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Constraints on High-Density Matter From Observations of Neutron Stars

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The cores of neutron stars contain matter at densities beyond nuclear saturation. Understanding such matter therefore feeds into understanding of neutron stars, and vice versa; studies of neutron stars can yield unique information about the state of high-density matter. Here we summarize current constraints based on observations of neutron stars. We focus on observations of neutron star binaries, because mass measurements are easiest and most precise for such systems. We highlight recent developments in observation and modeling of neutron star low-mass X-ray binaries that suggest that slowly rotating neutron stars may have maximum masses up to $\sim 2.2\,M_\odot$, implying a high-density equation of state that is relatively hard.

1. INTRODUCTION

High-density matter plays an important role in many exciting frontiers of astrophysics. These include the formation of primordial black holes [1–4], which are in turn a candidate for some fraction of dark matter [1,2]; understanding of core-collapse supernovae [5]; some gamma-ray burst models, such as those in which a phase transition releases the required energy [6]; and the expected waveforms of gravitational radiation from the inspiral of neutron stars in binaries [7]. Indeed, many aspects of the study of neutron stars touch on high-density physics, because matter in the cores of neutron stars is believed to attain several times nuclear saturation density, where there are substantial uncertainties in the state of matter. The state of the matter in the core also bears on issues such as whether various predicted strong-gravity general relativistic effects (such as the existence of unstable circular orbits) can be observed around neutron stars. Therefore, improved understanding of the equation of state (EOS) of cold dense matter has immediate application to many neutron star phenomena. Conversely, observations of these astronomical phenomena can be used to improve understanding of high-density matter. It is this latter direction that we will pursue.

As always in astronomy, a difficulty is that one cannot (with rare exceptions) perform controlled laboratory experiments to elucidate some aspect of physics, but must instead be content with passive observations. In the present case, this means one must look for macroscopic indicators of the microphysics of interest. That is, one must find ways in which the gross structure of neutron stars is affected by the state of matter beyond nuclear saturation density, and then find specific observational signatures that bear on

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that gross structure. There are several promising avenues:

- The maximum mass of a slowly rotating neutron star depends fairly sensitively on the equation of state beyond nuclear saturation (and at lower densities as well, but as the low-density state of matter is relatively well understood, the dominant uncertainties are at high densities). Qualitatively, if the high-density EOS is soft, the maximum mass is low. Softening is a consequence of any density-induced phase transition, e.g., to a kaon condensate [8] or a state of quark matter. The currently proposed equations of state imply maximum neutron star gravitational masses between $\sim 1.5\,M_\odot$ and $\sim 2.3\,M_\odot$. Note that definitive measurement of a high mass for even a *single* neutron star would rule out soft equations of state.

- Even if the mass of a given neutron star is not exceptionally high (e.g., $1.6\,M_\odot$), if its mass and radius can be measured simultaneously with precision then the EOS will be constrained significantly. This is because a given EOS implies a strict mass-radius relation for neutron stars.

- Similarly, the maximum spin rate of a neutron star depends on the EOS. For hard EOS, which predict a comparatively large radius for a given mass, the maximum spin rate is relatively low. Since neutron stars are by definition gravitationally bound, when the matter on the spin equator rotates at the Keplerian rate (including distortion due to the rotation itself), the "mass-shedding" limit is reached, and no further spinup is possible. The softer the EOS, the higher the maximum spin frequency. However, a very high observed spin frequency is needed to restrict the EOS; most realistic EOS allow spin frequencies up to >1500 Hz, if the mass is unknown [9], whereas the maximum known spin frequency is 642 Hz.

- If the moment of inertia can be measured for a given mass and spin frequency, the high-density EOS is likewise sharply constrained. This may be possible if clear evidence for frame-dragging is obtained [10,11].

- Finally, a more indirect measure of the state of matter at high densities would come from thermal emission. All else being equal, the presence of nonbaryonic matter in the cores of neutron stars allows new channels for neutrino generation [12]. These neutrinos escape directly and therefore cool the star more rapidly than if it cooled only via photon emission. Thus, precise knowledge of the surface temperature of a star of known mass and age is informative about the EOS at high densities as well as about phenomena such as superfluidity in baryon or quark phases [13]. At present, however, this is limited by lack of spectral resolution and uncertainties about the modeling of the emission.

In practice the first of these, mass measurements, has been the most restrictive. We therefore emphasize mass measurements. We start by reviewing the most reliable mass estimates, from timing of binary radio pulsars. We continue by discussing recent determinations of the masses of neutron stars in low-mass X-ray binaries, which may have accreted several tenths of a solar mass in their lifetimes, and which therefore may have much higher masses and hence be more constraining on the high-density EOS. In addition to presenting evidence of sources with $M > 1.9\,M_\odot$ from optical light curves, we show tantalizing hints that in one accreting source the neutron star has a mass $M \approx 2.15\,M_\odot$, strongly limiting the EOS. We conclude by speculating about future constraints that may be derived from current and planned instruments.

2. MASS MEASUREMENTS FOR NEUTRON STARS

Observations of binaries have historically given the cleanest mass measurements of astronomical objects. In most cases it is not possible to image the members of the binary, so it is necessary to derive information from line of sight motion. This motion can be inferred from either periodic variation of spectral lines or, in the case of pulsars, from periodic variation in time of arrival of pulses. Regardless of the nature of the binary, at least five quantities emerge from a fit to the periodic variation: the binary period P_b, the projected semimajor axis $a \sin i$ of the star being observed (where the orbital inclination to our line of sight is i, such that $i = 0$ is face on), the eccentricity e of the orbit, and the time T_0 and longitude ω_0 of periastron. If the observed star has mass m_1 and its companion has mass m_2, and we define $M = m_1 + m_2$ to be the total mass of the system, then the mass function f is defined to be

$$f = \frac{(m_2 \sin i)^3}{M^2} = \frac{4\pi^2}{GP_b^2}(a \sin i)^2 \, , \tag{1}$$

where G is Newton's constant. The mass function is a rigorous lower limit to m_2, and is the value m_2 takes when $\sin i = 1$ and $m_1 \to 0$. Note, however, that without further information it is impossible to determine the orbital inclination, and hence m_2 cannot be known with precision. In a binary containing a compact object and a main sequence star, i can be constrained by observations of the optical or infrared light curve. The main sequence companion is distorted away from a spherical shape by the gravity of the compact object. The amplitude of the resulting ellipsoidal light variations then depends on i: a face-on orbit will keep the same area of the companion towards us at all times, producing no variation, whereas an edge-on orbit will cause maximum variation. This method has been used to identify several black hole candidates [14]. Real precision in mass determination, however, requires a double degenerate binary with a millisecond pulsar, so that higher-order relativistic effects can resolve any ambiguity.

2.1. Timing of millisecond pulsars

Ordinary radio pulsars are powered by rotation of neutron stars. These stars slow down by emission of magnetic dipole radiation. A subclass of pulsars, the millisecond pulsars, rotate very rapidly (with periods P as low as 1.6×10^{-3} s) and have comparatively weak magnetic fields (surface fields of $10^8 - 10^{10}$ G), with the result that they spin down very slowly and hence are exceedingly accurate clocks. The intrinsic spindown rate, $\dot{P}$, is dimensionless and is typically in the range of 10^{-19} to 10^{-21}, meaning that the characteristic spindown time $P/2\dot{P}$ can be greater than the age of the universe. Accurate timing of such pulsations requires many minute corrections for, e.g., the motion of the solar system barycenter, but the current timing packages have been extensively tested and are reliable [15].

When a millisecond pulsar is in a binary system, the resulting modulation of the pulse time of arrival produces a clear signal. For example, the first extrasolar planets were detected around a millisecond pulsar in this fashion. The smallest planet in that system, which has approximately the mass of our Moon, is too small by a factor of 10^4 to be detected via spectral line modulation around main sequence stars with current technology.

For the purpose of assessing the masses of neutron stars, the advantage of such precision is that higher-order relativistic effects on the orbit can be detected and used to derive other parameters of the system and even test the predictions of general relativity. These effects include the advance of the periastron, transverse Doppler and gravitational redshifts, orbital decay due to emission of gravitational radiation, and the Shapiro time delay caused by propagation of the light through the curved spacetime around the star (see [15] for a review). If the companion to the pulsar is itself a neutron star, there are essentially no further corrections, because at the $\sim 10^{11}$ cm separations implied by orbital periods of hours, the $\sim 10^6$ cm radius neutron stars are effectively points and there are therefore no changes in the orbit due to tidal effects. In such a case the masses of both stars can be derived with high accuracy, and tests of general relativistic predictions to a relative precision of $\sim 10^{-3}$ are possible. For white dwarf companions there is more uncertainty, and for main sequence companions the tidal effects can be substantial and challenging to model, so the mass uncertainty increases with the size of the companion.

Some masses derived from pulse time arrival measurements are summarized in Table 1 (taken from [16]). These include only NS-NS and NS-WD binaries; binaries involving a main sequence star are discussed in the next section. All of these neutron star masses are consistent with $M \approx 1.4\,M_\odot$. It is important to note that this is the *gravitational* mass, not the baryonic mass; the gravitational redshift at the surface of a neutron star is approximately $0.2\,c^2$, so the baryonic masses are likely to be closer to $1.7\,M_\odot$. The baryonic mass of the iron core of a pre-supernova star is approximately the Chandrasekhar mass, $1.4\,M_\odot$, so some tenths of a solar mass of material must fall onto the cores of these stars. The NS-NS binaries are not likely to have undergone significant mass transfer, because the duration of the accretion phase in such systems is only a few million years, insufficient to accrete more than a tenth of a solar mass even at the maximum possible accretion rate. A NS-WD binary may or may not have been able to transfer much mass, depending on the evolutionary history of the system.

2.2. Masses of neutron stars in accreting binaries

Neutron stars that undergo active accretion for a prolonged period may have masses significantly greater than their birth masses. If a reliable gravitational mass $M > 1.8\,M_\odot$ can be established for a slowly rotating neutron star, the softest currently viable high-density equations of state will be eliminated. Mass determinations for such systems are distinct from those in millisecond pulsar binaries, because accretion torques introduce enough noise in the spin rates to confound accurate measurements of the required relativistic effects. Therefore, until recently (see below), the only information that has been extracted about such binaries has come from the light curve of the companion.

Stable accretion is possible in either of two regimes: high-mass X-ray binaries, in which the companion to the neutron star has a mass $M \gtrsim 10\,M_\odot$, and low-mass X-ray binaries, in which the companion has a mass $M \lesssim 1\,M_\odot$. In either case the maximum sustained

Table 1
Neutron star masses determined to within 10% from pulsar timing. Quoted uncertainties are 95% central limits. The first three pairs are double neutron star binaries; the last is a neutron star-white dwarf binary. From [16].

Star	Mass ($M_\odot$)
B1534+12	
Pulsar	1.339±0.006
Companion	1.339±0.006
B1913+16	
Pulsar	1.4411±0.0007
Companion	1.3874±0.0007
B2127+11C	
Pulsar	1.349±0.08
Companion	1.363±0.08
B1855+09	1.41±0.2

rate of roughly spherical accretion is the Eddington rate, at which the force on ionized hydrogen from accretion radiation equals the gravitational force. Aspherical accretion can in principle increase this limit by a small amount. This defines the Eddington luminosity, $L = 1.3 \times 10^{38}(M/M_\odot)$ erg s^{-1}. For a neutron star of mass $1.4\,M_\odot$, the implied maximum accretion rate is $\approx 10^{18}$ g s^{-1}, requiring $\approx 10^7$ yr to accrete a tenth of a solar mass. High-mass X-ray binaries are believed to sustain a high accretion rate for only a few million years, and hence neutron stars in those systems are likely to have close to their birth mass. It is therefore remarkable that extensive optical monitoring of the HMXB Vela X-1 implies a mass for the neutron star of $M = 1.86 \pm 0.17\,M_\odot$ [17]. This mass is determined from the binary motion including ellipsoidal light variations (see the introduction to § 2) and a model of the mass of the companion from its spectral type. This high a mass is significantly different from the masses derived from timing of millisecond pulsars. If the best estimate of $1.86\,M_\odot$ is correct, this restricts somewhat the range of high-density equations of state.

Low-mass X-ray binaries have active lifetimes of hundreds of millions of years, so accretion at a tenth or more of the Eddington rate, as is derived for many of these sources, can add several tenths of a solar mass to the neutron star. It is therefore these systems that are expected to yield the strongest constraints based on mass estimates. Recent measurements of the optical light curve of the LMXB Cyg X-2 indicate that, at the 95% confidence level, the mass of the neutron star in that system is $M > 1.88\,M_\odot$ [18]. Such a high mass would require significant three-body repulsive forces [19]. Moreover, interpretation of an LMXB phenomenon discovered only five years ago suggests that neutron stars with even higher masses exist in these systems. To understand this, we need to discuss the phenomenon of high-frequency quasiperiodic brightness oscillations (QPOs).

2.3. Constraints from kilohertz QPOs

Neutron stars in low-mass X-ray binaries have relatively weak surface magnetic fields, on the order of 10^{8-10} G instead of the $\sim 10^{12-13}$ G thought to be typical of newborn neutron stars such as the Crab pulsar. The reason for this is unclear. However, they afford excellent opportunities to study the effects of general relativity and to infer masses and radii of neutron stars, because the accretion disks in these systems extend close to the stellar surface rather than being truncated hundreds of radii away by a strong magnetic field. The primary emission from such accretion disks is in X-rays. In addition, rapid time variability up to orbital frequencies $\sim 1-2$ kHz carries valuable information about the physical state of the matter in strong gravity and at high densities. This was the motivation for the construction of the Rossi X-ray Timing Explorer (RXTE), which was launched in late 1995 and is dedicated to fast X-ray timing.

The major discovery with RXTE was the existence of kHz quasiperiodic brightness oscillations (QPOs) in neutron-star low-mass X-ray binary systems. To understand QPOs, consider first a perfectly periodic oscillation in the X-ray countrate, which is close to the usual conception of a pulsar. The resulting power density spectrum is a delta function. If instead other frequencies are present, or the amplitude or phase of the oscillation vary, the peak in the power density spectrum has some width and one speaks of a quasiperiodic oscillation. QPOs with ~kHz frequencies are seen from ~20 neutron star systems, and a number of trends have been established (see [20] for a review). These QPOs (1) often appear as two simultaneous peaks in a power density spectrum, (2) are relatively sharp and strong, (3) tend to increase in frequency as the inferred mass accretion rate increases, and (4) have a separation frequency that is close to the inferred spin frequency. A number of models have been proposed for this phenomenon (see [20] and references therein). However, many of the most important inferences about strong gravity and dense matter from these oscillations can be derived more generally, from the widely accepted idea that the upper peak frequency ν_2 is an orbital frequency.

There are two simple considerations that yield a constraint on the mass M and the radius R of a neutron star, from the observation of an orbital frequency $\nu_{\rm orb}$. First, the orbit is obviously outside the star. Second, general relativity predicts that there is a lower bound to the radii of stable circular orbits; this bound is called the innermost stable circular orbit (ISCO). The orbit generating the QPO must be outside the ISCO, because otherwise there would be rapid inspiral and any resulting oscillation would last at most a few cycles, leading to a broad QPO, in conflict with observations. As shown in [23], these limits constrain the mass of the star to be $M < 2.2(1+0.75j)(1000\ {\rm Hz}/\nu_2)\,M_\odot$ and the circumferential radius to be $R < 19.5(1+0.2j)(1000\ {\rm Hz}/\nu_2)$ km, to first order in the dimensionless spin parameter $j \equiv cJ/GM^2$, where J is the stellar angular momentum ($j \approx 0.1$ for a neutron star with a spin frequency of 300 Hz, and the external spacetime is unique to first order in j). The observation of a given orbital frequency also restricts the mass and radius to be inside a wedge in a $M-R$ diagram; the larger the frequency, the more restrictive the constraint. The highest frequency yet observed, 1330 Hz [21], rules out the hardest equations of state historically proposed for the cores of neutron stars, such as some tensor interaction equations of state [22]. This is the first time that an astrophysical observation has been able to rule out a hard equation of state. A QPO frequency greater than $\sim$ 1500 Hz in any source would rule out the hardest modern equations of state.

As first discussed in 1996, an even more major advance would occur if strong evidence were found for the existence of the ISCO. This is because the existence of unstable circular orbits is a qualitatively new feature of general relativity compared to Newtonian gravity, and is an essential component of models of accreting black holes on all scales as well as of inspiraling compact objects and other phenomena. Given that observed frequencies increase with increasing $\dot{M}$, but that $\nu_{\rm ISCO}$ is an upper limit to the frequency of the upper peak, it was expected that there would be a rollover in the slope of the $\nu - \dot{M}$ curve for both the upper and lower QPO peaks [23]. Moreover, the QPO frequency at which the curve rolls over must be a constant, for many observations of a given source. For a limiting frequency of $\nu_{\rm max}$, the implied gravitational mass of the source would be $M = 2.2\,M_\odot(1+0.75j)(1000\,{\rm Hz}/\nu_{\rm max})$ [23]. This may have been seen in RXTE data from 4U 1820–30 at $\nu_{\rm max}$=1090 Hz [24,25]. If confirmed, this would imply $M = 2\,M_\odot$ for a nonrotating star, but at the 290 Hz spin frequency inferred for 4U 1820–30 the implied mass rises to $2.15\,M_\odot$. This provides both a strong constraint on high density matter and an important test of a crucial prediction of strong-gravity general relativity. Given these dramatic implications, care must be taken in the interpretation of the data. The main question is whether the rollover really happens as a function of $\dot{M}$, or whether this effect is counterfeited by a spectral state change. Further observations and additional sources with this signature would help immensely. This is one of the many potential benefits of a future large-area timing instrument.

2.4. Summary of current constraints from neutron star masses

In relating mass measurements to the properties of high-density matter, it is convenient to have a parameterized family of models. Such a family has been constructed by [26] to fit the data of [19] (see equation (1) of [26]). This family has a single "softness" parameter s, which gives a best fit to the data of [19] at $s = 0.2$; larger values of s indicate a softer EOS, and hence a lower maximum mass for neutron stars. One can also explore the possibility of a mixed phase quark core [27,26], with a bag constant B [28]; [26] chose B=150 and 200 MeV fm^{-3} as representative.

These mass-radius relations, plus the mass measurements discussed above, are plotted for *nonrotating* neutron stars in Figure 1. The effect of rotation is to provide additional support against gravity, so rotating neutron stars can be more massive than nonrotating ones; for the neutron stars in low-mass X-ray binaries, which are thought to have spin frequencies $\sim$ 300 Hz [20], the maximum allowed mass is increased by $\approx$ 10% for the equations of state considered here. Rotation of the star does not affect the mass estimates for millisecond pulsars or from optical observations of companions (e.g., Vela X-1 and Cyg X-2), but it *does* affect the inference of mass from a rollover at the ISCO, as discussed above. If the signature of the ISCO in 4U 1820–30 is confirmed, the constraint $s < 0.2$ is approximately independent of rotation because the allowed and required masses are increased by roughly the same amount. We also see from Figure 1 that the existence of any quark matter phase in neutron stars decreases the maximum mass below the estimated mass of the neutron star in 4U 1820–30. Thus, as also emphasized by [26], if confirmed this mass rules out any significant phase transitions below a density $\approx$ 5 times nuclear saturation density.

3. Future Prospects

What improved constraints can be expected in the future? Serendipity has played an important role in mass determinations and EOS constraints, and may in the future as well. For example, the mass estimates of millisecond pulsars with compact companions are the most accurate available, so if a new one is discovered with a high mass this will be a rigorously reliable data point. However, one cannot mandate serendipity and the current selection of precisely known masses of millisecond pulsars is too small to draw sweeping conclusions.

Optical and infrared observations of neutron stars in accreting main sequence binaries are more promising. Since these stars, especially in low-mass binaries, may have accreted a few tenths of a solar mass, they will constrain more tightly the state of high-density matter. Extensive observing campaigns similar to those conducted for Vela X-1 and Cyg X-2 may be required. Potentially even more helpful is the analysis and continued observation of kilohertz QPOs from neutron star LMXBs. As discussed above, a QPO with a frequency just 150 Hz above the current maximum observed would strongly constrain the hardness of high-density matter. In addition, continued accumulation of data from 4U 1820–30 will help in assessing whether the observed frequency rollover is really a signature of the ISCO or is instead an effect of spectral change. Observation of another source with this signature would be valuable as a means of comparison, but this also appeals to serendipity.

A new way to estimate the masses and radii of neutron stars may come from the high spectroscopic resolution X-ray observations just now available from *Chandra* and *XMM*. If heavy elements are present in the atmospheres of cooling neutron stars, their line and edge opacities can leave a clear signature in the thermal emission from these stars [29]. If enough such features are detected then the mass, radius, temperature, surface composition, and surface magnetic field can be decoupled and determined with precision. However, the high surface gravity of neutron stars dictates that the lightest element present in abundance will rise to the surface and dominate the emergent spectrum. If this is hydrogen or helium it will be completely ionized and produce few if any detectable features.

In my view the best future method of obtaining information about neutron star masses and radii will be fitting of the waveforms of periodic X-ray oscillations from accreting neutron stars. A neutron star with a spin frequency of 300 Hz has a surface equatorial rotation velocity of $\approx 0.1\ c$ and hence produces strong Doppler shifts, whose magnitude depends on the stellar radius. General relativistic light deflection also affects the waveform of such oscillations. The amount of deflection depends on the mass to radius ratio, so combined with the information from Doppler shifts the mass and radius can be determined independently. To extract this information requires *periodic* oscillations caused by rotational modulation of a "hot spot". This occurs during thermonuclear bursts on neutron stars [30], and preliminary work is underway to estimate the masses and radii of the stars that exhibit brightness oscillations during bursts [31]. However, the collecting area of RXTE is insufficient to get enough data for precise constraints on mass or radius.

An X-ray timing satellite with ten times the collecting area of RXTE would be the first instrument able to characterize accurately the waveforms and energy-dependent phase lags of the high-frequency QPOs observed from compact objects in low-mass X-ray binaries. Measurements of the waveforms and energy-dependent phase lags of the burst oscillations with such an instrument could yield both the mass and radius of several neutron stars with 5-10% accuracy [32,33]. Combining the constraints from several neutron stars would allow an enormous advance in our understanding of high-density matter, as well as, for the first time, testing quantitatively general relativity in strong gravity.

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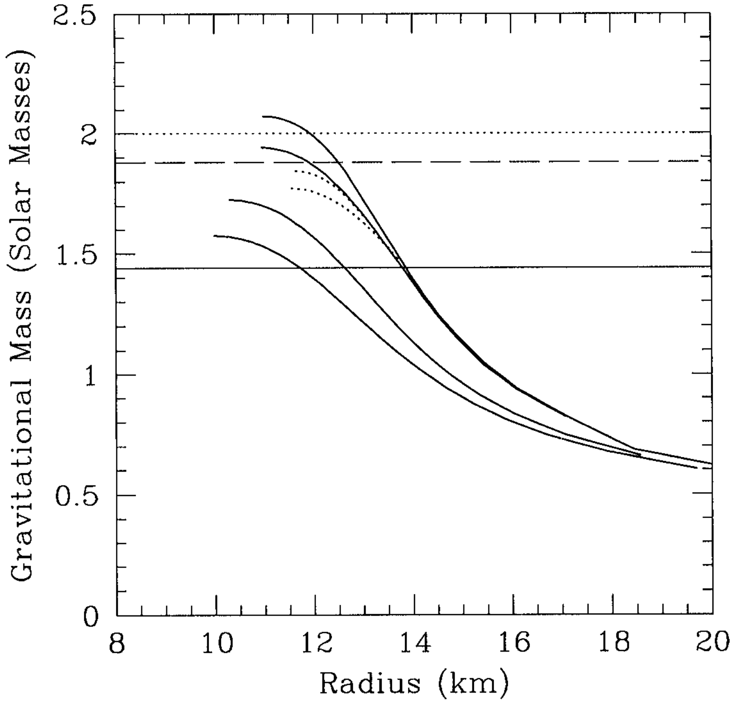

Figure 1. Summary of equation of state constraints from neutron star masses. The solid curves are the stable portions of the mass-radius relations predicted with the parameterized equations of state used by [26]: from bottom to top, their softness parameter s (see text) is 0.4, 0.3, 0.2, and 0.13. The dotted curves are the stable portions of the relations predicted using two representative values of the quark bag constant, B=150 (lower curve) and 200 MeV fm^{-3} [26]. The horizontal lines represent the highest neutron star masses inferred using various methods. The solid line at $M = 1.44\, M_\odot$ is the highest mass measured using millisecond pulsar timing; the dashed line at $M = 1.88\, M_\odot$ is the 95% lower limit to the mass of the accreting low-mass X-ray binary Cyg X-2, inferred from optical observations of the companion; and the dotted line at $M = 2\, M_\odot$ is the mass inferred from the frequency rollover at 1090 Hz [24] of the quasiperiodic brightness oscillations in 4U 1820–30. We stress that rotation of the neutron star is neglected in all these curves, to provide a common basis for comparison. Rotation at $\approx$ 300 Hz, which is likely for neutron stars in LMXBs, increases the maximum allowed mass for a given EOS by $\approx$10%. However, it also increases the *required* mass for the neutron star in 4U 1820–30 by close to the same amount, to $2.15\, M_\odot$, and hence the constraints on the softness parameter are unchanged to first order. Mass-radius data were kindly provided by Henning Heiselberg.