Lecture 5:

More Newton…

- Newton's Universal Law of Gravity
- Acceleration in circular orbits
- Weak equivalence principle
- Kepler's laws from Newtonian gravity
- The power of Newton's laws

…how old is the Earth?

- Hints from biology & geology
- Radioactive dating

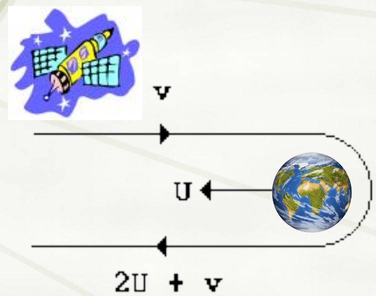
Reference frames and some puzzles…

- Real and fictitious forces

Quiz: Write your name and answer in the card

- Who was the first to observe phases in Venus?
- Why does the observation of phases in Venus disprove the geocentric system?

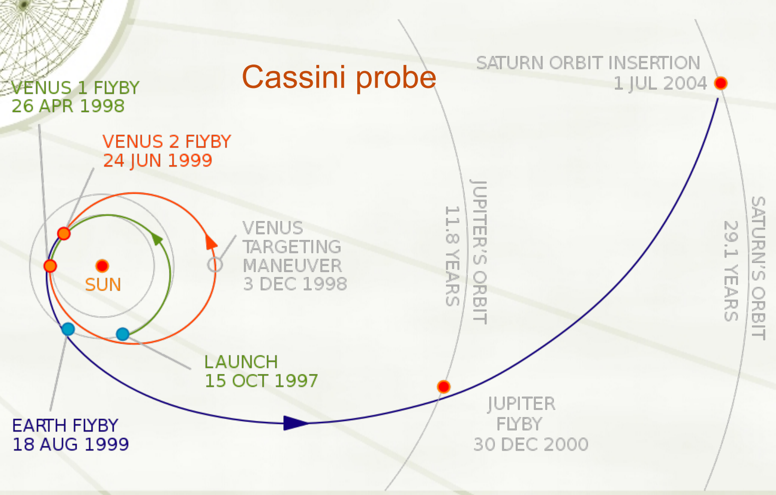
Gravitational slingshot



Gravity assists, commonly used to speed up interplanetary probes, are just another application of conservation of momentum

Gravitational slingshot

Cassini probe



RECAP

- **Newton's 1st law** – ***V*** = *constant* if ***F*** = *0*
- **Newton's 2nd law** – ***F*** = *Ma*
- **Newton's 3rd law** – for every action there is an equal and opposite reaction.
- **Galilean Transformation** – the "usual" velocity addition/subtraction rule for changing frames of reference.
- **Galilean Relativity** – the idea that the laws of nature are the same for a moving observer as for a stationary observer.

II: NEWTON'S LAW OF UNIVERSAL GRAVITATION

Newton's law of Gravitation: A particle with mass m_1 will attract another particle with mass m_2 and distance r with a force F given by

$$F = \frac{Gm_1m_2}{r^2}$$

- "G" is called the Gravitational constant (G = 6.67×10^{-11} N m^2 kg^{-2} in mks units)
- This is a **universal** attraction. Every particle in the universe attracts every other particle! Gravity often dominates in astronomical settings.

Newton's law of gravity

- **Newton's law of gravity** – Gravitational force between two particles with gravitational masses M_1 and M_2 separated by distance r is:

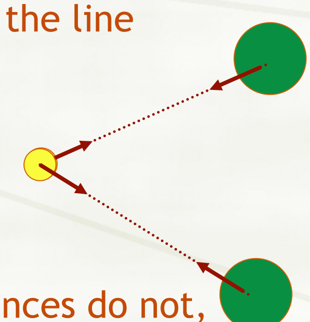
$$F = \frac{Gm_1m_2}{r^2}$$

- *Direction* of gravity is along the line connecting the two masses

- *Magnitude* of gravitational force between masses M_1 and M_2 separated by distance r is:

$$F = \frac{Gm_1m_2}{r^2}$$

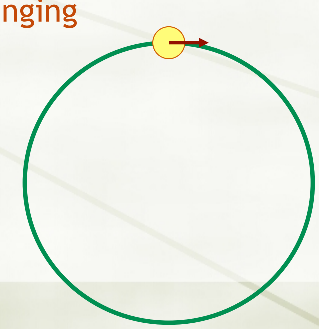
- *Direction* of gravity is along the line connecting the two masses



- If positions change but distances do not, *force changes because direction is changing*

Speed and velocity

- Velocity, as used in Newton's laws, includes both a speed and a direction. ***V*** and also ***F*** and ***a*** are *vectors*.
- Any *change in direction*, even if the speed is constant, *requires a force*
- In particular, motion at constant speed in a circle must involve a force at all times, since the direction is always changing



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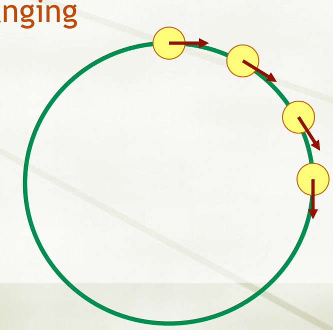
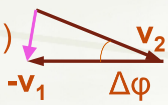
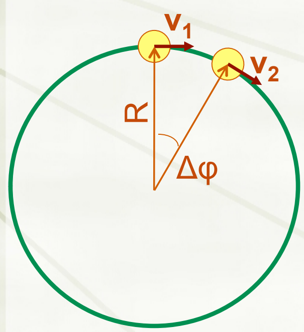


Acceleration in a circular trajectory

$\Delta \mathbf{v} = \mathbf{v}_2 + (-\mathbf{v}_1)$





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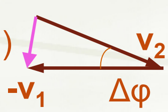
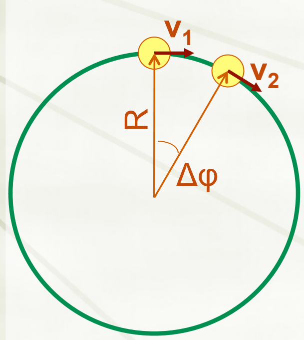


For small φ, $\Delta v \cong v\,\Delta\varphi$



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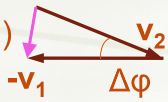

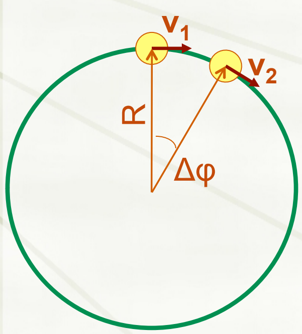

For small φ, Δv≅v Δφ

For constant speed v, the time it takes to go around is T = perimeter/v = $2\pi R/v$

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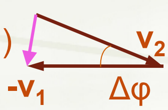

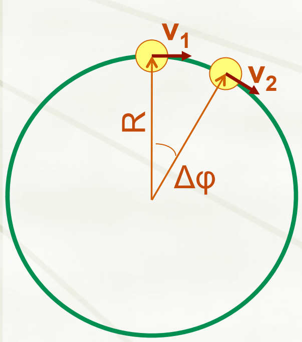

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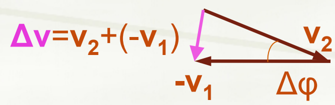

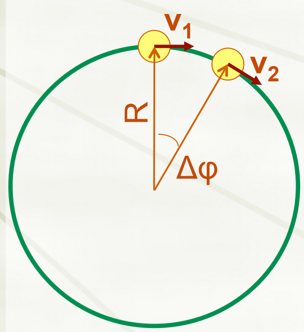

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So $\Delta v/\Delta t = v\,\Delta\varphi/\Delta t = v\, v/R$

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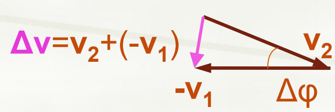

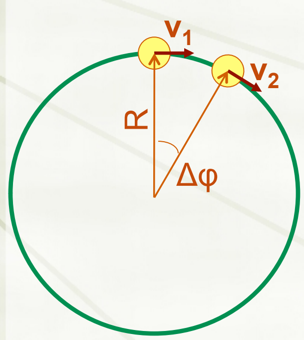

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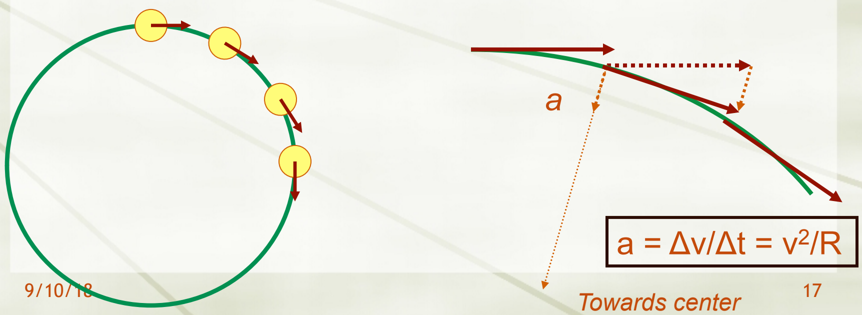
$$a = \Delta v/\Delta t = v^2/R$$

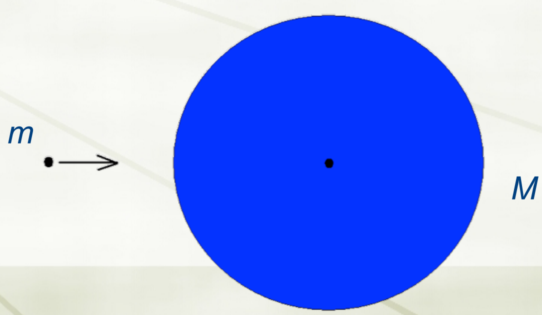
"Falling" in a circular orbit

- Newton arrived at the theory of gravity by imagining that the **same** force causes an apple to fall towards the Earth as the Moon to orbit Earth (continually "falling"), with a decrease in the magnitude of the force with distance
- In what sense is a body in orbit *"falling"?*
- *Direction* of acceleration (= rate of change of vector velocity) is always ***directly towards center of orbit***
- *Acceleration* must be toward center because *gravitational force* is toward center, and **F=ma** is a vector equation



More on gravity

- Newton's Law of Gravitation defines the "gravitational mass" of a body
- Using calculus, it can be shown that a spherical object with mass M (e.g. Sun, Earth) creates the same gravity as a particle of the same mass M at the sphere's center.



Inertial and gravitational mass: the weak equivalence principle

Newton's 2nd law says:

m_I=inertial mass

Newton's law of gravitation says:

m_G=gravitational mass

So, acceleration due to gravity is:

$$a = \left(\frac{m_G}{m_I}\right)\frac{GM}{r^2}$$

So, if the ratio (m_G/m_I) varies, the rate at which objects fall in a gravitational field will vary…

At the end of the last Apollo 15 moon walk (July 1971), Commander David Scott performed a live test of m_I/m_G for the television cameras.

Equivalence of inertial and gravitational mass

- ***Experimentally***, if all forces apart from gravity can be ignored, all objects fall at the same rate (first demonstrated by **Galileo**)
- So, m_I / m_G must be the *same* for all bodies
- And we can choose the constant "G" such that $m_I = m_G$, and $a = GM/r^2$
- This is the **weak equivalence principle**: gravity is equivalent to (indistinguishable from) any other acceleration.

"Weightlessness"

Apollo 10, in orbit (May 18-26, 1969)

No weight, or free-fall?

- Space Station orbits about 500km above Earth's surface. Radius of Earth is 6300km.
- Newton's inverse square law:
  - Gravitational acceleration at location of space station is 86% of what it is on the Earth's surface!
- So, why do the astronauts feel weightless?
  - The astronauts "fall" toward Earth at the same rate as the space station – another example of the equivalence principle.

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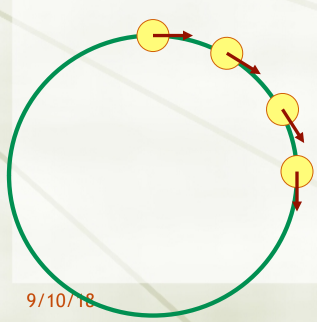
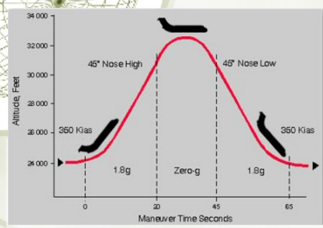


No need to go to space: NASA's vomit comet



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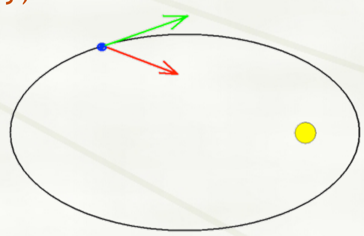
Kepler's laws explained!

- Kepler's laws of planetary motion
  - Can be derived from Newton's laws
  - Just need to assume that planets are attracted to the Sun by gravity (Newton's breakthrough).
  - Full proof requires calculus (or very involved geometry)



- If there were no gravity from Sun, a planet's natural state is to move in a straight line at constant velocity
- But, gravitational attraction by Sun is always making it swerve off course
- We're not going to prove this, but…
  - Newton's gravity law (*$1/R^2$*) is exactly what's needed to make this path be a perfect ellipse – hence Kepler's 1st law.
  - The fact that the force is always directed towards Sun gives Kepler's 2nd law (equal areas in equal times ⇔ conservation of angular momentum).

    Note: Kepler's 2nd law would be true for any "central" force, not just a $1/R^2$ law
  - Newton's 2nd law (F = ma) combined with his gravity law gives Kepler's 3rd law -- the relation between orbit period and semimajor axis

$$P^2 = \frac{4\pi^2}{GM_{sun}} R^3$$

Applications and impact of Newtonian physics

- With Newton's laws, it was possible to make new predictions about orbits of solar system bodies
  - Halley argued that several comet appearances separated by 76 years were actually the same comet, and predicted its recurrence in 1758
  - Planets have near-elliptical orbits, but they are not exact ellipses due to gravity of *other* planets
  - Herschel, in 1781, discovered Uranus; its orbit showed enough variations to predict there must be another as-yet-unknown planet, leading to discovery of Neptune in 1846
- Newton's laws can be applied to stars in galaxies, galaxies in clusters, etc., to understand orbits and "weigh" the system, since the mass is proportional to the inverse-square of the typical orbital period and cube of the orbital distance.
- As Newton's physics came to be widely known, there was a huge cultural impact. With the Universe describable by precise mathematical laws, it supported the idea of "rationality" in other arenas -- including architecture, government, history, etc. Key to shift in thought known as the Enlightenment. The universe is a giant machine! (?)

Newton lets us measure the mass of the Galactic center's black hole

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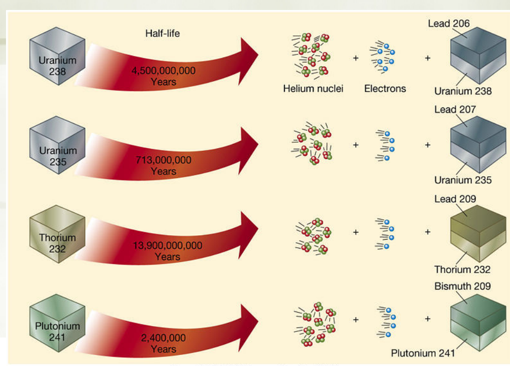
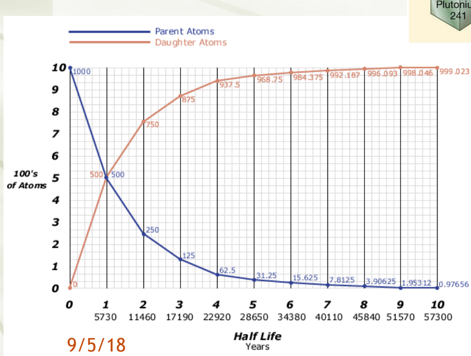
The time domain

- With Copernicus, Kepler, Galileo, and Newton, ideas of the vast magnitude of space beyond Earth opened up
- What about *time*?
  - From ~1700-1850, the idea took hold that Earth was very old
  - Important was the realization that strata of rock take their observed physical form due to weathering, volcanism, etc, acting over very long periods (Charles Lyell: Uniformitarianism)
  - Also important in appreciation of ancient earth was finding progressions of fossils of (often extinct) creatures in successive layers of rock
  - Physical progression in fossil features (invertebrates, fish, mammals) implied a biological transformation, presumed to be in response to environmental changes.
  - Charles Darwin (1859) published *The Origin of Species* to argue that evolution proceeds via natural selection. Much time would be required, since species are observed to change slowly. Darwin estimated at least 300 million years based on geological evidence.

“Red herrings” from physics, and a resolution

- In 1840-50’s physicists Kelvin and Helmholtz argued that the only possible way for the Sun to power itself was by gravitational contraction
- Comparing the rate energy is produced by the Sun (total observed luminosity) to the available gravitational energy, an age of 30 million years for the Sun was estimated
- Thus astronomical and geological/biological ideas were in conflict until the early 20th century
- With development of nuclear physics in early 1900’s, it was understood what was wrong with the Kelvin-Helmholtz argument: Sun’s energy source is *fusion, not gravity*
- Starting in 1920’s radioactive dating became possible
- Ages of meteorites, etc., establishes that the formation of the Solar system occurred about 4.6 billion years ago. Humans, 250,000 years (0.25 Myr); dinosaurs 100 Myr.





Radioactive dating

- Radioactive decay transforms one element into another, with a characteristic half-life
  - It is not affected by the physical conditions, although there are temperature considerations that depend on the mineral
- Age equation: $t=T_{1/2} \log(1+(D-D_0)/P)/\log2$
  - D is the number of stable "daughter" atoms now (D_0 in original composition)
  - P is the number of radioactive "parent" atoms now
  - $T_{1/2}$ is the half-life (time that takes to convert P/2 atoms into D/2 atoms)
- Reactions useful for planetary radioactive dating:
  - $^{238}U \rightarrow {}^{206}Pb$ ($T_{1/2}$ =4.47 billion years)
  - $^{235}U \rightarrow {}^{207}Pb$ ($T_{1/2}$ =704 million years)

Inertial and non-inertial frames of reference

- Newton's laws were clearly powerful. But they also led to some puzzles, particularly relating to reference frames.
- We have already come across idea of frames of reference that move with constant velocity. In such frames, Newton's laws (esp. N1) hold. These are called **inertial frames of reference**.
- Suppose you are in an accelerating car looking at a freely moving object (i.e., one with no forces acting on it). You will see its velocity changing because you are accelerating! *In accelerating frames of reference, N1 doesn't hold* – this is a **non-inertial frame of reference**.

Real and fictitious forces

- In non-inertial frames you might be fooled into thinking that there were forces acting on free bodies.
- Such forces are call "fictitious forces". Examples –
  - G-forces in an accelerating vehicle.
  - Centrifugal forces in amusement park rides.
  - The Coriolis force on the Earth.
- Fictitious forces point opposite to the direction of acceleration
- Fictitious forces are always proportional to the inertial mass of the body.

Next time…

- Principles of space and time

- *HW #1 is due next Thursday*
- *Start reading Ch. 6 for next week*