Challenges in the Measurement of Neutron Star Radii *

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The recent discovery of neutron stars near two solar masses has placed strong constraints on the properties of cold matter at a few times nuclear saturation density. Even tighter constraints would come from precise and accurate measurements of the radii of neutron stars of known masses, but current inferences are dominated by systematic errors. We summarize the current methods used to estimate neutron star radii and assess the prospects for reliable radii from future electromagnetic and gravitational wave observations.

1. Introduction

In the last three years, two neutron stars have been discovered that have gravitational masses of approximately two solar masses [1, 2]. Such high masses imply interesting constraints on the properties of cold matter beyond nuclear density [3]. Even stronger constraints would come from precise and reliable measurements of the radii of neutron stars of known masses [4], but such measurements are much more difficult. Indeed, as we emphasize here, all current neutron star radius estimates are dominated by systematic errors, and none are reliable enough to be used in the construction of models of high-density matter. However, current studies suggest that this situation will change with the advent of the next generation of X-ray instruments and of advanced gravitational wave detectors.

Here we discuss current and future neutron star radius constraints, with an emphasis on existing systematic errors and the possibilities for circumventing them. In Section 2 we give an overview of why radius measurements are difficult, and then summarize the reported constraints from observations of thermonuclear X-ray bursts. In Section 3 we discuss estimates of radii from fits to cooling neutron stars. In Section 4 we explore the prospects of tight constraints with future fits to waveforms from burst oscillations or the

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X-ray emission from non-accreting neutron stars with millisecond rotation periods. In Section 5 we examine likely constraints from gravitational wave observations, and in Section 6 we present our conclusions.

2. Constraints from X-ray bursts

As an overall perspective before we discuss particular radius estimates, let us consider how we measure the radius of an ordinary star, which is too distant for angular resolution. We find the luminosity L of the star by measuring its distance and flux and assuming that it emits isotropically. If we also measure the spectrum and assume blackbody emission, then the fitted color temperature $T_{\rm col}$ plus the luminosity give the radius via $R^2 = L/(4\pi\sigma T_{\rm col}^4)$. We can check our answer with direct angular measurements of the Sun and, more recently, with asteroseismological inference [5], and in both types of comparison we find that the inferred radius is fairly accurate.

When we apply the same procedure to neutron stars we find that the typical inferred radius is ~ 5 km, which is far smaller than the expected radius $R \sim 10-15$ km. This is because neutron star atmospheres are strongly scattering-dominated. To see how this can lead to misleading radius estimates, consider radiation that is generated deep in the neutron star atmosphere, where the radiation is in thermal equilibrium. If the radiation then encounters a scattering layer, then only a fraction $f_{\rm col}^{-4} < 1$ can escape, where $f_{\rm col} \sim 1.3 - 2$ (defined below) is called the color factor. The rest is eventually absorbed and re-thermalized, and thus increases the temperature. Thus the spectrum can be very accurately of the Planck form, but if the effective temperature $T_{\rm eff}$, so that the emergent flux is $\sigma T_{\rm eff}^4$, then the color temperature is $T_{\rm col} = f_{\rm col} T_{\rm eff}$. Thus the radius obtained with $R^2 = L/(4\pi\sigma T_{\rm col}^4)$ is lower than the true radius by the factor $f_{\rm col}^2$. Part of the challenge of such fits is thus to determine $f_{\rm col}$; this is made especially difficult because there are very few situations in which the data are sufficient to distinguish proposed model atmosphere spectra from each other [6].

The use of thermonuclear X-ray bursts to estimate radii via spectral fitting was first suggested by [7]. These bursts occur in accreting neutron star systems when the accreted matter (primarily hydrogen and helium) becomes unstable to runaway nuclear burning. Typically, the matter accumulated over hours to days is consumed in a few seconds (for the normal helium-driven bursts) to minutes (when hydrogen-helium burning occurs); see [8] for a recent summary. On much rarer occasions, perhaps once per year or two per source, far longer "superbursts" are observed that last for hours at the same flux as the normal bursts, and thus have $\sim 10^3$ times as many X-ray counts as normal bursts. These are thought to be caused by unstable carbon burning in a deep layer [9]. In both regular bursts and superbursts it can be that the surface radiative flux is temporarily slightly in excess of the Eddington flux. This leads to expansion of the photosphere (observationally, the color temperature drops sharply because the emitting area increases), and hence these are called photospheric radius expansion (PRE) bursts.

In the method proposed by [7], the assumptions are: (1) at the point in a PRE burst after the recontraction of the atmosphere when the fitted temperature is maximal (called the point of touchdown), the surface radiative flux is the Eddington flux, (2) at that point, and for the remainder of the burst, the entire surface emits uniformly, (3) the surface composition and spectrum and hence $f_{\rm col}$ are known, and $f_{\rm col}$ is often assumed to be constant, and (4) the distance to the source is known. When these assumptions are combined, they yield both the mass and the radius.

Until the launch of the *Rossi* X-ray Timing Explorer (*RXTE*), X-ray data were insufficiently precise for this method to be useful. However, analysis of *RXTE* data using the assumptions of [7] has yielded remarkably tight constraints for some sources. For example, [10] found 1σ fractional uncertainties of just 4% in the mass and radius of the neutron star in 4U 1820–30. If these numbers are reliable, then they revolutionize our understanding of neutron stars and dense matter.

Unfortunately, although it was entirely reasonable as an initial analysis to make all of the assumptions of [7], in fact the very tight constraints actually demonstrate that the model assumptions are inconsistent with the data. As pointed out by [11], the mass and radius inferred in this way involve the solution of quadratic equations. The discriminant of the quadratic must obviously be non-negative for the mass and radius to be real numbers. The problem is that the best values of the input quantities (flux at touchdown, distance to the source, color factor, and the area normalization during the cooling tail) often imply a negative discriminant. Indeed, [11] showed that given the prior probability distributions used by [10] in their analysis of 4U 1820–30 (Gaussians in the flux and area normalization, boxcar distributions in the distance and color factor), only a fraction 1.5×10^{-8} of the prior probability space yields real numbers for M and R.

Thus the conclusion is not that the mass and radius are known well, but that there are missing elements in the theoretical model. It seems likely, for example, that the surface emission is not uniform [12]. It might also be that the touchdown flux as defined above is not equal to the Eddington flux [13]. There are some promising approaches involving fits of the cooling data with sophisticated model atmosphere spectra [13], but there are also indications of non-surface emission when the neutron star cools below $\sim 50\%$ of its Eddington luminosity. If this emission is also present when the neutron star is closer to its Eddington luminosity, then spectral fits will not yield accurate radii. Thus current methods of neutron star radius inference from X-ray burst spectra do not produce reliable results.

3. Constraints from fits to cooling non-accreting neutron stars

Radius estimates using this method are, in principle, simpler than they are using the cooling tails of X-ray bursts because over the course of an individual observation (say, a 10^5 second X-ray data set) the properties of the source will not change. The assumptions in this case are that (1) all the emission is thermal and is generated deep within the star, (2) the composition and thus the emitting spectrum and angular distribution are known, and (3) the emission is uniform from the entire surface of the star. Both isolated neutron stars and sources that are thought to accrete transiently but also might go through nonaccreting phases (the so-called quiescent low-mass X-ray binaries, or qLMXBs) have been analyzed in this way.

Guillot et al. [14] recently analyzed five qLMXBs under the assumption that all five have the same radius (see Figure 3 in [1] to note that most equations of state predict a relatively constant radius for a broad range of masses). They also assume pure, nonmagnetic hydrogen atmospheres. The assumption of a hydrogen atmosphere seems at first to be plausible because heavier elements will sink rapidly given the very strong surface gravities of neutron stars. The assumption that magnetic effects are negligible is reasonable if the surface field strength is much less than 0.1 keV$m_e c/(\hbar e) \approx 10^{10}$ G; the inferred magnetic dipole moments of LMXBs suggest average surface field strengths of $\sim 10^{8-9}$ G, so this condition might be satisfied. With these assumptions, [14] find that the best single-radius fit to all the qLMXBs is only $R \sim 10$ km.

In contrast, work led by Trümper [15] on the isolated neutron star RXJ 1856.5–3754 suggests a much larger radius. This star has an X-ray spectrum that is well fit with a Planck function, but an optical spectrum that has a normalization that is six times higher than the low-energy extension of that function. [15] assume that the X-ray portion indicates a hot spot centered close to the rotational pole (so that there are no observed rotational modulations) and that the optical emission comes from the stellar surface as a whole. They argue that a blackbody fit provides a lower limit on the radius (for the reasons discussed above related to the color factor), and find that this lower limit is $R > 14$ km.

What, then, are the caveats for these methods? In both cases, a key point is that we do not know the surface composition with certainty. Even more critically, because these stars are cool and therefore the observed flux from them is low, very distinct surface models give comparably good statistical fits to the data. For example, a Planck spectrum, a nonmagnetic hydrogen atmosphere, a nonmagnetic helium atmosphere, a nonmagnetic carbon atmosphere, or magnetic versions of all of these usually give essentially equally good fits. However, the inferred radii are very different for these different fits. Nonmagnetic helium atmosphere fits to a cooling neutron star in M28 give $R \approx 14$ km, whereas $R \approx 9$ km for nonmagnetic hydrogen fits [16]. Although it is true that a neutron star atmosphere will segregate its atoms by mass within seconds, for a star that has not accreted for $\sim 10^2$ yr or more it has been suggested that diffusive burning could exhaust the hydrogen and lead to a helium or heavier-element atmosphere [17, 18]. On the other hand, if there is ongoing accretion at a low level, which would replenish the hydrogen, then the assumption that the only emission is thermal is incorrect. Indeed, such ongoing accretion may be necessary to explain short-term variability and the nonthermal spectral component seen in some transient LMXBs with higher duty cycles [19, 20]. This would also suggest that the emission need not be uniform across the entire surface, because even 10^{8-9} G surface magnetic fields could funnel matter towards the magnetic pole (and note again that if the magnetic pole is moderately close to the rotational pole, then the oscillation amplitude could easily be low enough to be undetectable; see [21, 22]).

For RXJ 1856.5–3754, other authors have suggested that the atmosphere is condensed rather than in a gas phase [11]. This would imply a smaller radius than the radius inferred from blackbody fits; the point is that although for bolometric spectra blackbody emission is the most efficient possible (and thus leads to the smallest radius possible), over a restricted range of wavelengths (such as the optical band) other spectra can yield more emission than a blackbody for a given effective temperature.

Thus at this stage the systematic errors are significant enough that radius estimates from fits to non-accreting neutron stars are not reliable and thus cannot be included in equation of state constraints.

4. Current and future constraints from waveform fitting

Another method that gives only moderate constraints at the moment but that appears to hold promise for future observations is fits to waveforms of rotating stars. Suppose that a neutron star has on it a hot spot that rotates with the star. A star that rotates at several hundred Hertz has a linear speed of up to ~ 0.2 c at its equator. Thus special relativistic effects contribute to the waveforms, and in particular produce asymmetry (the waveform will rise faster than it falls). General relativistic light deflection will also affect the curve; for example, whereas for Newtonian straight-line photon trajectories a distant observer will see half the star, when light deflection is included the observer can see more of the star, and for sufficiently compact stars could actually see multiple images of some parts of the star. Thus the mass and radius both affect the waveform, and hence one can hope that careful analysis of the waveform could be used to constrain the mass and radius.

A potential concern is that it has been suggested that for the oscillations in the tails of some X-ray bursts the phasing of high and low X-ray energies is backwards from what one expects in the rotating spot model [23]. In particular, in the rotating spot model one expects high energies to peak before low energies, but [23] found the opposite to be true in some bursters. 4U 1636–536 provided the clearest example of this apparent discrepancy. However, when Artigue et al. [24] re-analyzed the data from 4U 1636–536 they found that, in fact, all the bursts seen from it (including a superburst) can be fit with a rotating spot model that assumes the same stellar mass and radius, observer latitude, and distance for all bursts (as is physically required). They suggest that the stacking analysis of [23] might have led to the contrary result, and also note that statistical fluctuations are sufficient to account for the slight variety in phase behavior seen in different bursts.

Another difficulty is that many of the effects are degenerate with each other. Suppose, for example, that one sees a waveform with a small modulation amplitude. This could be because the spot is small and thus the flux from it is only a small fraction of the overall flux. It could be that the spot is large and thus there is little modulation. It could be that the emission from the star is nearly axisymmetric (e.g., because the spot center is near the rotational pole) or that the observer is looking nearly down the rotational pole. It could be that the star is very compact, and hence that light deflection smears out the waveform and reduces its amplitude. Thus although if one assumes knowledge of everything except the mass and radius (e.g., the rotational latitude of the spot and observer, the spot radius, etc.) tight mass and radius constraints are straightforward [25], real analysis will have to deal with the degeneracies.

To assess whether degeneracies can be overcome given a sufficiently high-count data set, Lo et al. [26] analyzed synthetic data in which a million X-ray counts came from the spot, and between 10^5 and 9×10^6 counts came from all other sources that were not commensurate with the rotation frequency (this includes instrumental background, emission from a disk or from the non-spot surface, and background sources). The million assumed counts are comparable to what would be seen with a future high-area X-ray instrument such as LOFT [27] when data from several bursts from the same star are combined. Lo et al. assumed that the spots are circular and emit uniformly, and that the surface emission pattern is a Hopf function, which is the appropriate beaming pattern for energy that is produced at a large optical depth and that comes to the surface via Thomson scattering.

Lo et al. found that if the spot center and observer inclination are both within $\sim 10^\circ$ of the equator, then it will be possible to constrain both the mass and radius of a star to within 10%. If in contrast the spot center and observer inclination are close to the rotational pole, no useful constraints will be obtained. A key factor in whether good constraints are possible is the presence of overtones in the waveform; near-polar emission produces very sinusoidal curves [22, 21], which do not carry the required information. There are bursters with overtones visible in the oscillations in their tails (e.g., XTE J1814–338 [28]), so prospects for useful constraints do exist. It is also possible that overtones will be more easily visible in the rise portion of bursts than in the tail portion, but as Lo et al. discuss the larger number of counts in burst tails than in burst rises probably makes the tails at least as good as the rises for analysis. Both the rises and tails will obviously be used to constrain neutron star radii.

Given our previous discussion of systematic errors in neutron star radius estimates, what is the situation for waveform analysis? Provisionally good news about this comes from the analysis of Lo et al. They explored the potential biases in fits of their standard model (Planck spectrum, uniform circular spots, Hopf beaming function) to data generated with different assumptions (Bose-Einstein spectrum with chemical potential $\mu = -kT$, spots strongly elongated along lines of latitude or longitude, isotropic beaming function). Encouragingly, they found that whenever the fit was both statistically good and constraining ($\Delta M/M,\ \Delta R/R \ll 1$), the fits were unbiased. Thus, at least for these systematic deviations, we will not fool ourselves with a good and constraining fit that is significantly biased. More work is needed.

The best current constraints using this method are by Bogdanov [29], who analyzed XMM-Newton data of the non-accreting binary millisecond pulsar J0437–4715. Using a two-spot model and assuming the beaming pattern appropriate for a nonmagnetic hydrogen atmosphere, he finds $R > 11.1$ km at 3σ significance. Much deeper observations of this pulsar and some other selected objects will be made with NASA's NICER mission [30].

5. Future constraints with gravitational wave observations

As the era of direct detection of gravitational waves draws nearer, it brings with it the prospect of constraints on the properties of the dense matter inside neutron stars that are entirely independent of the constraints from X-ray analyses. This new information will emerge from careful study of the gravitational radiation waveforms from the coalescence of two neutron stars, or of a neutron star and a black hole. The most valuable data will come from the point in the inspiral when the objects are close to merger, because when two compact objects are separated by a distance much greater than their radii, their orbits and thus their gravitational waveforms are those of point masses. Thus when the binary separation is large, the internal structure of a neutron star is irrelevant. At that stage, the quantity that can be measured best is the "chirp mass" $M_{\rm ch} = \eta^{3/5}M$, where $M = m_1+m_2$ is the total binary mass and $\eta = m_1m_2/M^2$ is the so-called symmetric mass ratio.

Breaking this degeneracy between the masses requires higher-order post-Newtonian effects in the waveform, which become more prominent when the stars are closer together. However, because η reaches its maximum of $1/4$ when $m_1 = m_2$, there is little variation in η over the expected mass ratios for neutron stars; for example, if $m_1 = 1.5\ m_2$, $\eta = 0.24$. Thus whereas the chirp mass can be very well determined, the individual masses will not be. Only at the frequencies $\nu_{\rm GW} \sim 1000 - 2000$ Hz close to merger can information in the waveforms help break these degeneracies.

At close separations, the waveforms will deviate from the waveforms of point mass orbits because of tidal effects. It was initially worried that non-linear three-mode couplings might produce significant deviations even at low frequencies $\nu_{\rm GW} < 100$ Hz, which would greatly complicate the detection of these events with template waveforms that assume point mass orbits [31]. However, a later analysis showed that when four-mode couplings are included these effects almost completely disappear, and higher-order couplings are also not important at low frequencies [32].

The focus has therefore been on the effects of tidal interactions as merger is approached. The very steep increase in tidal coupling strength with decreasing binary separation means that nearly all the information about tidal deformability (which is related to radius) emerges almost at the point of merger. There is therefore the concern that analytical models will not suffice, and thus that time-consuming numerical analyses will be needed. However, the current picture is encouraging: analytic studies using the effective one-body picture match numerical simulations all the way to merger within the accuracy of the latter [33, 34, 35, 36, 37]. The most recent studies indicate that neutron star radii could be constrained to 10% [38] for neutron star – neutron star coalescence at a sky- and orientation-averaged distance of ~ 150 Mpc as seen with full-sensitivity second-generation detectors such as Advanced LIGO and Advanced Virgo. The combination of data from multiple coalescences would lead to even better constraints with time [39, 40]. The lack of any current direct detections of gravitational waves means that, unlike with X-ray data, we do not know about any surprises Nature has in store for us. At a minimum, however, this method will have independent systematic issues from those in X-ray analysis and will thus produce highly valuable independent constraints. Even better constraints will be obtained if the sensitivity is improved at the high frequencies where the constraints are strongest. Photon squeezing seems a highly promising way to achieve such sensitivity [41].

6. Conclusions

Precise and reliable neutron star radii are coveted by nuclear physicists because of the unique information they will provide on the state of cold matter at a few times nuclear saturation density. Current measurements are beset with systematic errors that make existing estimates insufficiently reliable to use in the construction of high-density equations of state. However, there are reasons to be optimistic about the constraints from future X-ray observations and from gravitational wave detection. Such constraints will then be combined with laboratory measurements to give us important clues about the properties of matter in this extreme state.

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