

**DETECTING COALESCENCES OF INTERMEDIATE-MASS  
BLACK HOLES IN GLOBULAR CLUSTERS  
WITH THE EINSTEIN TELESCOPE**

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We discuss the capability of a third-generation ground-based detector such as the Einstein Telescope (ET) to detect mergers of intermediate-mass black holes (IMBHs) that may have formed through runaway stellar collisions in globular clusters. We find that detection rates of  $\sim 2000$  events per year are plausible.<sup>1</sup>

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The Einstein Telescope (ET), a proposed third-generation ground-based gravitational-wave (GW) detector, will be able to probe GWs in a frequency range reaching down to  $\sim 1$  Hz.<sup>2</sup> This bandwidth will allow the ET to probe sources with masses of hundreds or a few thousand  $M_{\odot}$  which are out of reach of LISA or the current ground-based detectors LIGO, Virgo, and GEO-600.

Globular clusters may host intermediate-mass black holes (IMBHs) with masses in the  $\sim 100 - 1000 M_{\odot}$  range (see Ref. 3 and references therein). If the stellar binary fraction in a globular cluster is sufficiently high, two or more IMBHs can form.<sup>4</sup> These IMBHs then sink to the center in a few million years, where they form a binary and merge via three-body interactions with cluster stars followed by gravitational radiation reaction (see<sup>4,5</sup> for more details). Therefore, the rate of IMBH binary mergers is just the rate at which pairs of IMBHs form in clusters. The rate of detectable coalescences is

$$R \equiv \frac{dN_{\text{event}}}{dt_o} = \int_{M_{\text{tot},\text{min}}}^{M_{\text{tot},\text{max}}} dM_{\text{tot}} \int_0^1 dq \int_0^{z_{\text{max}}(M_{\text{tot}},q)} dz \frac{d^4 N_{\text{event}}}{dM_{\text{tot}} dq dt_e dV_c} \frac{dt_e}{dt_o} \frac{dV_c}{dz}. \quad (1)$$

Here  $M_{\text{tot}}$  is the total mass of the coalescing IMBH-IMBH binary and  $q \leq 1$  is the mass ratio between the IMBHs;  $z_{\text{max}}(M_{\text{tot}}, q)$  is the maximum redshift to which the ET could detect a merger between two IMBHs of total mass  $M_{\text{tot}}$  and mass ratio  $q$ ;  $dt_e/dt_o = (1+z)^{-1}$  is the relation between local time and our observed time, and

$dV_c/dz$  is the change of comoving volume with redshift, given by

$$\frac{dV_c}{dz} = 4\pi D_H^3 [\Omega_M(1+z)^3 + \Omega_\Lambda]^{-1/2} \left\{ \int_0^z \frac{dz'}{[\Omega_M(1+z')^3 + \Omega_\Lambda]^{1/2}} \right\}^2. \quad (2)$$

We assume a flat universe ( $\Omega_k = 0$ ), and use  $\Omega_M = 0.27$ ,  $\Omega_\Lambda = 0.73$ ,  $H_0 = 72 \text{ km s}^{-1} \text{ Mpc}^{-1}$ , and  $D_H = c/H_0 \approx 4170 \text{ Mpc}$ , so that the luminosity distance can be written as a function of redshift as:<sup>6</sup>

$$D_L(z) = D_H(1+z) \left\{ \int_0^z \frac{dz'}{[\Omega_M(1+z')^3 + \Omega_\Lambda]^{1/2}} \right\}. \quad (3)$$

We make the following assumptions. **1.** IMBH pairs form in a fraction  $g$  of all globular clusters. **2.** We neglect the delay between cluster formation and IMBH coalescence. **3.** When an IMBH pair forms in a cluster, its total mass is a fixed fraction of the cluster mass,  $M_{\text{tot}} = 2 \times 10^{-3} M_{\text{cl}}$ , consistent with simulations.<sup>7</sup> The mass ratio is uniform in  $[0, 1]$ . We restrict our attention to systems with a total mass between  $M_{\text{tot,min}} = 100M_\odot$  and  $M_{\text{tot,max}} = 20000M_\odot$ . Thus,

$$\frac{d^4 N_{\text{event}}}{dM_{\text{tot}} dq dt_e dV_c} = g \frac{d^3 N_{\text{cl}}}{dM_{\text{cl}} dt_e dV_c} \frac{1}{2 \times 10^{-3}}. \quad (4)$$

**4.** The distribution of cluster masses scales as  $(dN_{\text{cl}}/dM_{\text{cl}}) \propto M_{\text{cl}}^{-2}$  independently of redshift. We confine our attention to clusters with masses ranging from  $M_{\text{cl,min}} = 5 \times 10^4 M_\odot$  to  $M_{\text{cl,max}} = 10^7 M_\odot$ . The total mass formed in all clusters in this mass range at a given redshift is a redshift-independent fraction  $g_{\text{cl}}$  of the total star formation rate per comoving volume:

$$\frac{d^3 N_{\text{cl}}}{dM_{\text{cl}} dt_e dV_c} = \frac{g_{\text{cl}}}{\ln(M_{\text{cl,max}}/M_{\text{cl,min}})} \frac{d^2 M_{\text{SF}}}{dV_c dt_e} \frac{1}{M_{\text{cl}}^2}. \quad (5)$$

**5.** The star formation rate as a function of redshift  $z$  rises rapidly with increasing  $z$  to  $z \sim 2$ , after which it remains roughly constant:<sup>8</sup>

$$\frac{d^2 M_{\text{SF}}}{dV_c dt_e} = 0.17 \frac{e^{3.4z}}{e^{3.4z} + 22} \frac{[\Omega_M(1+z)^3 + \Omega_\Lambda]^{1/2}}{(1+z)^{3/2}} M_\odot \text{ yr}^{-1} \text{ Mpc}^{-3}. \quad (6)$$

Rather than computing  $z_{\text{max}}(M_{\text{tot}}, q)$  [Eq. 1] for all values of  $M_{\text{tot}}$  and  $q$ , we rely on the following fitting formula for the luminosity-distance range  $D_{\text{L,max}}$  as a function of the redshifted total mass  $M_z = M_{\text{tot}}(1+z)$ , obtained by using the effective-one-body, numerical relativity (EOBNR) gravitational waveforms<sup>9</sup> to model the inspiral, merger, and ringdown phases of coalescence:

$$D_{\text{L,max}}(M_z) = (1.25 \text{ Gpc}) A \begin{cases} M_z^{3/5} & \text{if } M_z < M_0 \\ M_z^{11/10} M_z^{-1/2} & \text{if } M_z > M_0 \end{cases}, \quad (7)$$

where  $A = 4$ ,  $M_0 = 600M_\odot$  for  $q = 1$  and  $A = 2.25$ ,  $M_0 = 450M_\odot$  for  $q = 0.25$ . We use  $\rho = 8$  as the SNR threshold for a ‘‘single ET’’ configuration. We determine the sky-location and orientation averaged range by dividing the horizon distance by 2.26,<sup>10</sup> ignoring redshift corrections to this factor.

We can compute  $z(D_L)$  by inverting Eq. (3). For a given choice of  $M_{\text{tot}}$  and  $q$ , the maximum detectable redshift  $z_{\text{max}}(M_{\text{tot}}, q)$  is then obtained by finding a self-consistent solution of  $z(D_{L,\text{max}}(M_{\text{tot}}(1+z_{\text{max}}))) = z_{\text{max}}$ .

We substitute these expressions into Eq. (1) to obtain the rate of detectable coalescences. We carry out the integrals of  $M_{\text{tot}}$  and  $z$  in Eq. (1) for two specific values of  $q$ . For  $q = 1$ , we find the total rate to be  $R = 2.5 \times 10^5 g g_{\text{cl}} \text{ yr}^{-1}$ ; for  $q = 0.25$ , it is  $R = 2 \times 10^5 g g_{\text{cl}} \text{ yr}^{-1}$ . The range varies smoothly with  $q$ ; therefore, we estimate that the full rate, including the integral over  $q$  is

$$R = \frac{2 \times 10^{-3} g g_{\text{cl}} \text{ yr}^{-1}}{\ln(M_{\text{tot,max}}/M_{\text{tot,min}})} \int_{M_{\text{tot,min}}}^{M_{\text{tot,max}}} \frac{M_{\odot} dM_{\text{tot}}}{M_{\text{tot}}^2} \int_0^1 dq \quad (8)$$

$$\int_0^{z_{\text{max}}(M_{\text{tot}}, q)} dz 0.17 \frac{e^{3.4z}}{e^{3.4z} + 22} \frac{4\pi(D_H/\text{Mpc})^3}{(1+z)^{5/2}} \times \left\{ \int_0^z \frac{dz'}{[\Omega_M(1+z')^3 + \Omega_\Lambda]^{1/2}} \right\}^2$$

$$\approx 2000 \left(\frac{g}{0.1}\right) \left(\frac{g_{\text{cl}}}{0.1}\right) \text{ yr}^{-1},$$

where we arbitrarily chose  $g = 0.1$  and  $g_{\text{cl}} = 0.1$  as the default scalings.

Mergers between pairs of globular clusters containing IMBHs can increase this rate by up to a factor of  $\sim 2$ .<sup>11</sup> Ref. 1 contains additional details on coalescences involving intermediate-mass black holes as gravitational-wave sources for the ET.

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