10^{-15}\,\mathrm{au}^{-3}, \tag{1}$$

and the typical velocity dispersion is

$$v_{\mathrm{disp}} \sim \sqrt{\frac{GM}{R}} \approx 1.5\,\mathrm{km\,s}^{-1} \approx 0.3\,\mathrm{au\,yr}^{-1}. \tag{2}$$

Next, we consider a binary system inside the cluster with orbital separation (semimajor axis) a and orbital speed $v_{\mathrm{orb}} \sim \sqrt{Gm/a}$. If the binding energy of the binary is smaller in magnitude than the typical kinetic energy of PBHs in the cluster, the binary is called *soft*. Soft binaries are likely to be disrupted by interactions with single PBHs. In the opposite case the binary is called *hard*, and dynamical interactions with single PBHs will further tighten the binary. Thus, hard binaries in a cluster typically survive encounters and tend to become harder, whereas soft binaries tend to split (Heggie 1975; Hills 1975). The hard/soft boundary depends on the properties of the cluster and can be estimated as

$$a_{\mathrm{HS}} \sim \frac{R}{N} = 10^4\,\mathrm{au} \tag{3}$$

in our case. Even in the absence of primordial binaries, binaries will generally form throughout three-body interactions over the lifetime of the cluster. This happens on the time-scale

$$\tau_{3\mathrm{B}} \sim \frac{N^2 R^{3/2}}{(GNm)^{1/2}} \sim N^2 \tau_{\mathrm{cross}} \simeq 150\,\mathrm{Myr}, \tag{4}$$

where $\tau_{\mathrm{cross}} = R/v_{\mathrm{disp}} = 0.7\,\mathrm{Myr}$ is the cluster crossing time-scale.

Alternatively, binaries can form via gravitational two-body captures. This can happen when two PBHs pass close enough to each other to emit GW radiation. If the energy released in GWs during the passage exceeds the total initial kinetic energy, the two PBHs become bound. The cross-section of two-body captures is given by

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[1]Clusters with $M < 400\,M_\odot$ are expected to evaporate in a few Gyr after their formation, and thus are not included in the merger rate derivation.

(Quinlan & Shapiro 1989)

$$\sigma_{2B} = 5 \times 10^{-9} \left(\frac{m}{30\,\mathrm{M_\odot}} \right)^2 \left(\frac{v_{\mathrm{disp}}}{1.5\,\mathrm{km\,s^{-1}}} \right)^{-18/7} \mathrm{pc}^2, \qquad (5)$$

while the typical time-scale is $\tau_{2B} = (n\sigma_{2B}v_{\mathrm{disp}})^{-1} = 90 \times 10^3\,\mathrm{Gyr} \gg \tau_{3B}$. Therefore, in the case we consider, three-body interactions will be the dominant binary formation mechanism.

Once formed, hard binaries necessarily interact with single PBHs in the cluster. These 2+1 interactions happen on a time-scale of

$$\tau_{2+1} \sim \frac{v_{\mathrm{disp}}}{nGma}, \qquad (6)$$

so the initial time-scale for strong interactions is $\tau_{2+1}(a_{\mathrm{HS}}) \approx 10\,\mathrm{Myr}$. Each interaction carries away a significant fraction of the binary orbital energy and the interloper PBH is ejected with a speed $\sim v_{\mathrm{orb}}$. Because in this example the cluster is composed of a single-mass species, conservation of linear momentum of the binary – interloper system implies that the binary must be ejected with a speed $\sim v_{\mathrm{orb}}/2$. This speed needs to be compared to the escape speed from a cluster, which is typically a few times the velocity dispersion of the cluster (e.g. Binney & Tremaine 2008). This implies that the recoil kicks will eject the binary once its orbital speed reaches $\sim 10 v_{\mathrm{disp}}$. Since $v_{\mathrm{orb}} \sim v_{\mathrm{disp}}$ at the hard–soft boundary, and $v_{\mathrm{orb}} \propto a^{-1/2}$, the minimum semimajor axis at the ejection a_{eject} is approximately 2 orders of magnitude smaller than a_{HS} before being ejected. At that time, the 2+1 interaction time-scale is $\tau_{2+1}(a_{\mathrm{eject}}) \sim 1\,\mathrm{Gyr}$. Binaries tighter than $\sim 0.01 a_{\mathrm{HS}}$ can only remain in the cluster if GW hardening takes over as the dominant forcing mechanism before the binary reaches a_{eject} and can be ejected. The time-scale of GW hardening a circular binary composed of $30 + 30\,\mathrm{M_\odot}$ PBHs is (Peters 1964)

$$\tau_{\mathrm{GW}}(a, e = 0) \simeq 6 \times 10^{20}\,\mathrm{yr} \left(\frac{a}{100\,\mathrm{au}} \right)^4. \qquad (7)$$

If the binary is highly eccentric, with eccentricity $e \to 1$, τ_{GW} becomes

$$\tau_{\mathrm{GW}}(a, e) \simeq \frac{768}{425}(1 - e^2)^{7/2}\,\tau_{\mathrm{GW}}(a, e = 0). \qquad (8)$$

The ultimate fate of the binary is essentially determined by a comparison between τ_{2+1}, the Hubble time τ_H, and τ_{GW}:

(i) if $\tau_{2+1} + \tau_{\mathrm{GW}}(e = 0) < \tau_H$ for some $a \in (a_{\mathrm{HS}}, a_{\mathrm{eject}})$, the binary will merge inside the cluster through a sequence of 2+1 interactions and GW emission;

(ii) if $\tau_{2+1} + \tau_{\mathrm{GW}}(e = 0) > \tau_H$ but $\tau_{2+1} < \tau_H$ at a_{eject}, the binary may either merge inside the cluster if 2+1 interactions happen to drive it to a sufficiently high eccentricity to reduce τ_{GW} at ejection or may be ejected; and

(iii) if both $\tau_{2+1} > \tau_H$ and $\tau_{\mathrm{GW}} > \tau_H$ at some $a \in (a_{\mathrm{HS}}, a_{\mathrm{eject}})$, the binary will remain in the cluster and stall at the orbital separation at which $\tau_{2+1} > \tau_H$.

The $N \sim 30$ regime pushes the range of validity of analytical scalings; fortunately, this regime is readily amenable to numerical simulations, which we introduce in the next section.

3 N-BODY SIMULATIONS

In this section, we describe a suite of simulations of clusters with total mass 10^2–$10^3\,\mathrm{M_\odot}$ performed using REBOUND, an N-body open source code (Rein & Liu 2012). Specifically, we model three types of clusters:

(i) $15 \times 30\,\mathrm{M_\odot}$ PBHs,

(ii) $5 \times 30 + 30 \times 10\,\mathrm{M_\odot}$ PBHs, and

(iii) $35 \times 30\,\mathrm{M_\odot}$ PBHs,

such that types (i) and (ii) have the same total mass of $450\,\mathrm{M_\odot}$, while (ii) and (iii) both consist of 35 PBHs. We set the size of all clusters to be 1 pc. Note that our clusters are more compact than the reference example of a $450\,\mathrm{M_\odot}$ DM halo with a velocity dispersion of $0.15\,\mathrm{km\,s^{-1}}$ (Bird et al. 2016), which implies that the virial radius of the reference example is $\sim 100\,\mathrm{pc}$. This changes the approximate numerical values of the scalings derived in the previous section to $a_{\mathrm{HS}} \sim 10^6\,\mathrm{au}$, $a_{\mathrm{eject}} \sim 10^4\,\mathrm{au}$, $\tau_{3B} \sim 150\,\mathrm{Gyr} \ll \tau_{2B} \sim 10^9\,\mathrm{Gyr}$, $\tau_{2+1}(a_{\mathrm{HS}}) \sim 10\,\mathrm{Gyr}$, and $\tau_{2+1}(a_{\mathrm{eject}}) \sim 1000\,\mathrm{Gyr}$. Our simulations are essentially scale free; other than the two-body capture, which has an additional length-scale set by the gravitational radius of the PBH masses, the results can be re-scaled to an arbitrary cluster size (for re-scaling to the reference example from Bird et al. 2016, which corresponds to multiplying lengths by 10^2, dividing speeds by 10, and multiplying time-scales by 10^3). In the following, we refer to all quantities as originally set in our simulations as 'simulated', and we also report 're-scaled' quantities to compare with the reference example.

In the simulations, we consider three values of the initial virial ratio of the cluster, defined as

$$V = \frac{\sum_i m_i v_i^2}{2 \sum_{ij} Gm_i m_j / r_{ij}}, \qquad (9)$$

with r_{ij} being the separation between PBHs i and j. We set V to 0.5, 0.3, and 0.1, allowing us to model clusters with a range of initial properties; however, we find that clusters promptly virialise to $V = 0.5$.

In total, we performed 90 simulations: 10 for each combination of the three types of cluster (i, ii, and iii) and the three initial virial ratios (0.5, 0.3, and 0.1). Our simulations are purely Newtonian; we do not consider additional forces or effects such as general relativity or tidal forces. We draw the initial positions of PBHs from a uniform space–density distribution and velocities from a Maxwellian distribution with the scale parameter equal to the velocity dispersion of the cluster v_{disp}. We evolve clusters for 100 Myr (100 Gyr of re-scaled time) using the IAS15 integrator (Rein & Spiegel 2015). We record the formation of PBH binaries and multibody bound systems and their properties.

4 RESULTS

After 100 Myr of simulation time, we find that the PBH clusters are significantly spread out. Specifically, we find that the median distance of a PBH from the centre of mass of the cluster is more than an order of magnitude larger than it was initially. We illustrate an example for each type of cluster with $V = 0.5$ in Fig. 1. Each line represents the distance of PBHs from the cluster's barycentre as a function of time (each coloured line represents a single PBH): from top to bottom for the cluster of a type (i), (ii), and (iii). The thick black line, representing the median distance of PBHs, clearly shows that clusters expand. The majority of the (coloured) lines closely intertwine for the first part of the simulation indicating that the cluster stays bound. The arched lines represent objects that are kicked out of the cluster with speed lower than the escape speed, such that they fall back into the cluster after reaching a maximum distance. The nearly vertical lines indicate PBHs ejected from the cluster. Note that we do not remove ejected PBHs from the simulation, so the cluster's barycentre drifts as the cluster evolves.
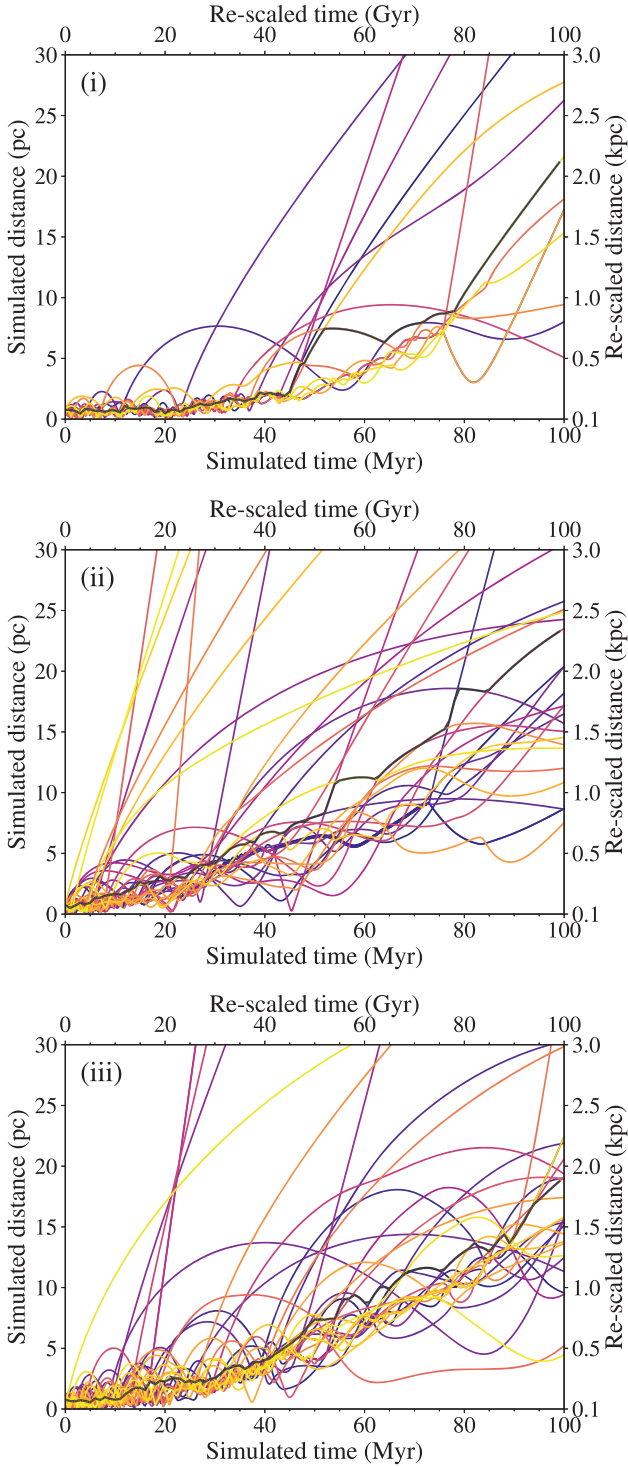
Figure 1. Distance from the barycentre of the cluster as a function of time: each coloured line represents one PBH. The thick black line represents the median distance. The bottom *x*-axis and left *y*-axis represent our fiducial simulation; the top *x*-axis and right *y*-axis correspond to clusters re-scaled to the reference example from Bird et al. (2016). From the top to the bottom, we show the results for clusters with (i) 15×30 M$_\odot$, (ii) $5 \times 30 + 30 \times 10$ M$_\odot$, and (iii) 35×30 M$_\odot$ PBHs and virial ratio $V = 0.5$.

In Fig. 2, we plot the separation between each pair of PBHs in the cluster, i.e. $|\mathbf{r}_i - \mathbf{r}_j|$ where $\mathbf{r}$ is the position vector of the object in the cluster's barycentre reference frame. The black solid horizontal line represents the hard/soft boundary, a_{HS}. Consequently, the
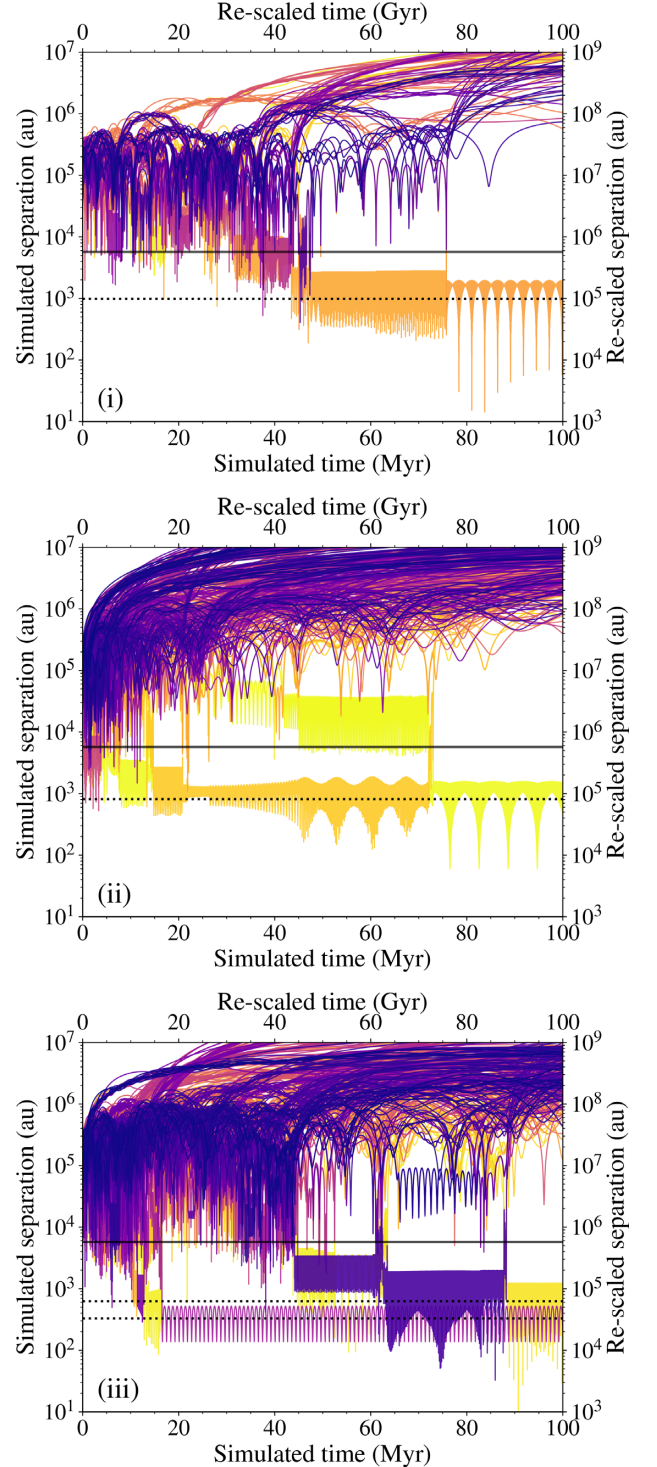
Figure 2. Separation between two PBHs as a function of time: each line represents a PBH pair. The clusters are the same as in Fig 1. The horizontal black solid line shows the separation between hard and soft binaries a_{HS}. The dotted horizontal lines indicate the semimajor axes of hard binaries formed in our simulations. These three examples illustrate that the typical outcome in our simulations is the formation of a few hard binaries.

coloured lines that lie below represent hard PBH binaries. The dotted horizontal lines indicate the semimajor axis of hard binaries at $t = 100$ Myr. These three examples illustrate that the typical outcome in our simulations is the formation of a few hard binaries. We also find that in clusters of type (ii), composed of a mix of PBHs
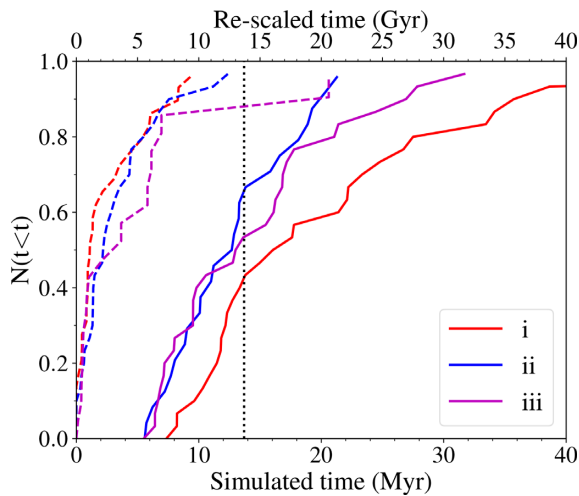
Figure 3. Cumulative distribution of the time when the density of the cluster drops by a factor of 20 (the solid lines), which implies a drop in the merger rate due to two-body captures by an order of magnitude, and of the formation time of the first hard binary (the dashed lines). The vertical dotted line marks $t = 13.7$ Gyr for the re-scaled simulation.

of mass 10 and $30 \, M_\odot$, the more massive PBHs tend to form binaries, consistent with expectations for mass segregation and substitutions during 2+1 interactions.

Binaries represent an energy source in a cluster, and dynamical interactions with binaries cause the cluster to heat up, expand and in some cases evaporate (Heggie & Hut 1993). For example, the binding energy of the hard binary in the top panel of Fig. 2 with a separation of $\sim 0.1 \, a_{HS}$ is ~ 67 per cent of the binding energy of the entire cluster. This means that to harden this binary through 2+1 interactions, which transfer energy from the binary to other cluster members, the cluster expanded by a factor of about 3. Thus, the density and typical speed of the PBHs in the cluster decrease, respectively, by $1/3^3$ and $1/\sqrt{3}$. Therefore, the merger rate by gravitational two-body captures drops by more than an order of magnitude ($\Gamma_{2B} \propto n^{31/42}$, see Section 2). If the hard binary is ejected without having time to share its energy with the cluster, the expansion rate of the cluster would be more moderate than in this particular example.

In order to suppress the merger rate due to two-body captures by an order of magnitude the density needs to drop by a factor of 20. We use this scaling to estimate how long it takes to suppress the two-body capture merger rate by an order of magnitude in our simulations. Specifically, we trace the evolution of the density of PBHs in the cluster with time. We use the density within half-mass radius, the radius measured from the cluster's barycentre containing half the total mass of the cluster. In Fig. 3, we plot the cumulative distributions of the time when $\rho(t)/\rho(t = 0) = 1/20$ (the solid lines) and of the formation time of the first hard binary (the dashed lines) across simulations. We find that $\gtrsim 90$ per cent of clusters form the first hard binary within 10 Myr. Thereafter, the hard binary heats the cluster and causes the density to decrease by a factor of 20 within 40 Myr. Thus, all simulated clusters evaporate in far less than a Hubble time.

By re-scaling these results to the reference example, we find that the formation of the first hard binary occurs within 10 Gyr, and that by $t = 13.7$ Gyr (the dashed vertical line in Fig. 3) the cluster density and hence the rate of mergers through two-body captures drops significantly in 40–60 per cent of the clusters. We find that the

density drops by a factor between 2 and 1100, with a median of 20. This corresponds to a drop in the merger rate of about 10.

Ali-Haïmoud et al. (2017) argued that clusters containing ~ 15 PBHs should form by redshift $z \sim 300$ (rather than $z \sim 30$ as estimated by Bird et al. 2016), and therefore their density should be $1000 \times$ higher (or cluster virial radius $10 \times$ smaller) than in Bird et al. (2016). These initial conditions change the approximate numerical values of the scalings derived in Section 2 to $a_{HS} \sim 10^5$ au, $a_{eject} \sim 10^4$ au, and $\tau_{3B} \sim 5$ Gyr $\ll \tau_{2B} \sim 5 \times 10^6$ Gyr. Our numerical results can re-scaled by multiplying lengths by ~ 10 and times by ~ 30. Using these scalings, we find that the formation of the first hard binary occurs within 300 Myr, while the density drop that would significantly suppress two-body captures happens within 13.7 Gyr for all simulated clusters.

Finally, in Fig. 4 we summarize the eccentricities and semimajor axes of hard binaries at the end of simulation: the circles are binaries that are not bound to any other PBH, the triangles and the squares represent hard binaries, respectively, in triples and in multiple systems. The typical outcome is the formation of binaries that are not bound to other PBHs, with occasional formation of triples, and only in a few cases the formation of multiple systems. Binary semimajor axes ranges between 10^2 and 10^4 au with medians of 940, 700, and 330 au (the dashed vertical lines in the top panel of Fig. 4), respectively, for clusters (i), (ii), and (iii). Only PBH binaries below and to the left of the black solid line, obtained by numerically integrating equation (5.14) of Peters (1964) for $30 + 30 \, M_\odot$ systems, can merge in < 13.7 Gyr. As all of the binaries lie significantly above the black line; none will merge due to GW emission within the Hubble time. Even for the tightest binary with $a = 100$ au, $1 - e$ needs to be as small as 10^{-4} (i.e. $e = 0.9999$) to merge in a Hubble time; however, the eccentricity would need to reach $1 - e \approx 2 \times 10^{-6}$ once this binary is re-scaled to the Bird et al. (2016) cluster size. High eccentricity can be induced by the Kozai–Lidov (KL) mechanism if the binary is in a hierarchical triple system. We discuss this mechanism further in Section 5.1.

5 DISCUSSION AND CONCLUSIONS

We carried out a set of simulations to investigate the dynamical evolution of clusters composed of 15 and 35 PBHs of 30 and $10 \, M_\odot$ with an initial radius of ~ 1 pc. We find that in the considered regime hard binaries form via three-body interaction within 10 Myr. Subsequently binaries further harden via 2+1 interactions until they reach orbital separations of $\sim 10^2$–10^3 au and eccentricities of $0 - 0.9999$ at $t = 100$ Myr. Binaries with these orbital separation and eccentricities require more than a Hubble time to merge via GW radiation (cf. Fig. 4).

Meanwhile, binaries act as a heat source in the cluster, driving its expansion and ultimate evaporation as energy is transferred from the binaries to the cluster through 2+1 interactions. The expansion of the cluster lowers the rate of mergers through two-body captures by an order of magnitude or more by 14 Myr (cf. Fig. 3), or by the age of the Universe if the cluster is re-scaled to the preferred density of Bird et al. (2016) and Ali-Haïmoud et al. (2017).

At the same time, the 2+1 interaction rate drops for tight binaries and is further lowered as the density drops in expanding clusters (equation 6), ultimately becoming longer than the age of the Universe long before these binaries can merge through the emission of GWs. Consequently, binaries stall at the orbital separations represented in Fig. 4. Next, we discuss two additional mechanisms that could enhance the dynamical merger rate, but find that these are unlikely to play a significant role for PBH clusters.
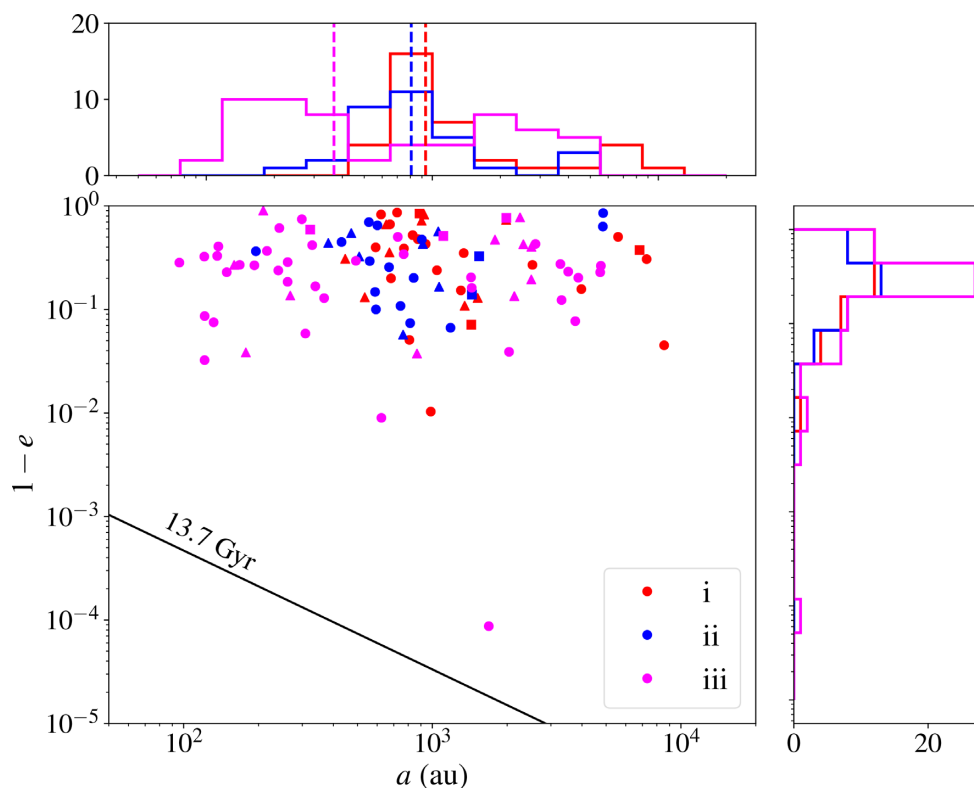
Figure 4. Properties of the binaries formed in our simulations: the circles are binaries that are not bound to any other PHB, the triangles are triples, and the squares are multiple systems. The red, blue, and magenta denote simulations of clusters of types (i), (ii), and (iii), respectively. The black solid line shows the merger time of 13.7 Gyr for a $30 + 30$ M$_\odot$ PBH binary. In the top and the right-hand panels, we show, respectively, distributions of semimajor axis and eccentricities. The dashed vertical lines indicate median values of a. To re-scale to the Bird et al. (2016) cluster size, the abscissa must be scaled by a factor of 100.

Therefore, we conclude that not accounting for cluster expansion through heating by binaries led Bird et al. (2016) to overestimate the PBH merger rate. The actual merger rate of PBHs in such clusters is likely to be well below $1\,\mathrm{Gpc^{-3}\,yr^{-1}}$, so they do not contribute appreciably to the total binary black hole merger rate inferred from GW observations. Ali-Haïmoud et al. (2017) pointed out that clusters composed of $\lesssim 3 \times 10^3$ PBHs form earlier in the Universe at a higher density and should rapidly evaporate due to weak gravitational interactions. However, the scaling relations (equation 99 of Ali-Haïmoud et al. 2017) are not necessarily valid in the small-N regime considered in this paper. Our simulations show that clusters of 15–35 PBHs do expand and evaporate because of the formation of hard binaries, and thus represent a useful numerical check of the approach of Ali-Haïmoud et al. (2017).

5.1 Mergers induced by Kozai–Lidov mechanism

One rapid path to very high binary eccentricity, which reduces the GW merger time-scale significantly, is through secular KL oscillations (Kozai 1962; Lidov 1962). This mechanism operates in hierarchical triples: two PBHs orbiting each other in a relatively tight inner binary plus a tertiary PBH orbiting the binary on a much wider outer orbit. In this configuration, the two orbits torque each other and exchange angular momentum, but not energy. Therefore, the orbits can change their eccentricities and relative inclination (typically on time-scales much longer than their orbital periods), but not their semimajor axes. Kozai (1962), Lidov (1962), and subsequent work show that in Newtonian gravity between point

masses, there is always a relative inclination of the outer to the inner orbit such that the inner binary can evolve to $e \sim 1$ from an arbitrarily small initial eccentricity through large-amplitude oscillations of the eccentricities and inclinations. The time required to drive the inner binary from its minimum to maximum eccentricity is of the order of

$$\tau_{\mathrm{KL}} \sim \frac{16}{15} \frac{a_2^3}{a_1^{3/2}} \sqrt{\frac{M_1}{Gm_3^2}} (1 - e_2^2)^{3/2}, \tag{10}$$

where a_1 and a_2 are, respectively, the semimajor axes of the inner and outer binary, M_1 is the total mass of the inner binary, m_3 is the mass of the tertiary and e_2 is the eccentricity of the outer binary (e.g. Lidov & Ziglin 1976). For binaries formed in our simulation this time-scale is smaller than a Hubble time for only $\sim$36 per cent of the triples. When re-scaling to the reference clusters of Bird et al. (2016), this time-scale becomes $1000\times$ longer, so we can conclude that for most triples, KL oscillations are inefficient at driving up inner binary eccentricities.

However, it is challenging to definitively rule out a contribution of KL oscillations to the PBH merger rate. Triples will form generically as a result of 2+2 interactions, with the expected ratio of such interactions to 2+1 interactions of order the ratio of the number of binaries to the number of single stars in the cluster, or $\sim$0.1 for our typical clusters. The presence of triples in the simulated clusters is illustrated in Fig. 4. Only one of the triples formed in the 30 simulated clusters shown in Fig. 4 has a KL time-scale below the age of Universe after re-scaling. This triple has re-scaled inner and outer binary semimajor axes of 4.2×10^4 and 2.8×10^6 au, respectively;

its inner binary eccentricity would have to grow to a very large eccentricity $1 - e \approx 4 \times 10^{-7}$ in order for it to merge through GW emission in the age of the Universe. It is therefore possible that hierarchical triples could form and be driven to sufficiently high eccentricities by KL oscillations to merge in a small fraction of PBH clusters.

5.2 Mergers due to dynamical inspirals

In a cluster containing binaries, 2+1 interactions can be efficient in dynamically forming highly eccentric inspiraling binaries. A significant fraction of the binary-single encounters result in resonant interactions, in which the three PBHs wander for a long time on chaotic orbits and approach each other repeatedly (Heggie 1975). In particular, during these chaotic encounters two PBHs can pass sufficiently close to capture through the emission of GW radiation and even merge while the system is still in resonance. This last outcome is rare. However, the cross-section of binaries is larger than that of a single PBH for two-body capture. In particular, the cross-section of 2+1 interactions is larger for binaries with larger orbital separations, and the overall cross-section for dynamical inspirals scales as $a^{2/7}$ with the target binary semimajor axis for equal-mass binaries (e.g. Gültekin, Miller & Hamilton 2006; Samsing, MacLeod & Ramirez-Ruiz 2014; Samsing et al. 2019). Thus, wide binaries formed in our simulations can potentially open another merger channel.

The cross-section of these encounters can be estimated using equation (36) from Samsing et al. (2014):

$$\sigma_{\text{insp}} \simeq 5 \times 10^{-8} \left(\frac{a}{10^3 \text{au}} \right)^{2/7} \left(\frac{m}{30 \, \text{M}_\odot} \right)^{12/7} \left(\frac{v_{\text{disp}}}{1.5 \text{km s}^{-1}} \right)^{-2} \text{pc}^2. \tag{11}$$

We can compare this cross-section to the cross-section for direct two-body capture given in equation (5), using the Bird et al. (2016) cluster parameters: $m = 30 \, \text{M}_\odot$, $v_{\text{disp}} = 0.15 \, \text{km s}^{-1}$, $a = a_{\text{HS}} \sim 10^6$ au (the widest stable binaries provide the greatest contribution to dynamical interactions). With these values, $\sigma_{2\text{B}} \approx 2 \times 10^{-6}$ pc^2, while $\sigma_{\text{insp}} \approx 4 \times 10^{-5}$ pc^2, a factor of 20 greater. However, the fraction and hence number density of binaries is only ~ 0.1 of that of single stars for typical simulated clusters (see Fig. 2), reducing the relative contribution for this channel by a factor of ~ 10. Consequently, the overall rate of captures during three-body interactions is comparable to the rate of direct two-body captures for cluster parameters of interest. Both rates will drop as $n v_{\text{disp}}^{-18/7} \sim n a^{2/7} v_{\text{disp}}^{-2} \sim R^{-12/7}$ as the cluster expands.

In this work, we re-examined the merger rate of PBH binaries formed in the subdominant late Universe channel by accounting for the first time for the full spectrum of dynamical interactions, including binary formation and subsequent cluster heating and expansion. We find that the contribution of $\lesssim 10^3 \, \text{M}_\odot$ PBH clusters to the binary black hole merger rate (if no initial binaries are present in the cluster) falls by an order of magnitude or more, to well below 1 Gpc^{-3} yr^{-1}. If PBHs comprise only a fraction f_{pbh} of the DM, the event rate will scale as $f_{\text{pbh}}^{53/21}$ (Bird et al. 2016). Gravitational captures would thus make a minimal contribution to the observed rate of black hole mergers, making detections of merging binary black holes formed through this process unlikely. Consequently, if PBHs are formed in such low-mass clusters but not in tight binaries, GW observations cannot observe PBHs or constrain their contribution to the DM content of the Universe.

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