AN UPPER LIMIT TO THE VELOCITY DISPERSION OF RELAXED STELLAR SYSTEMS WITHOUT MASSIVE BLACK HOLES

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ABSTRACT

Massive black holes have been discovered in all closely examined galaxies with high velocity dispersion. The case is not as clear for lower-dispersion systems such as low-mass galaxies and globular clusters. Here we suggest that above a critical velocity dispersion ~40 km s^{-1}, massive central black holes will form in relaxed stellar systems at any cosmic epoch. This is because above this dispersion primordial binaries cannot support the system against deep core collapse. If, as previous simulations show, the black holes formed in the cluster settle to produce a dense subcluster, then given the extremely high densities reached during core collapse the holes will merge with each other. For low velocity dispersions and hence low cluster escape speeds, mergers will typically kick out all or all but one of the holes due to three-body kicks or the asymmetric emission of gravitational radiation. If one hole remains, it will tidally disrupt stars at a high rate. If none remain, one is formed after runaway collisions between stars, and then it tidally disrupts stars at a high rate. The accretion rate after disruption is many orders of magnitude above Eddington. If, as several studies suggest, the hole can accept matter at that rate because the generated radiation is trapped and advected, then it will grow quickly and form a massive central black hole.

Key words: accretion, accretion disks – binaries: general – black hole physics – galaxies: bulges – galaxies: clusters: general – gravitation

1. INTRODUCTION

Observations over the last two decades have revealed central massive black holes in all sufficiently well observed massive galaxies (e.g., Gültekin et al. 2011). However, the case is not as clear for lower-mass galaxies or globular clusters, and indeed although there is evidence for black holes in some low-mass galaxies (Greene et al. 2010; Kuo et al. 2011), there are examples of galaxies that clearly do not have black holes that follow the standard mass–velocity dispersion (M–σ) relation (Merritt et al. 2001; Gebhardt et al. 2001), and the case for globular clusters is far from clear (e.g., Gerssen et al. 2002; McNamara et al. 2003; Baumgardt et al. 2003; Strader et al. 2012).

Here we approach this question by focusing on the velocity dispersion rather than the mass of a stellar system. In Section 2 we show that above a critical velocity dispersion $\sigma_{\rm crit} \sim 40$ km s^{-1}, the total binding energy in primordial binaries that can be tapped in three- and four-body interactions is significantly less than the total binding energy of the system as a whole, and hence if such systems are dynamically relaxed, they will undergo deep core collapse essentially unhindered by dynamical heating from binaries (thus leading to one of the scenarios discussed by Begelman & Rees 1978 in the context of more massive clusters). We note that the galaxies seen thus far without massive black holes have velocity dispersions below this limit (e.g., NGC 205 has $\sigma = 39$ km s^{-1} and M33 has $\sigma = 24$ km s^{-1}; see Gültekin et al. 2009 and references therein). In Section 3 we discuss the evolution of binary-free systems. Previous studies have demonstrated that the black holes in such systems sink rapidly to the center and interact mostly with each other in a dense subcluster. This leads to three paths, all of which culminate in the formation of a massive black hole: (1) For sufficiently high escape speed systems dynamical interactions result in runaway merging of the black holes into a massive hole. For lower escape speed systems either one or zero black holes remain after ejection of merged pairs due to asymmetric emission of gravitational radiation during

coalescence or Newtonian recoil from interactions of black holes with dynamically formed binaries. (2) If one black hole remains, then it tidally disrupts ordinary stars and consumes the remnant disks quickly, hence growing rapidly into a massive black hole; other growth mechanisms, such as the accretion of nascent gas or winds, are insignificant. (3) If no black holes remain, then runaway collisions form a massive star that evolves into a black hole, and this first black hole grows via accumulation of tidally disrupted stars. Thus, once binary support is removed, massive black hole formation is assured as long as holes consume tidal remnants quickly. In Section 4, we determine the minimum mass of a black hole formed via these paths and discuss the implications of this scenario.

2. VELOCITY DISPERSION THRESHOLD FOR DEEP CORE COLLAPSE

Stellar systems that are in virial equilibrium evolve via two-body interactions over their relaxation time, which for a star of mass m in a system of velocity dispersion σ at a location with an average stellar-mass density ρ is

$$t_{\rm rlx} \approx \frac{0.3}{\ln \Lambda} \frac{\sigma^3}{G^2 \rho m} \qquad (1)$$

(Spitzer 1987), where $\ln \Lambda \sim$ 5–10 is the Coulomb logarithm. The evolution of an isolated stellar system is toward a greater concentration of stars in the center balanced by a greater expansion of the cluster on the outskirts; there is a productive analogy with thermodynamics in which this behavior can be seen as the gradual increase of cluster entropy (the greater phase space accessed by the outer stars more than makes up for the diminished phase space accessed by the stars in the core). It was demonstrated several decades ago that if all the stars are single (as opposed to being in binary or multiple systems), then over a timescale that scales with the relaxation time at the half-mass radius for a typical star (where the multiple is ~15 for an

initially Plummer sphere of equal-mass stars but is ∼0.2 if there is a broad initial mass function; see Portegies Zwart & McMillan 2002), the core becomes so dense that it loses thermal contact with the rest of the cluster and the core undergoes a collapse such that the number density in the inner portions scales as $n \sim r^{-2.2}$ (Lynden-Bell & Eggleton 1980; Cohn 1980). If we take present-day nuclear star clusters as an example, then from Figure 1 of Merritt (2009) we find that most have half-mass relaxation times less than few × 10^10 yr and thus are candidates to collapse within a Hubble time if they had broad initial mass functions and no central massive object to supply energy.

Binaries are the key to sustaining a cluster against this collapse. When number densities become high enough that binary–single interactions are common, such interactions can harden the binary and hence inject energy into the cluster that decreases its density. Many calculations (see, e.g., Hénon 1961; Heggie 1975 for pioneering work) have shown that binaries that are initially hard (meaning that their binding energy exceeds the kinetic energy of a typical single star) tend to harden via binary–single interactions, whereas initially soft binaries tend to soften and eventually break up. Consistent with this expectation, globular clusters have a significantly smaller binary fraction than is seen in the field (e.g., Rubenstein & Bailyn 1997; Milone et al. 2012).

In principle, even a very small number of binaries could have enough binding energy to hold off the collapse of a cluster. Consider, for example, a reasonably rich globular cluster with a velocity dispersion of 10 km s⁻¹. A binary of two solar-mass stars near contact, with an orbital radius of 0.01 AU, has ∼10³ times the binding energy per mass that a single cluster star has in kinetic energy, so if 0.1% of stars are in such binaries, the energy to hold off cluster collapse appears to be present. White dwarfs are 100 times smaller yet, so it might seem that if there is one near-contact white dwarf binary in a cluster of 10⁵ stars, its binary interactions could successfully oppose core collapse.

This is of course not true, for two reasons. First, as the semimajor axis of a binary shrinks, its close interactions with single stars have a greater and greater chance of destroying the single star or one of the binary stars; hence, the kinetic energy of recoil is not shared with the cluster (Davies et al. 1994). As an example, a tight white dwarf binary cannot eject a main-sequence star in this way. Second, even if a three-body interaction is clean, a star that is thrown completely from the cluster cannot share its kinetic energy with the cluster and the only expansion of the core comes from the comparatively minor effect that the core now has lost one star's worth of mass.

The available binding energy from binaries is thus limited; clusters having higher velocity dispersions have a more limited available binding energy. As we now argue, this means that above a velocity dispersion $\sigma_{\rm crit} \sim 40$ km s⁻¹, the binaries cannot hold off core collapse. It should be noted that the velocity dispersion of a cluster will evolve as a function of time, with velocity dispersions being somewhat larger in the past when the cluster was more massive (e.g., Giersz & Heggie 2009; Küpper et al. 2010). The effect could be particularly enhanced for clusters containing multiple stellar populations where a large fraction of the first generation of stars are lost (D'Ercole et al. 2008). However, the velocity dispersion at later times will be more relevant to the discussion in this paper, as this is when core collapse may typically be possible (i.e., on timescales longer than the half-mass relaxation time).

As a first estimate of the available binding energy for a binary with initial semimajor axis a_0, we assume that the eccentricity

distribution of binaries with a given semimajor axis is a thermal distribution $P(e < e_0) = e_0^2$ truncated at the maximum eccentricity $e_{\rm max}$ allowed for pericenter distances greater than some minimum $r_{\rm p,min}$ (this could be the pericenter distance at which stars collide), $a(1 - e_{\rm max}) = r_{\rm p,min}$ for a semimajor axis a. Thus, a fraction $e_{\rm max}^2$ of orbits are allowed, and the binding energy that can be released from semimajor axis $a + da$ to a is weighted by $e_{\rm max}^2(a) = 1 - 2r_{\rm p,min}/a + (r_{\rm p,min}/a)^2$. Thus, the total available binding energy from an initial semimajor axis a_0 with stars of mass m is

$$E_{\rm bind,tot}(a_0) = \int_{r_{\rm p,min}}^{a_0} \frac{Gm^2}{2a^2} e_{\rm max}^2(a) da. \tag{2}$$

This gives

$$E_{\rm bind,tot}(a_0) = \frac{Gm^2}{2r_{\rm p,min}} \left[\frac{1}{3} - \frac{r_{\rm p,min}}{a_0} + \left(\frac{r_{\rm p,min}}{a_0} \right)^2 \right.$$
$$\left. - \frac{1}{3} \left(\frac{r_{\rm p,min}}{a_0} \right)^3 \right]. \tag{3}$$

For $a_0 > 10r_{\rm p,min}$, $E_{\rm bind,tot}$ is roughly constant at $Gm^2/6r_{\rm p,min}$, whereas it decreases rapidly below $10r_{\rm p,min}$, so for simplicity we will approximate $E_{\rm bind,tot}$ as zero below $10r_{\rm p,min}$ and $Gm^2/6r_{\rm p,min}$ above it.

Our next step is to note that for stars formed in a low-density environment, there is roughly one binary per single star, and the binary semimajor axes are approximately equally distributed in $\ln a$ from 0.01 AU to ∼10⁴ AU (Popova et al. 1982). In an environment where binaries beyond a certain semimajor axis are ionized by binary–single encounters, the fraction of binaries will be decreased. For example, if we begin with six single stars and six binaries and ionize the ones larger than 1 AU, we now have fourteen single stars and two binaries. If as above we now only concentrate on the binaries larger than 0.1 AU = $10r_{\rm p,min}$, this represents $f \sim 7\%$ of the stars in the system. Thus, the binary binding energy per *all* stars in the system is

$$e_{\rm bin}/{\rm star} = fGm^2/(6r_{\rm p,min}). \tag{4}$$

This is to be compared with the binding energy per star in the cluster, which by the virial theorem equals the kinetic energy per star in the cluster, or

$$e_{\rm cluster}/{\rm star} = \frac{1}{2}m\sigma^2 \tag{5}$$

for a velocity dispersion σ. The point at which $e_{\rm bin}/{\rm star} < e_{\rm cluster}/{\rm star}$ is the point at which core collapse is theoretically possible. From the numbers above, if the single stars and the binary components both have masses ≈1 $M_\odot$, this happens when $\sigma \sim 40$ km s⁻¹, meaning that interactions with binaries of semimajor axis ≳0.5 AU have positive total energy and are thus soft, and core collapse can proceed. If the initial distribution of binary binding energies is extremely unusual, e.g., if most stars are formed in binaries with semimajor axes less than 0.5 AU, then the supply of binary energy would be greater and the threshold velocity dispersion could in principle be raised. Barring such an unexpected distribution, however, the threshold should be robust.

Indeed, work by Chernoff & Huang (1996) suggests that there may be less binary energy available than we derive above. They take into account that, rather than simply resetting

the eccentricity of a binary, a binary–single encounter can be resonant and hence for a given interaction there is a greater chance to get to a very small separation. From their Figure 4 we infer that for solar-type stars and $\sigma = 40\,\mathrm{km\,s^{-1}}$ a typical energy $\Delta E \approx 6 \times 10^{46}$ erg can be extracted from an initially hard binary, whereas Equation (3) gives roughly an order of magnitude larger energy. Thus, at $\sigma = 40\,\mathrm{km\,s^{-1}}$, and perhaps at a slightly lower velocity dispersion, the energy that can be extracted from primordial binaries is significantly less than the binding energy of the cluster; hence, such clusters can undergo core collapse without being impeded significantly (three-body binary formation and two-body tidal capture are also insignificant; see Hut & Bahcall 1983 and Fabian et al. 1975, respectively).

3. PATHS TOWARD MASSIVE BLACK HOLE FORMATION

We now evaluate the paths toward massive black hole formation that we mentioned in the introduction: runaway merging of black holes, tidal disruption of stars by a single remaining black hole, and formation of a new black hole from runaway collisions of stars, followed by tidal disruption of stars by that black hole. We first address the relevant timescales. In all three paths, the overwhelmingly longest phase is the initial progression to core collapse. To see this, note that the time to core collapse is a multiple less than unity (~ 0.2 for systems with a broad mass distribution; see Portegies Zwart & McMillan 2002) of the relaxation time of the nuclear star cluster, which from Figure 1(a) of Merritt (2009) is $t_{\mathrm{rlx}} \sim 10^9\,\mathrm{yr}(M/10^6\,M_\odot)$ with a spread of a factor ~ 10. There is some evidence that nuclear star clusters obey a similar M–σ relation to that seen for higher-mass black holes. There is some observational evidence for this; e.g., Figure 2 from Ferrarese et al. (2006) indicates that nuclear star clusters might have the same M–σ slope as has been found for black holes (see Gültekin et al. 2009 for a recent discussion of this relation) but offset so that the mass is a factor of ~ 10 higher than the black hole mass would be. If we loosely equate the velocity dispersion of the cluster with that of the surrounding bulge, this gives a cluster mass of

$$M_{\mathrm{cl}} \approx 10^6\,M_\odot(\sigma/40\,\mathrm{km\,s^{-1}})^4 \qquad (6)$$

based on the scalings of Gültekin et al. (2009). Thus, clusters with $\sigma \lesssim 100\,\mathrm{km\,s^{-1}}$ have a chance to undergo core collapse within a Hubble time. Figure 1 illustrates the paths we consider.

This step is necessary for all three paths we discuss. For a collapsed core, the self-similarity arguments of Lynden-Bell & Eggleton (1980) and the classic simulations of Cohn (1980) show that if all the stars are treated as point masses and no three-body binary formation is allowed, then the density of a single-mass system evolves toward an $n \propto r^{-2.2}$ configuration. This is quite close to a singular isothermal sphere $n \propto r^{-2}$; hence, we will assume that the velocity dispersion is nearly constant in the collapsed region. As a result, the relaxation time scales roughly as ρ^{-1}, so the evolution timescale is shorter by orders of magnitude in the core of the cluster than it is in the cluster as a whole.

As a result, once the core collapses, all three paths are traveled in a time much shorter than the time to collapse. For example, runaway mergers between black holes (or, in the third path, runaway collisions between stars) occur roughly on the core relaxation timescale because when the number density is not yet sufficient for frequent mergers or collisions, further relaxation will increase the density on the relaxation time until interactions

Velocity Dispersion

40 km/s   100 km/s

Binaries prevent core collapse.

Runaway BH merger? Unlikely: ejection nearly certain.

Relaxation time too long for core collapse.

One BH remaining? Growth to MBH via tidal disruptions.

No BH remaining? Runaway stellar collisions produce BH, growth to MBH via tidal disruptions.
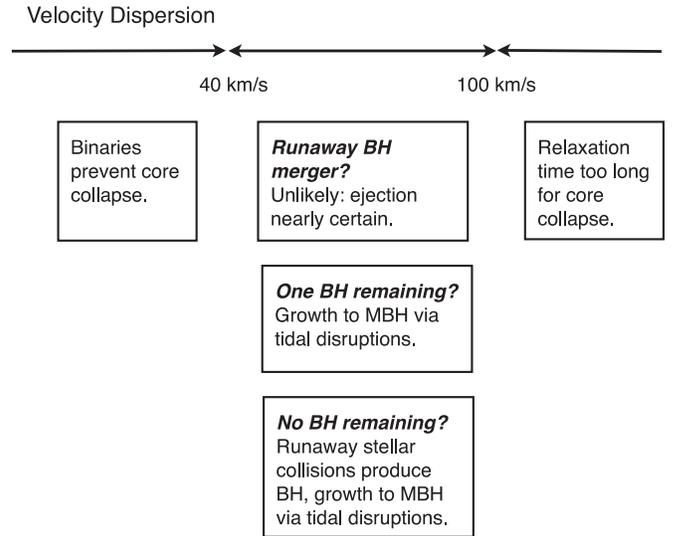
Figure 1. Paths toward massive black hole formation in a stellar cluster. At velocity dispersions less than $\sim 40\,\mathrm{km\,s^{-1}}$, heating from binaries prevents full core collapse. At velocity dispersions greater than $\sim 100\,\mathrm{km\,s^{-1}}$, a typical nuclear star cluster will have too long a relaxation time for its core to collapse in a Hubble time, although a massive black hole can form in other ways. At intermediate velocity dispersions, full core collapse will occur and will likely result in either zero or one remaining stellar-mass black hole. In the latter case, the hole will grow via tidal disruption of stars; in the former, the stars will undergo runaway collisions that produce a black hole, which will then grow via tidal disruption of stars.

are frequent. Thus, the only limiting factor is the initial collapse time. We also note that unlike in the scenario of runaway collapse of young massive clusters proposed by Portegies Zwart & McMillan (2002), the time window for runaway collisions of stars to form a single black hole (in the third path) is not millions of years, but billions of years. The reason is that when the cluster is young enough that initially all stars are on the main sequence, supernovae from the most massive stars begin at ~ 2.5 Myr and proceed for many stars, causing the core to lose a large amount of mass to the ejecta and therefore expand and lower the number density. In contrast, in our picture the evolution to core collapse is much later, perhaps billions of years; hence, the remaining stars are of low mass and thus only the collision product will be massive enough to explode. Very little mass is lost, so the density remains high.

In addition to the general core collapse, in a multimass system there is considerable mass segregation. This means that the stars in the core will tend to be toward the massive end, perhaps $\sim 1\,M_\odot$ after billions of years. In addition, of the objects likely to be present after a long time, stellar-mass black holes will be by a factor of a few to several the most massive. Many studies (e.g., Mackey et al. 2008) have concluded that the black holes then form a dense subcluster in which the holes interact mainly with themselves. If, as in our scenario, there are no binaries, then the holes can reach extremely high density in the center of the subcluster and capture each other via emission of gravitational radiation in initially hyperbolic two-body encounters. From Quinlan & Shapiro (1989), the critical pericenter for a two-body gravitational wave capture between two black holes with a total mass $M = m_1 + m_2$ and a reduced mass $\mu = m_1 m_2/M$ is

$$r l r_{\mathrm{p,GW}} = 8.5 \times 10^8\,\mathrm{cm}(M/20\,M_\odot)^{5/7}$$
$$\times (\mu/5\,M_\odot)^{2/7}(\sigma/40\,\mathrm{km\,s^{-1}})^{-4/7}, \qquad (7)$$

and hence their gravitationally focused cross section is

$$\Sigma_{bh} = 2\pi r_p GM/\sigma^2 \approx 9 \times 10^{23}\,\mathrm{cm}^2 (M/20\,M_\odot)^{12/7}$$
$$\times (\mu/5\,M_\odot)^{2/7}(\sigma/40\,\mathrm{km\,s^{-1}})^{-18/7}. \qquad (8)$$

When two black holes capture each other in this way, their inspiral is extremely rapid: from Peters (1964), the inspiral time is

$$rla/(da/dt) = \frac{5}{64}\frac{c^5 a^4 (1-e^2)^{7/2}}{G^3 \mu M^2(1+73e^2/24+37e^4/96)}$$
$$\approx 10^5\,\mathrm{yr}(a/R_\odot)^4(1-e^2)^{7/2}, \qquad (9)$$

where the approximation is for $e \approx 1$ and our given number assumes $m_1 = m_2 = 10\,M_\odot$. For $a \approx 1\,R_\odot = 7 \times 10^{10}$ cm and $e > 0.99$ (so that $r_p < r_{p,GW}$), the inspiral time is therefore less than 0.1 years, and for a fixed r_p the inspiral time scales as $a^{1/2}$ so that even for an initial $a = 100$ AU the inspiral time is just a few years.

When the holes do merge, they emit gravitational radiation that is in general asymmetric, meaning that the remnant single black hole will recoil relative to its original center of mass. Studies of black hole recoil (Baker et al. 2007, 2008; Lousto & Zlochower 2008, 2009; Lousto et al. 2010, 2012; van Meter et al. 2010) show that although kicks from the coalescence of nonspinning black holes are limited to $<200\,\mathrm{km\,s^{-1}}$, rapidly spinning black holes can produce remnants that travel at thousands of kilometers per second relative to their original center of mass. Thus, in this environment, unlike in the conditions that may exist in the $z > 10$ universe (Davies et al. 2011), mergers that are restricted to comparable-mass black holes are most likely to lead to an ejection of the remnant.

As pointed out to us by S. Sigurdsson (2012, private communication), for low velocity dispersions the ejection of black holes is likely to be dominated by encounters with hard binaries formed by the interaction of three initially hyperbolic black holes. Hut (1985) finds that the rate of formation of "immortal" binaries by this process (i.e., binaries that are not later softened and ionized) is $\dot{n}_{3B} = 126 G^5 m^5 n^3/\sigma^9$, where he assumes objects of identical mass m and σ is the three-dimensional velocity dispersion. Thus, the ratio of the formation rate per volume of these binaries to the rate of gravitational wave capture of black holes by each other, assuming equal masses, is

$$\frac{\dot{n}_{3B}}{\dot{n}_{BHBH}} \approx 200(n/10^{10}\,\mathrm{pc}^{-3})(m/10\,M_\odot)^3(\sigma/40\,\mathrm{km\,s^{-1}})^{-52/9}. \qquad (10)$$

Thus, for low to moderate velocity dispersions and high number densities, binary–single ejections are likely to dominate. The result will be similar to the case in which only double black hole mergers occur: there will either be zero or one hole left. For the rest of this section we concentrate on ejections by mergers.

We set up a simple simulation of the evolution of a black hole subcluster with no binaries. We assume that initially there are either 100 or 101 black holes; note that even if all mergers eject the remnant, having an odd number initially guarantees that one will survive because with no binaries the interactions are pairwise. The distribution of black hole masses is not well established, and the distribution of their spins is even less so, but as an illustrative example we draw the initial masses of the black holes from the range $[5, 30]\,M_\odot$, with a distribution
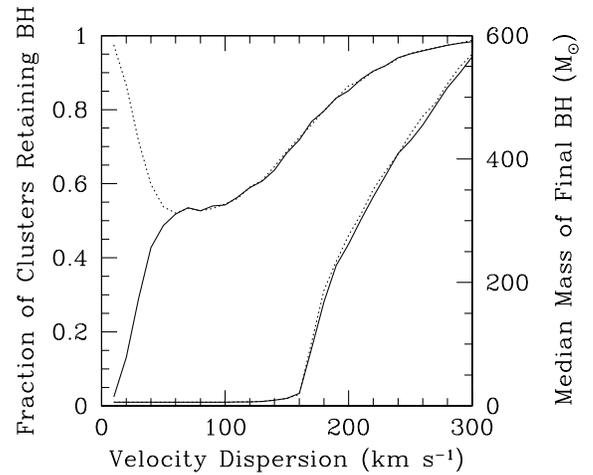
Figure 2. Fraction of clusters of a given velocity dispersion that retain a black hole after a succession of mergers (upper left curves) and, if a black hole is left, the median mass of the remaining black hole (lower right curves). For this figure we ignore the effects of hard binaries formed by the interactions of three initially hyperbolic black holes (see the text); hence, there is a difference between cases with an initially even and an initially odd number of black holes. The solid curves are for 100 initial black holes, and the dotted curves are for 101 initial black holes; the asymmetry in retained fraction at low velocity dispersions is because if every black hole merger results in an ejection, an initially even number will leave behind no black holes, whereas an initially odd number will leave behind one. We assume an escape speed that is four times the velocity dispersion. This figure demonstrates that retained runaway mergers leading to massive seeds are only likely for velocity dispersions $\gtrsim 200\,\mathrm{km\,s^{-1}}$.

$dN/dM \propto M^{-2}$, and the initial spins are drawn uniformly from the range $cJ/(GM^2) = [0, 1)$.

We simulate the evolution of the cluster interaction by interaction using the rejection method: we select two black holes randomly, compute the cross section Σ of the interaction, divide by the largest possible cross section Σ_{max} (which is the cross section of capture by the two most massive black holes in the sample), and then compare that ratio with a uniform random deviate $x \in [0, 1)$. If $x < \Sigma/\Sigma_{max}$, we accept the interaction; otherwise, we draw again.

If the interaction is between two black holes, then we use the recent Lousto et al. (2012) formula for the kick. If the kick is greater than the escape speed $v_{esc} = 4\sigma$ (typical of a core-collapsed cluster), we assume that the remnant has been ejected from the cluster and thus remove both black holes from the sample. Otherwise, we assume that the remnant remains; hence, we sum the masses of the holes and estimate the spin of the remnant following the prescription given in Rezzolla et al. (2008).

Figure 2 shows the results. Here we plot the fraction of clusters that retain a black hole after subcluster evolution, as well as the median mass of the final black hole if one remains, as a function of the velocity dispersion σ of the cluster. For each velocity dispersion we performed 10^4 simulations. For low escape speeds, almost all mergers between black holes eject the remnant; hence, retention depends on whether the initial number of holes is even or odd. As the escape speed increases, so does the probability that a merger will not eject the remnant; for $v_{esc} \lesssim 100\,\mathrm{km\,s^{-1}}$ it is most probable that this happens when the spins of the holes are low and their masses are close to each other (note from symmetry that there is zero recoil from the merger of equal-mass nonspinning holes). As the escape speed increases further, mergers between black holes of different masses can be retained, until at $v_{esc} \gtrsim 800\,\mathrm{km\,s^{-1}}$ a runaway occurs and a single victorious black hole is usually the result.

From these simulations we can argue that for clusters with velocity dispersions $\lesssim 100$ km s^{-1} a runaway is unlikely, but that there is roughly an equal chance of leaving behind either one or zero holes (depending largely on the parity of the initial number until $\sigma \gtrsim 60$ km s^{-1}). When there is a black hole left behind, it is likely for $\sigma \lesssim 100$ km s^{-1} to be at the low end of the mass distribution ($\sim 5\,M_\odot$) because such black holes are initially more common. In addition, lower-mass black holes have a lower cross section for capture and hence an enhanced probability of survival.

The subsequent evolution has the following two possibilities:

One black hole remains. Then, as we discuss in the next section, the black hole will sit near the center of the high number density distribution of stars. Tidal disruptions will add a few tens of percent of the stellar mass to the hole, mostly within a few weeks or less of the initial disruption, and hence the hole will grow quickly. Given that interactions with the stars cannot eject the hole from the cluster, it will become a massive black hole in a short timescale.

No black holes remain. In this case, the stars will undergo runaway collisions with themselves, leading to the production of a massive star that will then become a black hole (e.g., Portegies Zwart & McMillan 2002). The situation then reduces to the previous case because the time needed to produce a *second* black hole, which could potentially eject the first, is significantly larger than the time needed for the first hole to increase its mass to the point that it can no longer be ejected.

We now discuss these possibilities in greater depth.

3.1. Interactions between Stars and a Black Hole

Although the central density after core collapse is formally infinite, the finite number of stars means that this translates to a few stars in a small region near the core. For example, if we consider the inner ~ 10 solar-type stars after core collapse and continue to assume a constant velocity dispersion, then they are in a region $r = GM/\sigma^2 \sim 5\,\mathrm{AU}(M/10\,M_\odot)(\sigma/40\,\mathrm{km\,s}^{-1})^{-2}$ in radius, with a resulting number density of $n > 10^{14}$ pc^{-3}. Even the inner 1000 stars are in a region with $n > 10^{10}$ pc^{-3}, so interactions will be common and rapid.

Stellar tidal disruption by black holes. A promising mechanism for such runaway growth is tidal disruption of stars by stellar-mass black holes. The critical pericenter for tidal disruption of a star of mass m and radius R by a black hole of mass M is

$$r_{\mathrm{p,tidal}} = (3M/m)^{1/3}R \ . \tag{11}$$

Thus, the gravitationally focused cross section for tidal disruption, assuming that the black hole mass greatly exceeds the stellar mass, is

$$\Sigma_{\mathrm{tidal}} \approx 10^{26}\,\mathrm{cm}^2 (M/10\,M_\odot)^{4/3}(\sigma/40\,\mathrm{km\,s}^{-1})^{-2} \tag{12}$$

for solar-type stars. This is roughly an order of magnitude greater than the star–star collision cross section discussed later, and two orders of magnitude larger than the black hole—black hole capture cross section. Moreover, the rate is nonlinear in the mass of the black hole ($\Sigma \propto M^{4/3}$). Thus, the conditions for a runaway exist.

If tidal disruption does occur, then the mass will be force-fed to the black hole at an extremely super-Eddington rate. Studies suggest that fallback initially occurs over several times the internal dynamical time of the disrupted star (Evans & Kochanek 1989), which is several hours for a solar-type star. The accretion rate is therefore many millions of times the Eddington rate of a

stellar-mass black hole. Analyses of such supercritical accretion (e.g., Maraschi et al. 1976; Begelman 1979; Jaroszynski et al. 1980; Popham et al. 1999; Ohsuga et al. 2005) indicate that the matter will indeed flow into the hole at that rate, but that most of the photon luminosity that is generated will be advected in with the very optically thick matter (hence, although the accretion rate is tremendously super-Eddington, the luminosity could be limited to Eddington or slightly higher). Thus, it is expected that within a matter of days, i.e., much shorter than any other relevant timescale, most of the bound remainder of the star will flow onto the hole. If this is the case, then the majority of the accretion will finish without harassment from additional encounters by stars. If, on the contrary, the accretion rate is actually limited to the Eddington rate, then the time needed to accrete most of the matter is much longer than the time to the next encounter, and the disk might be disrupted, leading to negligible growth of the hole.

The unbound remnant of the star will be thrown outward at speeds comparable to the orbital speed at tidal disruption, which is ~ 800 km s^{-1} $(M/10\,M_\odot)^{1/3}$ for a solar-type star. This is much larger than the escape speed, so the wind will depart ballistically unless it runs into many times its own mass in gas in the cluster. However, given that the virial temperature of the cluster is $\sim 10^5\,\mathrm{K}(\sigma/40\,\mathrm{km\,s}^{-1})^2$ and that cooling is extremely efficient at that temperature, the total amount of gas in the cluster at a given time will be small even though its escape speed is sufficient to retain winds from red giants or (earlier, when more massive stars existed) planetary nebulae. Thus, we assume that the unbound gas simply escapes from the cluster. The ratio of unbound gas to gas that accretes onto the black hole is rather uncertain. The initial disruption leaves about half the mass bound (Evans & Kochanek 1989), but shocks upon the return of the bound matter might unbind additional mass. In a recent study by Strubbe & Quataert (2009), they consider different ejection fractions ranging from $f_{\mathrm{esc}} = 0.5$ (corresponding to negligible return shocks) to $f_{\mathrm{esc}} = 0.8$ (corresponding to powerful return shocks). In our scenario, the upshot is that because a single black hole will grow, its growth will eject up to a few times its own mass in stellar debris. Until this reaches at least hundreds, and probably thousands, of solar masses, this will be such a small fraction of even the core mass that we expect it to have a minor effect on the dynamical evolution.

Star–star collisions. At the velocity dispersions we consider, these collisions are likely to lead to mergers with little mass loss because $\sigma \sim 40$ km s^{-1} is much less than the escape speed ~ 600 km s^{-1} of a solar-type star. For the same reason, these collisions are gravitationally focused, with a cross section $\Sigma = \pi r_p^2(1 + 2GM_{\mathrm{tot}}/(r_p\sigma^2)) \approx 2\pi(GM_{\mathrm{tot}}/\sigma^2)r_p$ for a pericenter distance r_p and a total mass between the stars of M_{tot}. The relevant pericenter distance is the sum of the stellar radii, which is $2\,R_\odot \approx 0.01$ AU for two solar-type stars; hence, for two such stars $\Sigma \approx 1.5 \times 10^{25}$ cm$^2(\sigma/40\,\mathrm{km\,s}^{-1})^{-2}$. The characteristic time of interaction is then $\tau = 1/(n\Sigma\sigma) \approx 10^6$ yr$(n/10^{10}\,\mathrm{pc}^{-3})^{-1}(M_{\mathrm{tot}}/2\,M_\odot)^{-1}(\sigma/40\,\mathrm{km\,s}^{-1})^{-1}$. Note that as a result even for the inner $\sim 10^3$ stars the collision time for solar-type stars is much less than their $\sim 3 \times 10^7$ yr Kelvin–Helmholtz time, and hence the stars will not be able to radiate their collisional energy before the next collision. However, because the velocity dispersion is <0.1 times the stellar escape speed, the energy added is minor and most of the pressure holding up the collision product stems from gravitational contraction rather than either collision energy or nuclear energy; these are thus not stars in the standard sense and need not have

luminosities as high as those of main-sequence stars of the same mass.

In addition, on the main sequence, stellar radii increase with increasing mass; hence, the rate of interactions increases more than linearly with increasing stellar mass. An additional factor is that more massive stars tend to sit closer to the center of the potential, where the number density of objects is greater. The conditions are thus ripe for a runaway, and indeed runaway merging of stars has been proposed as a mechanism for the generation of supermassive stars that later evolve into intermediate-mass black holes (Portegies Zwart & McMillan 2002; Freitag et al. 2006). It has been suggested that the high wind rates expected for high-mass stars can severely limit the growth of supermassive stars (Glebbeek et al. 2009). Note, however, that these wind rates are based on extrapolations of winds for main-sequence stars, and as indicated above, the collision products will be substantially larger and less luminous than main-sequence stars. Indeed, the collision products are more likely to be a "bag of cores" than an actual star, where an extended gaseous envelope engulfs an ensemble of stellar cores.

We do note that although Glebbeek et al. (2009) argue that winds may prevent the formation of intermediate-mass black holes, they find that runaway collisions produce stars massive enough to evolve to normal stellar-mass black holes, at least. Thus, for our purposes we assume that star–star collisions will lead to black hole production.

The question is then whether the first black hole that forms has enough time to consume many stars so that by the time the next black hole forms, the first one is so massive that any black-hole–black-hole merger will produce a weak recoil that retains the remnant in the cluster. We argue that this is in fact the case: the first black hole to form will be at the center of the mass distribution, where the number density is the highest. If this is, for example, in the region occupied by the inner $\sim$100 stars, then the number density is such that tidal disruptions of stars by even a $10\,M_\odot$ black hole occur on average once per few hundred years, and the interaction time scales as $M^{-4/3}$. Thus, the hole will double its mass every few thousand years, i.e., in a time vastly shorter than the lifetime of even the most massive stars. We note that although the segregation of the black holes to the center of the cluster and their ejection lead to some flattening of the stellar number density near the center of the cluster (see, e.g., Aarseth 2012 and in particular his Figure 8 for a recent N-body simulation), we expect that when most of the black holes have been ejected the stars near the core, which have a very short relaxation time, will regrow the cusp. Hence, the first black hole formed due to runaway stellar collisions will be able to increase its mass by a large factor before any other new-generation black hole forms.

3.2. Minimum Mass of Central Black Hole

We consider here the evolution of a cluster with a velocity dispersion large enough to guarantee core collapse. If a black hole grows in the cluster, what is a rough approximation to its minimum mass? We will approach this question in two different ways. First, we will determine the mass of a black hole nailed to the center of an $n \propto r^{-2}$ core-collapse cluster such that dynamical processes around the black hole can supply enough heat to help forestall further core collapse. Second, we will apply the criterion that the wander radius of the black hole must be less than its radius of influence, under the assumption that otherwise the number of stars bound to the black hole would be much less and hence its heating influence would be reduced.

We will assume as before that the mass of the nuclear star cluster is related to its velocity dispersion by $M_{\rm cl} \approx 10^6\,M_\odot(\sigma/40\,{\rm km\,s}^{-1})^4$. For a core-collapse cluster with $n \propto r^{-2}$, the velocity dispersion is the same at all radii and the gravitational binding energy is $E_{\rm bind} = (1/2)M_{\rm cl}\sigma^2$ from the virial theorem.

This energy must be compared with the available energy (as defined before) from dynamics around the central black hole. The available energy per unit mass around the black hole that we found previously is $GM_{\rm BH}/(6r_{\rm p,min})$, where $r_{\rm p,min}$ is the minimum pericenter distance of an orbit that can last long enough for significant dynamical interactions. For a black hole, the relevant time is the time for gravitational radiation to cause the object to spiral in; this time scales as $T \sim (mM_{\rm BH}^2)^{-1}r_p^4$, roughly, so for a fixed T we have $r_{\rm p,min} \propto M_{\rm BH}^{1/2}$. We used $r_{\rm p,min} \approx 0.01$ AU for $10\,M_\odot$ (giving an inspiral time of a few million years), so we will adopt $r_{\rm p,min} = 0.1$ AU$(M_{\rm BH}/10^3\,M_\odot)^{1/2}$.

If the distribution of stars around the black hole is a steep cusp, then the stellar mass in the radius of influence of the black hole equals the mass of the black hole (this need not be true if the density distribution has a core profile; see Equation (14) of Lasota et al. 2011). When we compare the available dynamical energy of the stars around the black hole with the binding energy of the cluster, we find that

$$M_{\rm BH} \gtrsim 500\,M_\odot(\sigma/40\,{\rm km\,s}^{-1})^4 \qquad (13)$$

is required for the stars around the black hole to provide sufficient energy to hold off collapse.

We can also approach this from a different angle. A finite-mass black hole will not be nailed to the center of the cluster. Instead, it will wander due to stochastic dynamical interactions. If the wander radius is less than its radius of influence, then we can suppose that it is near the center of the stellar distribution where encounters are frequent, but if the wander radius is larger, then this need not be the case and heating could be less efficient. Thus, a different criterion is $r_{\rm wander} < r_{\rm infl,BH}$. Suppose that there is a nearly constant density core in the inner 10% of the cluster; then the scale height of a species in a cluster is inversely proportional to the square root of its mass (from energy equipartition arguments), and hence for this system we expect

$$r_{\rm wander} \sim r_{\rm cl}(\langle m\rangle/M_{\rm BH})^{1/2} . \qquad (14)$$

Here $\langle m\rangle$ is the average mass of a star. The cluster radius is $r_{\rm cl} = GM_{\rm cl}/\sigma^2$, and the radius of influence of the black hole is $GM_{\rm BH}/\sigma^2$; hence, the wander criterion is

$$0.1(GM_{\rm cl}/\sigma^2)(\langle m\rangle/M_{\rm BH})^{1/2} \lesssim GM_{\rm BH}/\sigma^2$$
$$M_{\rm BH} \gtrsim (0.01M_{\rm cl}^2\langle m\rangle)^{1/3} \qquad (15)$$
$$M_{\rm BH} \gtrsim 2\times10^3\,M_\odot(\sigma/40\,{\rm km\,s}^{-1})^{8/3},$$

where in the last line we assume $\langle m\rangle = 1\,M_\odot$.

Recall that these are *lower limits* on the mass of the central black hole. The mass could be considerably greater depending on long-term accretion of stars or gas.

4. DISCUSSION AND CONCLUSIONS

We have discussed the evolution of a relaxed cluster that has a velocity dispersion $\sigma \gtrsim 40$ km s^{-1}, which is large enough to render binaries insignificant, but that does not initially contain a massive central black hole. We argue that a massive hole will

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inevitably form if it can swallow tidal debris rapidly: interactions in the black hole subcluster will leave either zero or one hole. In the case of zero, a black hole will form from the product of runaway stellar merging. In either case, the hole will feed quickly from the remnants of the stars it tidally disrupts and hence will grow until it has significant dynamical effects on the cluster and thus slows its own growth. It is not guaranteed that the holes will then follow the same M–σ relation that exists for higher velocity dispersion systems. It is also not guaranteed that clusters with lower velocity dispersions will *not* have black holes, but it is possible that massive black hole formation is prevented as long as binaries have a significant heating effect (see Gill et al. 2008 for a numerical exploration of the heating due to binaries or a massive central object).

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