THE ASTROPHYSICAL JOURNAL, 578:325–329, 2002 October 10

IMPLICATIONS OF THE NARROW PERIOD DISTRIBUTION OF ANOMALOUS X-RAY PULSARS AND SOFT GAMMA-RAY REPEATERS

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Received 2002 February 2; accepted 2002 May 23

ABSTRACT

The spin periods of 10 observed anomalous X-ray pulsars and soft gamma-ray repeaters lie in the very narrow range of 6–12 s. We use a point-likelihood technique to show that the observed period clustering is not simply a statistical fluctuation, and we quantify the constraints it imposes on the birth period and on the final period of such systems. We consider a general law for their spin evolution described by a constant braking index. We find that, for positive values of the braking index, the observed clustering requires an upper cutoff period that is very close to the maximum observed period of $\simeq 12$ s. We also show that the constraint on the birth period depends very strongly on the assumed value of the braking index n, ranging from a few milliseconds for $n \gtrsim 2$ to a few seconds for $n \lesssim 2$. We discuss possible ways of tightening these constraints based on similarities with the population of radio pulsars and by future observations of such sources with current X-ray observatories.

Subject headings: pulsars: general — stars: neutron — X-rays: stars

1. INTRODUCTION

In the last few years, evidence for the existence of neutron stars with ultrastrong magnetic fields, or magnetars, has become very compelling. The discovery of rapid spin-down in the pulsations observed from soft gamma-ray repeaters (SGRs; e.g., Kouveliotou et al. 1998, 1999) gave support to the suggestion by Thompson & Duncan (1995) that the very energetic bursts observed from these sources require the presence of 10^{15} G magnetic fields. Furthermore, the lack of detectable companions to the anomalous X-ray pulsars (AXPs; e.g., Mereghetti, Israel, & Stella 1998; Hulleman, van Kerkwijk, & Kulkarni 2000), combined with their rapid spin-down rates and spectral properties (see, e.g., Özel 2001), favor the magnetar interpretation.

A striking feature of all magnetar candidates (SGRs and AXPs) is that their periods lie in a relatively narrow range, between 6 and 12 s. This property has been discussed since the original study of Mereghetti & Stella (1995) and was addressed by Colpi, Geppert, & Page (2000) in the context of models with magnetic field decay (see also Chatterjee & Hernquist 2000 for a discussion of the period clustering expected in a variant of accretion models for AXPs). However, a quantitative, statistical analysis of the constraints imposed on magnetar models by this period clustering is still lacking. Indeed, the observed narrow period distribution might simply be the result of statistical coincidence, given the small number of known sources.

The purpose of this paper is to quantify the constraints on the period evolution of magnetar candidates imposed by the tightness of the observed period distribution, using a point-likelihood technique. In § 2 we review the detections of each of the 10 magnetar candidate sources and focus, in particular, on selection effects that could restrict the range of periods that can be discovered. In § 3 we perform a likelihood analysis to determine the allowed range of periods using a mathematical model for the period evolution of AXPs that has broad applicability and of which a dipole spin-down law is a special case. In § 4 we discuss the implications of these results and explore the potential for improving these constraints using future observations of AXPs and SGRs with current X-ray telescopes.

2. OBSERVATIONS AND SELECTION EFFECTS

In this section we describe briefly the observations that led to the discovery of the 10 AXPs and SGRs with measured spin periods, in order to assess the observational selection effects that could have affected their period distribution. Even though no systematic searches for AXPs and SGRs have been performed to date, we argue that these discovery observations were not confined to periods on the order of ~6–12 s and hence no observational selection effects can account for the observed period clustering. This conclusion is strengthened by the fact that accretion-powered pulsars in X-ray binaries with periods significantly larger than a few seconds have been routinely discovered with observations made with the same instruments that have discovered AXPs and SGRs.

2.1. *Anomalous X-Ray Pulsars*

Two of the AXPs were known persistent X-ray sources before pulsations were detected in their X-ray emission. The object 4U 0142+61 ($P = 8.69$ s) was discovered as a persistent source in an early all-sky survey. Subsequent power-spectral analysis of *pointed* observations of this source with *EXOSAT* revealed the pulsations (Israel, Mereghetti, & Stella 1994). The search was performed using *EXOSAT* ME data that had a timing resolution of 1 s, yielding a lower limit on the search period of only 2 s. The duration of the observation was 12 hr, and the upper limit on the search period was of the order 10^4 s.

The object 1E 1048.1−5937 ($P = 6.44$ s) was serendipitously discovered as an X-ray source with *Einstein*. The pulsations were discovered by Seward, Charles, & Smale

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(1986), who searched in both *Einstein* and *EXOSAT* data but did not report the period range of their searches.

The remaining three AXPs were discovered in searches for pulsed X-ray sources, and hence the strategy followed in their observations could have introduced significant selection effects. The object 1E 2259+586 ($P = 6.98$ s) was discovered using *Einstein* data, following a systematic search for a pulsar in the supernova remnant G109.1−1.0 (Fahlman & Gregory 1981). The range of periods searched was 0.1–200 s.

The object 1E 1841−045 ($P = 11.77$ s) was discovered with *ASCA*, following a systematic search for pulsations from all point sources within the supernova remnant Kes 73 (Vasisht & Gotthelf 1997). The search was performed with three timing resolutions: 488 μs, 32 ms, and 0.5 s, and power spectra were produced for each timing resolution. As a result, the minimum period at which pulsations could be detected was $\leqslant$1 s. Moreover, the search was performed over timescales of 96, 10, and 1 minutes and thus was sensitive to pulsations with periods $\gg$100 s.

The object 1RXS J170849.0−400910 ($P = 10.99$ s) was discovered by *ASCA* in a survey of the Galactic plane and was later identified with a *ROSAT* source (Sugizaki et al. 1997). The observation was performed with a timing resolution of 62.5 ms in the high bit-rate mode and 0.5 s in the medium bit-rate mode. As a result, searches for pulsations were sensitive to periods $\leqslant$1 s. The highest period searched was quoted as $\simeq$600 s.

The object AX J1845−0258 ($P = 6.97$ s) was discovered with *ASCA* in the distant Milky Way (Gotthelf & Vasisht 1998) in a pointed observation toward the supernova remnant Kes 75, even though it is possibly associated to the remnant G29.6+0.1 (Gaesnler, Gotthelf, & Vasisht 1999). For computational efficiency, the search was performed only for long periods, i.e., for $1\ \mathrm{s} < P < 100$ s. Note that the source flux has decreased dramatically since its discovery (Vasisht et al. 2000), and hence its identification as an AXP is not secure.

2.2. *Soft Gamma-Ray Repeaters*

SGRs are identified through their recurrent γ-ray bursts and not because of their pulsations. However, pulsations have been detected in *all* four securely identified SGRs. In SGR 1900+14, pulsations have been observed both in the quiescent emission and during bursts at a period of $P = 5.16$ s (Hurley et al. 1999). In SGR 1806−20 ($P = 7.47$ s) and possibly in SGR 1627−41 ($P = 6.4$ s) pulsations have been detected only during the quiescent emission (Kouveliotou et al. 1998; Woods et al. 1999; see, however, Hurley et al. 2000), whereas in SGR 0525−66 ($P = 8$ s) pulsations have been observed only during bursts (Barat et al. 1979). All these searches were performed with data obtained using *ASCA* or *RXTE* and, therefore, were not limited to periods only comparable to those observed.

3. ANALYSIS OF PERIOD CLUSTERING

The period clustering of AXPs and SGRs has often been attributed to a general prediction of a large class of spin-evolution models in which the spin period derivative decreases with increasing period. In these models, the objects evolve quickly through the small periods, making their detection improbable at these periods and their steady state period distribution insensitive to the birth values. However, because the objects spend increasingly more time at long periods, the observed cutoff toward high periods can be used for placing a very strong constraint on the maximum period at which they are detectable. In this section we quantify the above statement using a point-likelihood technique.

3.1. *Analytical Setup*

In order to model the period distribution of magnetar candidates, we assume a general braking law of the form

$$\dot{\Omega} = -\kappa\Omega^n , \qquad (1)$$

where Ω is the spin frequency of the stars and n is the braking index. For $n = 3$ and $\kappa \sim B^2$, equation (1) corresponds to the standard spin-down law for an inclined magnetic dipole of strength B. In order to avoid introducing unnecessary complications to our model, we assume that all systems are born with the same initial period $P_{\rm in}$ and become undetectable when they reach period P_f.

The evolution of the period distribution function $f(P)$ between $P_{\rm in}$ and P_f is described by the conservation law

$$\frac{\partial f(P)}{\partial t} + \frac{\partial}{\partial P}\left[\dot{P} f(P)\right] = 0 , \qquad (2)$$

where we have assumed that there is no evolution of the factor κ. In steady state, the distribution of systems over spin period is then $f(P) \sim \dot{P}^{-1}$ or

$$f(P) = CP^{n-2} , \qquad (3)$$

where the constant C is calculated from the requirement that $f(P)$ is normalized or

$$C = \begin{cases} (n-1)(P_f^{n-1} - P_{\rm in}^{n-1})^{-1} , & n \neq 1 , \\ \ln\left(-P_f/P_{\rm in}\right) , & n = 1 \end{cases} \qquad (4)$$

(see also Phinney & Blandford 1981). Note that in deriving this period distribution we have made two assumptions. First, we have assumed that the AXP and SGR population has reached a steady state, which is justified by the fact that their ages are significantly smaller than the age of the Galaxy. Second, we have assumed that the observed sample is not dominated by a local population of systems that has been the result of a recent burst of magnetar formation. This is justified by the fact that magnetar candidates have been observed throughout the Galactic disk and their relative distances are comparable to or larger than the light-travel distance for a duration equal to their ages.

We now use this period distribution to estimate the best values for the initial and final period, using a likelihood analysis and given the fact that m systems have been detected with measured periods P_j; $j = 1, \ldots, m$. (We assume that all periods are measured to arbitrary precision.) To this end, we subdivide the available parameter space into infinitesimally small bins of width ΔP, such that each bin has either zero or one data point in it. The likelihood of the data given the model is then simply

$$\mathscr{F}(P_j|P_{\rm in}, P_f) = \prod_{j=1}^{m} f(P_j)\Delta P = C^m(\Delta P)^m \prod_{j=1}^{m} P_j^{n-2} . \qquad (5)$$

If we assume two prior probability distributions $\mathscr{G}(P_{\rm in})$ and

$\mathscr{G}(P_f)$ for the parameters, then the posterior probability distribution is proportional to the above likelihood. Following the standard Bayesian approach, we then obtain the posterior probability distribution for an individual parameter (e.g., $P_{\rm in}$) by integrating the full multidimensional posterior distribution over all parameters but the one of interest, i.e., "marginalizing" over the remaining parameters. In practice, the posterior distribution of, e.g., $P_{\rm in}$ is

$$\mathscr{P}(P_{\rm in}|P_j) = \frac{\int \mathscr{F}(P_j|P_{\rm in}, P_f)\mathscr{G}(P_{\rm in})\mathscr{G}(P_f)dP_f}{\int\int \mathscr{F}(P_j|P_{\rm in}, P_f)\mathscr{G}(P_{\rm in})\mathscr{G}(P_f)dP_f dP_{\rm in}} . \quad (6)$$

For the prior probability distribution over $P_{\rm in}$ we assume a flat distribution in $\log P_{\rm in}$ between 10^{-3} s and the minimum observed period of $P_{\rm min} = 6.44$ s, which does not imply any particular period scale. We chose 10^{-3} s as our lowest acceptable initial spin period because this is comparable to the fastest neutron star spins allowed by general relativity and modern equations of state (Cook, Shapiro, & Teukolsky 1994). Similarly, for the prior probability distribution over P_f, we assume a flat distribution in $\log P_f$ between the maximum observed period $P_{\rm max} = 11.77$ and 100 s. The upper acceptable final spin period is arbitrary and very weakly affects the results for positive values of the braking index.

Using equations (3) and (4), we derive the posterior probability distribution of, e.g., the initial period $P_{\rm in}$, to be

$$\mathscr{P}(P_{\rm in}) = \frac{P_{\rm in}^{-1}\int_{P_f} P_f^{-1}(P_f^{n-1} - P_{\rm in}^{n-1})^{-m}dP_f}{\int_{P_{\rm in}} P_{\rm in}^{-1}\int_{P_f} P_f^{-1}(P_f^{n-1} - P_{\rm in}^{n-1})^{-m}dP_f dP_{\rm in}} , \quad (7)$$

where we have shown for simplicity only the expression for $n \neq 1$. Note that because we are not testing the hypothesis that the period distribution of sources follows a power law but we are simply estimating parameters, the posterior distributions depend only on the range of observed periods and not on their specific values.

3.2. *Numerical Results*

Figure 1 shows the posterior probability distributions over $\log P_{\rm in}$ and $\log P_f$ for a dipole spin-down law (i.e., for $n = 3$) and for the 10 magnetar candidates discussed in § 2. Clearly, for both parameters, the most likely values are the extremes of the observed period range. However, the shapes of the probability distributions are very different for the two parameters.

For the dipole spin-down law assumed, the systems spend increasingly more time at increasingly longer periods. Therefore, the absence of any observed systems with periods larger than 12 s requires a rather rapid turnoff at periods comparable to the highest observed period. As a result, the probability distribution over $\log P_f$ is very sharply peaked. On the other hand, for this spin-down law, the initial period $P_{\rm in}$ is nearly unconstrained. Systems that appear now at periods of ≃6–12 s have spent very little time slowing down from $\sim 10^{-3}$ to ~ 1 s and, therefore, there is little information about their initial periods imprinted on their current period distribution.

As is apparent from the above discussion, the constraints on the initial and final periods of AXPs and SGRs depend strongly on the braking index. This is shown in Figure 2, where the 68%, 90%, and 95% confidence levels of $P_{\rm in}$ and P_f are plotted against the assumed braking index n. For $n > 2$ the final period is strongly constrained to lie very close to the maximum observed period, while the initial period can lie in a large range of values. On the other hand, the situation is reversed for $n < 0$. In both cases, the flattening of the confidence limits at the extreme values of allowed periods is caused by the assumed prior distributions that are bounded. This is a physical bound for the case of the initial period, as discussed above, but it is artificial for the case of the final period.

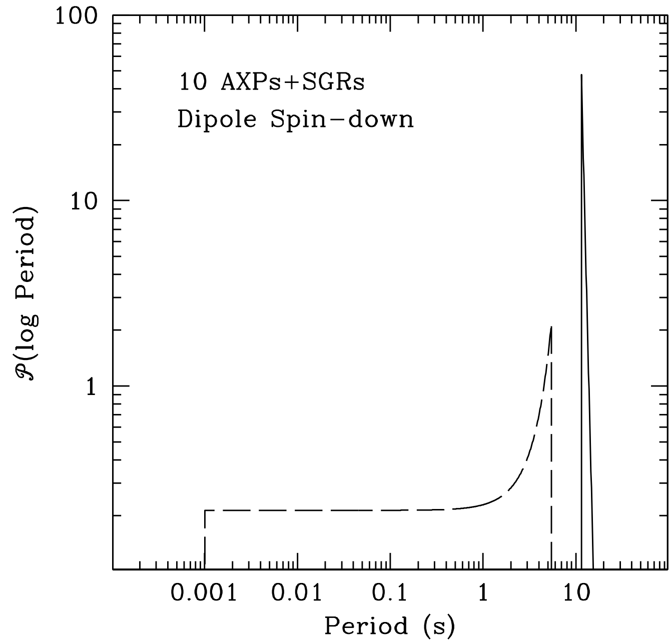

FIG. 1.—Posterior probability distributions over the initial (*dashed line*) and final (*solid line*) spin periods calculated for the 10 AXPs and SGRs discussed in § 2 and for a dipole spin-down law.

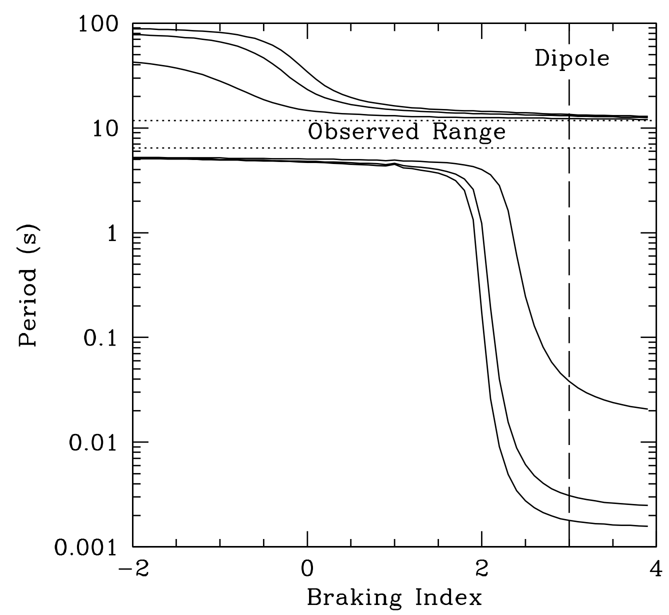

FIG. 2.—The 68%, 90%, and 95% confidence levels of the initial (*lower part of the diagram*) and final (*upper part of the diagram*) periods of AXPs and SGRs for different values of the braking index.

4. DISCUSSION

The narrow range of observed periods of magnetar candidates can be used to constrain their birth periods, the periods at which they cease to be active, or both, depending on the value of their braking index. For positive values of the braking index, we showed in § 3 that the final periods must lie very close to the maximum observed period of $\simeq$12 s. At the same time, if the braking index is $n \lesssim 2$, then the birth periods must be $\simeq$5 s, i.e., very slow. On the other hand, if $n \gtrsim 2$, then the birth periods are largely unconstrained from the analysis of the observed clustering.

It is interesting to note that braking indices of young radio pulsars have been measured to be from as low as 1.81 ± 0.07 (PSR B0540−69; Zhang et al. 2001) to as high as 2.837 ± 0.001 (PSR B1509−58; Kaspi et al. 1994), bracketing the value of $n = 2$ that separates a strong constraint on $P_{\rm in}$ from a very weak constraint. However, slow radio pulsars have second period derivatives (and hence braking indices) that are variable and are larger by factors of $\sim 10^2$–10^4 than what is predicted by simple magnetic braking (see, e.g., Cordes & Helfand 1980). Such anomalously large braking indices are thought to be the result of glitches and of timing noise, both of which are known to occur at least in AXPs, which can have instantaneous braking indices of order $\sim 10^3$ (Kaspi, Lackey, & Chakrabarty 2000; Woods et al. 2000; Kaspi et al. 2001).

Our analysis, however, and in particular the constraint on the birth periods, depends on the value of the average braking index, to the extent that the spin evolution of the magnetar candidates is described approximately by a law of the form of relation (1) and not on any instantaneous index. For the case of 20 moderate-aged radio pulsars, Johnston & Galloway (1999) computed braking indices integrated over $\simeq$5–20 yr and found values in the wide range of $-220 \lesssim n \lesssim 35$. This result is not surprising, given that the braking indices of such pulsars computed over shorter timescales vary rapidly and often change sign. It shows, however, that unfortunately little progress can be expected in constraining the braking indices of magnetar candidates and hence their birth periods.

On the other hand, the strong constraint derived here on the maximum period may provide some insight into the physical mechanism that might be causing it. For example, comparing the inferred *equatorial* dipole magnetic fields and spin periods of magnetar candidates with those of radio pulsars indicates that the *empirically drawn* death line of radio pulsars, when extrapolated toward higher magnetic fields, appears to be unrelated to the maximum period of magnetar candidates.

An exponentially decaying magnetic field, as discussed by Colpi et al. (2000), provides a plausible explanation for both the period clustering and the young ages of magnetar candidates. It is worth noting, however, that the period derivatives of the AXPs with periods close to 12 s are comparatively large (see, e.g., Özel, Psaltis, & Kaspi 2001) and, hence, if the characteristic timescale for field decay were larger than the dipole spin-down timescale then these systems would quickly evolve to larger periods, in contrast to the constraints derived above. As a result, the long-term period evolution of the currently known AXPs and SGRs must be dictated by the decay of their magnetic field and not by dipole radiation. Since the decay of the magnetic field may be erratic, the above conclusion appears consistent with the large and variable braking indices observed in AXPs and SGRs (see, e.g., Kaspi et al. 2000, 2001; Woods et al. 2000).

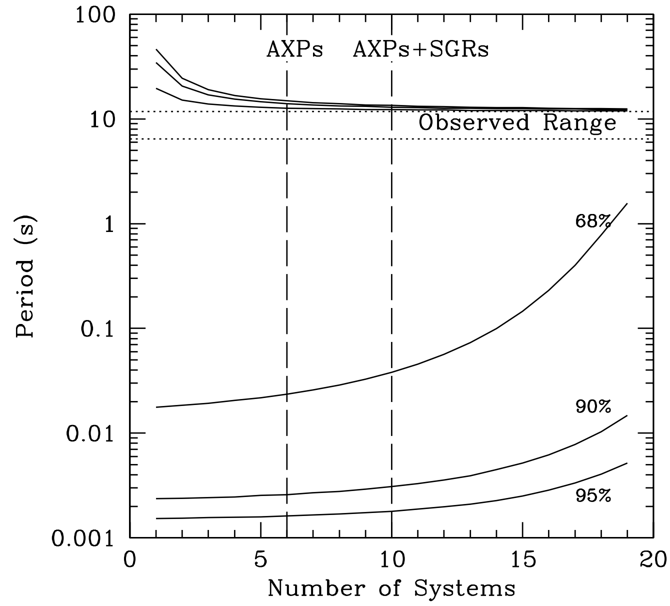

Fig. 3.—Same as Fig. 2, but for magnetar candidates as a function of their total number for a dipole braking law.

Finally, we address the dependence of our results on the number of magnetar candidates that we considered. In our analysis, we chose to assume that both AXPs and SGRs are formed and evolve in the same way. We show in Figure 3, however, where we use the dipole spin-down law as an example, that the resulting constraints depend rather weakly on the number of systems that are known within *the same* period range. Indeed, even considering simply the six known AXPs would be enough to reach similar conclusions.

Figure 3 also shows that increasing the sample of magnetar candidates with similar periods even by a factor of 2 will only affect our results mildly: the constraint on the birth period will become as high as $\sim$2 s, but only at the 68% level. However, the detection of even a single magnetar candidate with a period larger than $\simeq$12 s will change the constraint on P_f. Such a detection does not require detectors with fast timing capabilities and is, therefore, possible, if such systems exist, with current X-ray observatories such as *Chandra* and *XMM/Newton*.

We thank the participants of the Aspen Summer Workshop on X-Ray Astronomy, where this work was initiated. We also thank Feryal Özel for many useful discussions and comments on the manuscript as well as B. Gaensler, S. Mereghetti, P. Woods, and the referee M. Colpi for their suggestions. D. P. thanks the Astronomy Department of the University of Maryland for its hospitality. D. P. acknowledges the support of NSF grant PHY 00-70928. M. C. M. acknowledges the support of NASA grant NAG 5-9756 and NSF grant AST 00-98436.

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