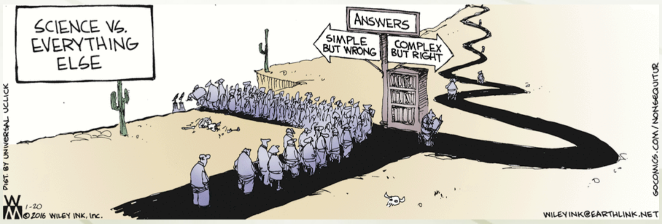
The Expanding Universe II

- The basic Cosmological Principles
- The geometry of the Universe
  - The scale factor *R* and curvature constant *k*
  - Comoving coordinates
- Einstein's initial solutions



I : BASIC COSMOLOGICAL ASSUMPTIONS

- Germany 1915:
  - Einstein just completed theory of GR
  - Explains anomalous orbit of Mercury perfectly
  - Schwarzschild is working on black holes etc.
  - Einstein turns his attention to modeling the universe as a whole…
- How to proceed… it's a horribly complex problem

How to make progress…

- Proceed by ignoring details…
  - Imagine that all matter in universe is "smoothed" out
  - i.e., ignore details like stars and galaxies, but deal with a smooth distribution of matter
- Then make the following assumptions
  - Universe is **homogeneous** – every place in the universe has the same conditions as every other place, on average.
  - Universe is **isotropic** – there is no preferred direction in the universe, on average.

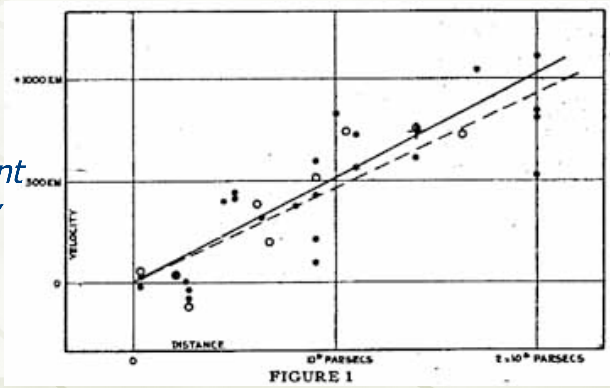
Observational evidence for homogeneity and isotropy

- Let's look into space… see how matter is distributed on large scales.
- "Redshift surveys"…
  - Make 3-d map of galaxy positions
  - Use redshift and Hubble's law to determine distance

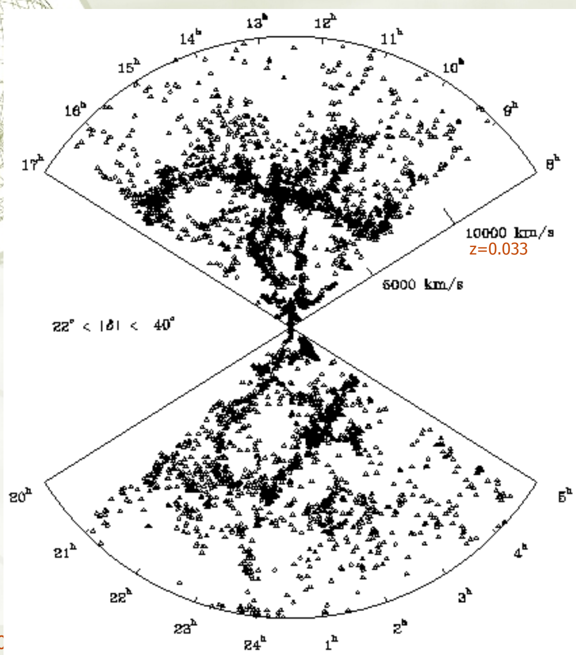
You can think of these as Hubble's plot on steroids

- By plotting redshift versus distance, Hubble (1929) found a linear relationship!

Apparent velocity = c×z

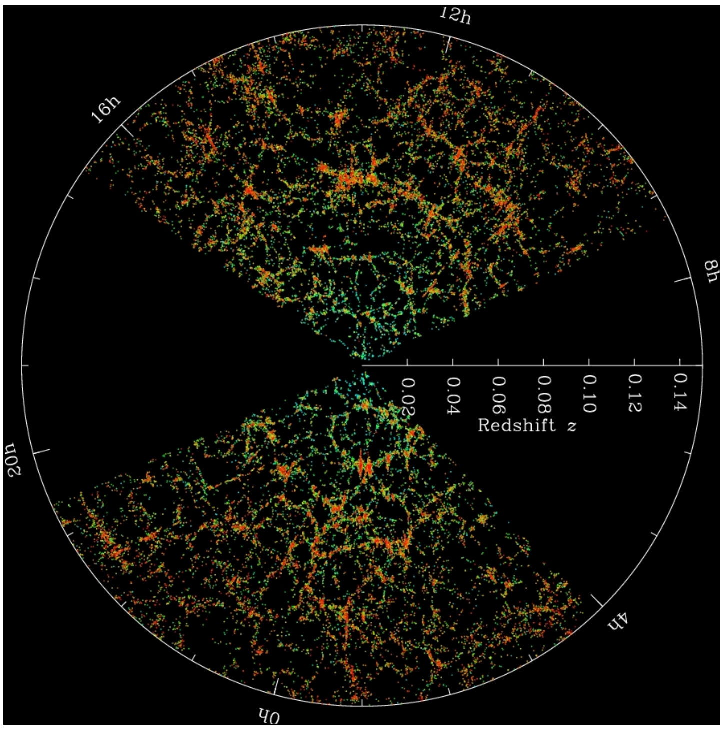


Distance



Each point is a bright galaxy

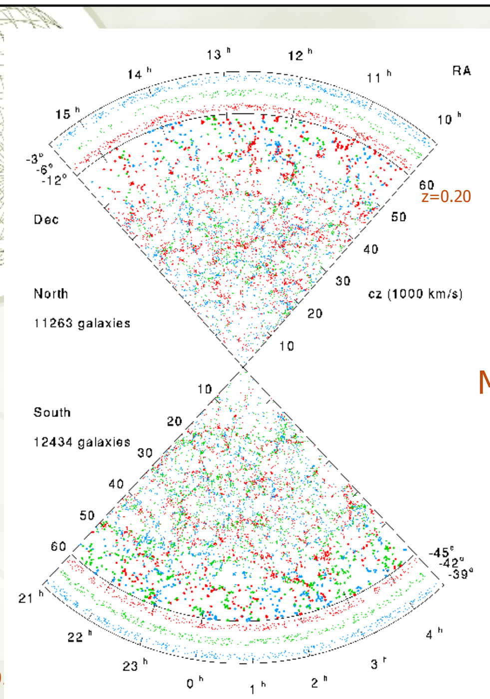
CfA redshift survey



Sloan Digital Sky Survey

Galaxies color coded by the age of their stars

http://www.sdss.org



Las Campanas Redshift survey

Max distance ~ 10^9 pc !

Homogeneous?

- There is clearly large-scale structure
  - Filaments, clumps
  - Voids and bubbles
- But, homogeneous on very large-scales.
- So, we have the…
- **The Generalized Copernican Principle…** there are no special points in space within the Universe. The Universe has no center!
- These ideas are collectively called the **Cosmological Principles**.

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II : POSSIBLE GEOMETRIES FOR THE UNIVERSE

- The Cosmological Principles constrain the possible geometries for the space-time that describes Universe on large scales.
- The problem at hand - to find curved 4-d space-times which are both homogeneous and isotropic…
- Solution to this mathematical problem is the Friedmann-Robertson-Walker (FRW) metric.

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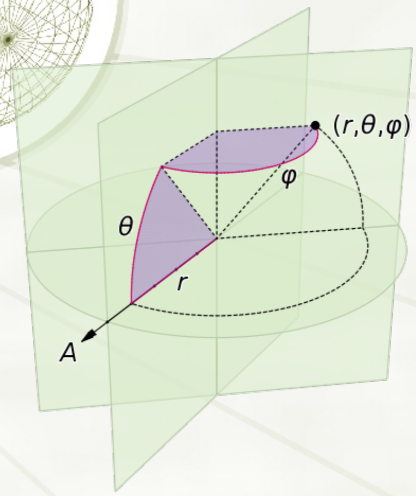
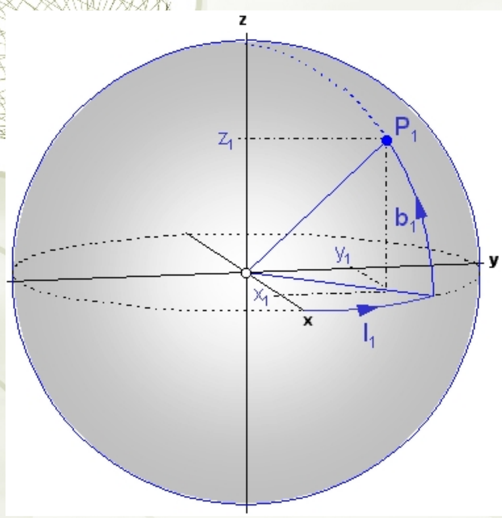
Coordinates



- Coordinates are just recipes to get from here (the origin) to there.
- Spherical coordinates tell you how to get there using one distance and two angles.

Coordinates



- Coordinates are just recipes to get from here (the origin) to there.
- Cartesian (rectangular) coordinates use three distances to do the same.

Friedmann-Robertson-Walker metric

- A "metric" describes how the space-time intervals relate to local changes in the coordinates
- We are already familiar with the formula for the space-time interval in flat space (generalized for arbitrary space coordinate scale factor *R*):
- In terms of radius and angles instead of x,y,z, this is written:

$$\Delta s^2 = (c\Delta t)^2 - R^2\left(\Delta r^2 + \Delta\theta^2 + \sin^2\theta(\Delta\varphi)^2\right)$$

- General solution for isotropic, homogeneous *curved* space is:

$$\Delta s^2 = (c\Delta t)^2 - R^2\left(\frac{\Delta r^2}{1-kr^2} + \Delta\theta^2 + \sin^2\theta(\Delta\varphi)^2\right)$$

- And in fact, in general the scale factor may be a function of time, i.e. *R(t)*

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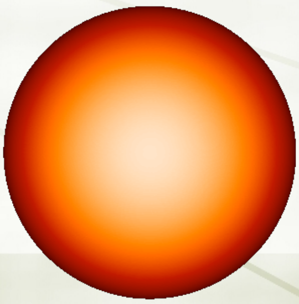
Curvature in the FRW metric

- This introduces the **curvature constant**, k
- Three possible cases…

Spherical spaces (closed; k=+1)



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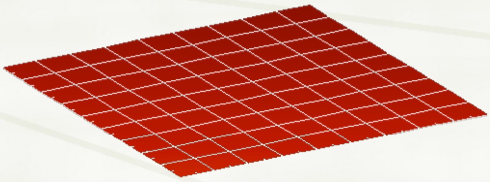
Curvature in the FRW metric

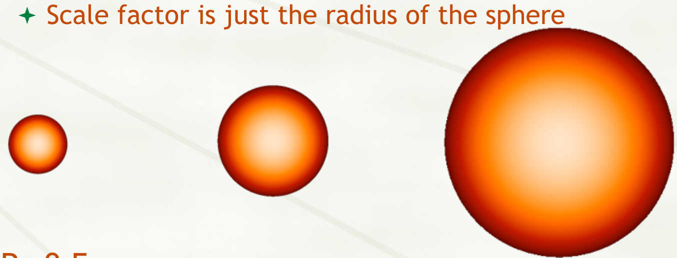
Flat spaces (open; k=0)



Hyperbolic spaces (open; k=-1)

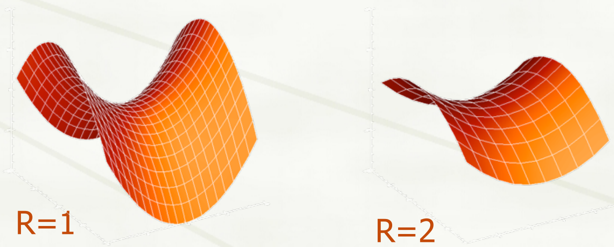
Meaning of the scale factor, *R*.

- Scale factor, *R*, is a central concept!
  - *R* tells you how "big" the space is…
  - Allows you to talk about changing the size of the space (expansion and contraction of the Universe - even if the Universe is infinite).
- Simplest example is k=+1 case (sphere)
  - Scale factor is just the radius of the sphere



The scale factor R

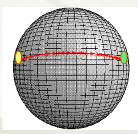
- What about k=-1 (hyperbolic) universe?
  - Scale factor gives “radius of curvature”

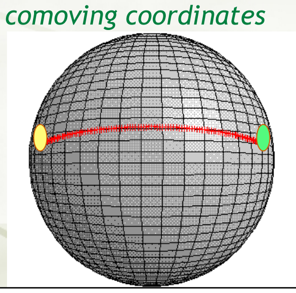


- For k=0 universe, there is no curvature… shape is unchanged as universe changes its scale (stretching a flat rubber sheet)

Co-moving coordinates.

- What do the coordinates x,y,z or r,θ,φ represent?
- They are positions of a body (e.g. a galaxy) in the space that describes the Universe
- Thus, Δx can represent the separation between two galaxies
- But what if the size of the space itself changes?
- e.g. suppose space is sphere, and has a grid of coordinates on surface, with two points at a given latitudes and longitudes θ_1,φ_1 and θ_2,φ_2
- If the sphere expands, the two points would have the same latitudes and longitudes as before, but *distance between them would increase*
- *Coordinates defined this way are called* comoving coordinates





- If a galaxy remains at rest relative to the overall space (i.e. with respect to the average positions of everything else in space) then it has fixed co-moving coordinates.
- Consider two galaxies that have fixed co-moving coordinates.
  - Let's define a "co-moving" distance D
  - Then, the real (proper) distance between the galaxies is $d = R(t) \times D$

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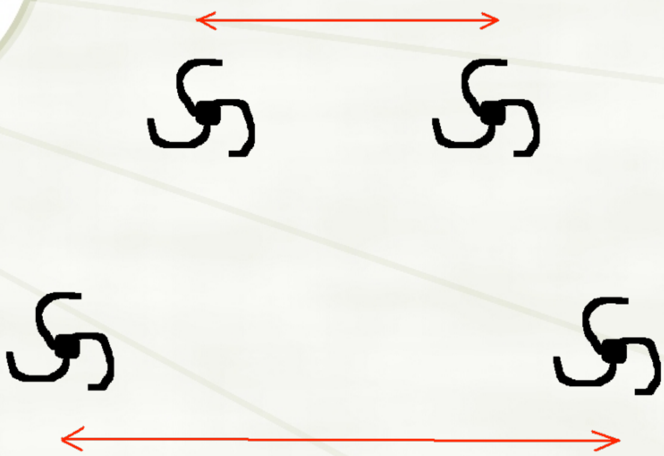

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Cosmological redshift, z

- If galaxies move apart, z describes a Doppler shift from the expansion velocity
- More fundamentally, it comes from the change in metric scaling, $R(t)$
- It's more like the gravitational redshift than a Doppler shift
- Since it's relativistic, it affects time as well as length

III : THE DYNAMICS OF THE UNIVERSE – EINSTEIN'S MODEL

- Back to Einstein's equations of GR

$$\underline{\underline{\mathbf{G}}} = \frac{8\pi G}{c^4}\underline{\underline{\mathbf{T}}}$$

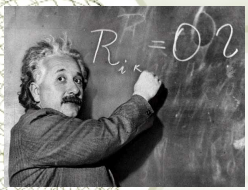
"G" describes the space-time curvature (including its dependence with time) of Universe… here's where we plug in the FRW geometries.

"T" describes the matter content of the Universe. Here's where we tell the equations that the Universe is homogeneous and isotropic.

- Einstein plugged the three homogeneous/isotropic cases of the FRW metric formula into his equations of GR to see what would happen…
- Einstein found…
  - That, for a static universe (*R(t)=constant*), only the **spherical** case worked as a solution to his equations
  - If the sphere started off static, it would rapidly start collapsing (since gravity attracts)
  - The only way to prevent collapse was for the universe to start off expanding… **there would then be a phase of expansion followed by a phase of collapse**

A bit of scientific sociology

- So… Einstein could have used this to **predict** that the universe must be either expanding or contracting!
- … but this was **before** Hubble discovered expanding universe (more soon!)- everybody thought that universe was static (neither expanding nor contracting).
- So instead, Einstein modified his GR equations!
  - Essentially added a repulsive component of gravity
  - New term called "**Cosmological Constant,**" Λ
  - Could make his spherical universe remain static
  - BUT, it was unstable… a fine balance of opposing forces. Slightest push could make it expand violently or collapse horribly.



A stroke of genius?

- Soon after, Hubble discovered that the universe **was** expanding!
- Einstein called the Cosmological Constant “Greatest Blunder of My Life”!
- ….but recent work suggests that he may have been right (more later!)