Lecture 15 : Cosmological Models I

- Standard cosmological models - 3 types
- Hubble time and other terminology
- The Friedmann equation
- The Critical Density and Ω

"IT WAS A LOT EASIER TO KEEP AN EYE ON THINGS BEFORE THE BIG BANG. EVERYTHING WAS ALL IN ONE PLACE THEN."

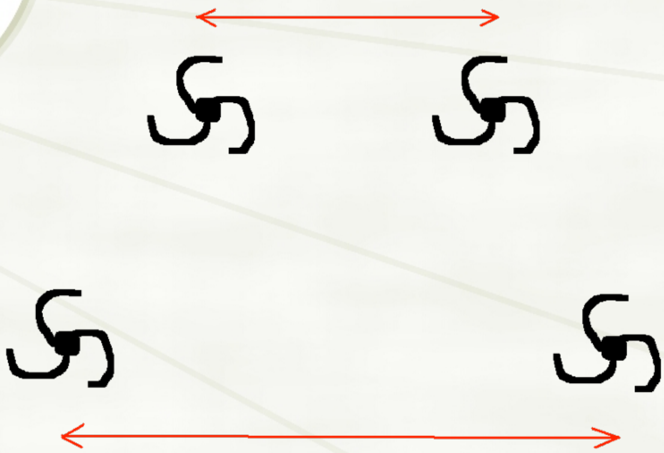
Reading for lectures 15 &16: Chapter 11

The FRW metric

- According to GR, the possible space-time intervals in a homogeneous, isotropic Universe are the FRW metric forms with *k=0* (flat), *k=1* (spherical), *k=-1*(hyperbolic):

$$(\Delta s)^2 = (c\Delta t)^2 - R(t)^2\left(\frac{(\Delta r)^2}{1-kr^2} + (\Delta\theta)^2 + \sin^2\theta(\Delta\varphi)^2\right)$$

- The scale factor *R(t)* describes the relative expansion of space itself as a function of time.
- For *k=1*, maximum *r=1* ; for *k=0* or *k= -1*, *r=0 to ∞*



The Hubble parameter

- Both physical distances between galaxies and wavelengths of radiation vary proportional to *$R(t)$*:
  - $d(t) = D_{comoving} R(t)$
  - $\lambda(t) = \lambda_{emitted} R(t)/R(emitted)$
- Observed redshift of radiation from distant source is related to scale factor at emission time (t_{em}) and present time (t_0) by

  $$1+z = \lambda(t_0)/\lambda(t_{em}) = R(t_0)/R(t_{em})$$

- Hubble observed that Universe is currently expanding; expansion can be characterized by $H=(dR/dt)/R= \Delta R/(R\times\Delta t)$
- For nearby galaxies, $v=d\times H_0$, where the present value of the Hubble parameter is approximately

  H_0 =70 km/s/Mpc

- But how does *R(t)* (and *H*) change in time? And what is the value of the curvature *k*?
- Need to solve Einstein's equation!

$$\underline{\underline{\mathbf{G}}} = \frac{8\pi G}{c^4}\underline{\underline{\mathbf{T}}}$$

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- First we will discuss all of the possible types of solutions that *could* exist for Einstein's equation
- Later we will discuss which solution or solutions appear to hold in *real* Universe, based on current observations

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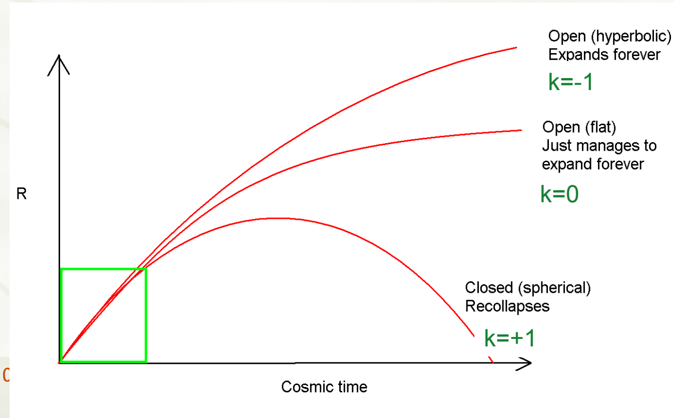
STANDARD COSMOLOGICAL MODELS

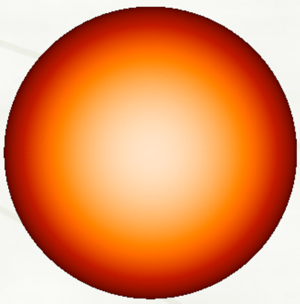
We'll start with the answer, and then explain it…

- In general Einstein's equation relates geometry to dynamics
- That means curvature must relate to evolution
- It turns out that there are three possibilities…

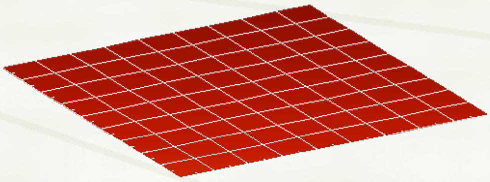


Types of spaces

Spherical space (closed=finite; k=+1)



Flat spaces (open=infinite; k=0)



Hyperbolic spaces (open=infinite; k=-1)

Important features of standard models…

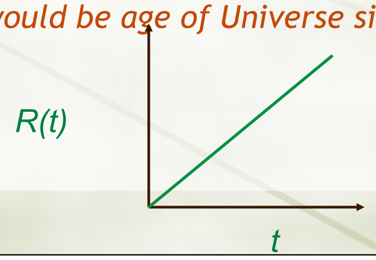
- All models begin with $R \rightarrow 0$ at a finite time in the past
  - This time is known as the **BIG BANG**
  - Space and time come into existence at this moment… there is no time before the big bang!
  - The big bang happens everywhere in space… not at a point!

- There is a connection between the geometry and the dynamics
  - Closed (k=+1) solutions for universe expand to maximum size then re-collapse
  - Open (k=-1) solutions for universe expand forever
  - Flat (k=0) solution for universe expands forever (but only just barely… almost grinds to a halt).

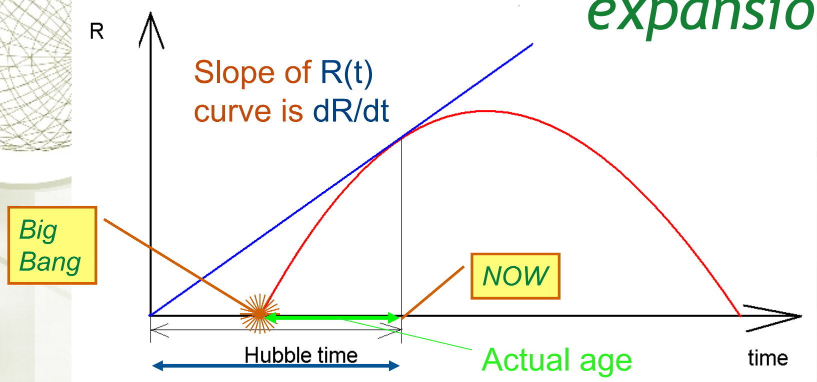
Hubble time

We can relate this to observations…

- Once the Hubble parameter has been determined accurately from observations, it gives very useful information about age and size of the expanding Universe…
- Recall Hubble parameter is ratio of rate of change of size of Universe to size of Universe: $H = \frac{1}{R}\frac{\Delta R}{\Delta t} = \frac{1}{R}\frac{dR}{dt}$
- *If* Universe ***were*** expanding at a constant rate, we would have $\Delta R/\Delta t = constant$ and $R(t) = t \times (\Delta R/\Delta t)$; then would have $H = (\Delta R/\Delta t)/R = 1/t$
- *ie $t_H = 1/H$ would be age of Universe since Big Bang*



Hubble time for nonuniform expansion



- **Hubble time** is $t_H=1/H=R/(dR/dt)$
- Since rate of expansion varies, $t_H=1/H$ gives an estimate of the age of the Universe
- This tends to overestimate the age of the Universe since the Big Bang compared to the actual age

Terminology

- **Hubble distance,** $D=ct_H$ (distance that light travels in a Hubble time). This gives an approximate idea of the size of the observable Universe.
- **Age of the Universe**, t_{age} (the amount of cosmic time since the big bang). In standard models, this is always less than the Hubble time.
- **Look-back time**, t_{lb} (amount of cosmic time that passes between the emission of light by a certain galaxy and the observation of that light by us)
- **Particle horizon** (a sphere centered on the Earth with radius ct_{age}; i.e., the sphere defined by the distance that light can travel since the big bang). This gives the edge of the actual observable Universe.

Friedmann Equation

- **Where do the three types of evolutionary solutions come from?**
- Back to Einstein's eq…. $\underline{\underline{\mathbf{G}}} = \frac{8\pi G}{c^4}\underline{\underline{\mathbf{T}}}$
- When we put the FRW metric in Einstein's equation and go though the GR, we get the **Friedmann Equation**… this is what determines the dynamics of the Universe

$$\left(\frac{dR}{dt}\right)^2 = \frac{8\pi G}{3}\rho R^2 - kc^2$$

- What are the terms involved?
  - G is Newton's universal constant of gravitation
  - *dR/dt* is the rate of change of the cosmic scale factor
    This is same as *ΔR/Δt* for small changes in time
    In textbook, symbol for *dR/dt* is $\dot{R}$ *(pronounced "R-dot")*
  - ρ is the total energy density ÷c^2; this equals mass/volume for "matter-dominated" Universe
  - *k* is the geometric curvature constant (= *+1,0,-1)*

- If we divide Friedmann equation by R^2 , we get:

$$H^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{R^2}$$

- Let's examine this equation…
- H^2 must be positive… so the RHS of this equation must also be positive.
- Suppose density is zero (ρ=0)
  - Then, we **must** have negative k (i.e., k=-1)
  - So, empty universes are open and expand forever
  - Flat and spherical Universes can only occur in presence of (enough) matter.

Critical density

- Now, suppose the Universe is flat ($k=0$)
  - Friedmann equation then gives

$$H^2 = \frac{8\pi G}{3}\rho$$

  - So, this case occurs if the density ρ is exactly equal to the **critical density:**

$$\rho_{crit} \equiv \frac{3H^2}{8\pi G}$$

  - "Critical" density means "flat" solution for a ***given** value of* H, which is the most easily observed parameter

- In general, we can define the **density parameter:**

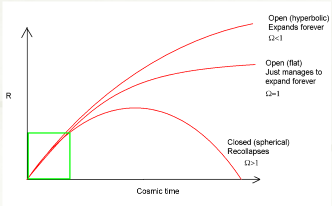
$$\Omega \equiv \frac{\rho}{\rho_{crit}} \equiv \frac{8\pi G\rho}{3H^2}$$

- Can now rewrite Friedmann's equation, moving the curvature term to the other side of the equation and dividing by H^2
- We get:

$$\Omega = 1 + \frac{kc^2}{H^2R^2}$$

Omega in standard models

$$\Omega = 1 + \frac{kc^2}{H^2R^2}$$



- Thus, within context of the standard model:
  - $\Omega<1$ if $k=-1$; then universe is hyperbolic and will expand forever
  - $\Omega=1$ if $k=0$; then universe is flat and will (just manage to) expand forever
  - $\Omega>1$ if $k=+1$; then universe is spherical and will recollapse
- Physical interpretation:

  If there is more than a certain amount of matter in the universe ($\rho>\rho_{critical}$), the attractive nature of gravity will ensure that the Universe recollapses!

T-shirt version

DENSITY IS DESTINY

Value of critical density

- For present best-observed value of the Hubble constant, H_0=70 km/s/Mpc *
  critical density, $\rho_{critical}=3H_0^2/(8\pi G)$, is equal to
  $\rho_{critical}=10^{-26}$ kg/m^3 ; i.e. 6 H atoms/m^3
- Compare to:
  - ρ_{water} = 1000 kg/m^3
  - ρ_{air}=1.25 kg/m^3 (at sea level)
  - $\rho_{interstellar\ gas}$ =2 × 10^{-21} kg/m^3

* Planck finds H_0=68 km/s/Mpc

Next time…

- Deceleration parameter
- Beyond the standard models
  - Cosmological constant models
- Solutions for special cases