LECTURE 6: PRINCIPLES OF SPACE AND TIME

- Real and fictitious forces (review)
- Cosmological principle
- Coordinate systems and reference frames
- Galilean relativity
- Cracks in Newton's theory
- The speed of light problem

This week: please read Chapter 6 of text

- If there were no gravity from Sun, a planet's natural state is to move in a straight line at constant velocity
- But, gravitational attraction by Sun is always making it swerve off course
- We're not going to prove this, but…
  - Newton's gravity law ($1/R^2$) is exactly what's needed to make this path be a perfect ellipse – hence Kepler's 1st law.
  - The fact that the force is always directed towards Sun gives Kepler's 2nd law (equal areas in equal times ⇔ conservation of angular momentum).

    Note: Kepler's 2nd law would be true for any "central" force, not just a $1/R^2$ law
  - Newton's 2nd law (F = ma) combined with his gravity law gives Kepler's 3rd law -- the relation between orbit period and semimajor axis

$$P^2 = \frac{4\pi^2}{GM_{sun}} R^3$$

Applications and impact of Newtonian physics

- With Newton's laws, it was possible to make new predictions about orbits of solar system bodies
  - Halley argued that several comet appearances separated by 76 years were actually the same comet, and predicted its recurrence in 1758
  - Planets have near-elliptical orbits, but they are not exact ellipses due to gravity of *other* planets
  - Herschel, in 1781, discovered Uranus; its orbit showed enough variations to predict there must be another as-yet-unknown planet, leading to discovery of Neptune in 1846
- Newton's laws can be applied to stars in galaxies, galaxies in clusters, etc., to understand orbits and "weigh" the system, since the mass is proportional to the inverse-square of the typical orbital period and cube of the orbital distance.
- As Newton's physics came to be widely known, there was a huge cultural impact. With the Universe describable by precise mathematical laws, it supported the idea of "rationality" in other arenas -- including architecture, government, history, etc. Key to shift in thought known as the Enlightenment. The universe is a giant machine! (?)

The time domain

- With Copernicus, Kepler, Galileo, and Newton, ideas of the vast magnitude of space beyond Earth opened up
- What about *time*?
  - From ~1700-1850, the idea took hold that Earth was very old
  - Important was the realization that strata of rock take their observed physical form due to weathering, volcanism, etc, acting over very long periods (Charles Lyell: Uniformitarianism)
  - Also important in appreciation of ancient earth was finding progressions of fossils of (often extinct) creatures in successive layers of rock
  - Physical progression in fossil features (invertebrates, fish, mammals) implied a biological transformation, presumed to be in response to environmental changes.
  - Charles Darwin (1859) published *The Origin of Species* to argue that evolution proceeds via natural selection. Much time would be required, since species are observed to change slowly. Darwin estimated at least 300 million years based on geological evidence.

"Red herrings" from physics, and a resolution

- In 1840-50's physicists Kelvin and Helmholtz argued that the only possible way for the Sun to power itself was by gravitational contraction
- Comparing the rate energy is produced by the Sun (total observed luminosity) to the available gravitational energy, an age of 30 million years for the Sun was estimated
- Thus astronomical and geological/biological ideas were in conflict until the early 20th century
- With development of nuclear physics in early 1900's, it was understood what was wrong with the Kelvin-Helmholtz argument: Sun's energy source is *fusion, not gravity*
- Starting in 1920's radioactive dating became possible
- Ages of meteorites, etc., establishes that the formation of the Solar system occurred about 4.6 billion years ago. Humans, 250,000 years (0.25 Myr); dinosaurs 100 Myr.

Inertial and non-inertial frames of reference

- Newton's laws were clearly powerful. But they also led to some puzzles, particularly relating to reference frames.
- We have already come across idea of frames of reference that move with constant velocity. In such frames, Newton's laws (esp. N1) hold. These are called **inertial frames of reference**.
- Suppose you are in an accelerating car looking at a freely moving object (i.e., one with no forces acting on it). You will see its velocity changing because you are accelerating! *In accelerating frames of reference, N1 doesn't hold* – this is a **non-inertial frame of reference**.

Real and fictitious forces

- In non-inertial frames you might be fooled into thinking that there were forces acting on free bodies.
- Such forces are call "fictitious forces". Examples –
  - G-forces in an accelerating vehicle.
  - Centrifugal forces in amusement park rides.
  - The Coriolis force on the Earth.
- Fictitious forces point opposite to the direction of acceleration
- Fictitious forces are always proportional to the inertial mass of the body.

Cosmological principle

- The physical universe is described in terms of its properties in space and time
- Confined to Earth, we are unable to visit all of space (or obtain information from all of time).
- From Copernicus onward, it was realized that *Earth does not hold a special place*
- In formulating cosmological models, we extend this idea to the much larger scale of the whole Universe
  - Fundamental principle is that any place in the Universe is "just about the same" as any other place. The Universe is essentially *uniform* (on large scales). It is both:
  - *Isotropic:* Uniform in all *directions*

    e.g., surface of a sphere is isotropic, surface of a cylinder is anisotropic
  - *Homogeneous:* Similar conditions at all *locations*
  - Notes:
    1. isotropy as seen from every location implies homogeneity
    2. Homogeneity everywhere need not imply isotropy; e.g. surface of smooth, infinitely long cylinder is homogeneous but not isotropic

- Cosmological principle means that physical laws are assumed to be the same everywhere, too
- The cosmological principle of isotropy and homogeneity, like other scientific hypotheses, is testable by confrontation with data.
- So far, observations support this hypothesis

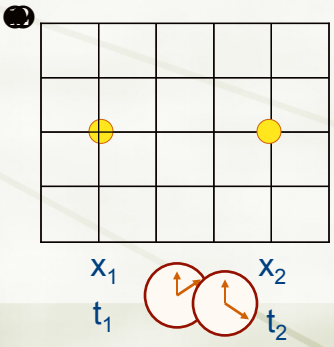
Coordinate systems

- Scientific observations involve making measurements
- fundamental measurements are always of events in terms of their coordinates in space and time
- Space-time coordinates are often written as (x,y,z,t)
- Coordinates are convenient labels, not fundamental attributes of space and time
  - We are free to choose whatever units we want (e.g. m, km, foot,…), and whatever coordinate origin we want
  - What matters is the *intervals* in time and space, not absolute numbers. For Event 1 at (x_1,y_1,z_1,t_1) and Event 2 at (x_2,y_2,z_2,t_2), the time interval is $\Delta t=t_2-t_1$, and using the Pythagorean theorem generalized to 3D, the space interval (distance) is

$$\Delta s = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

Velocities and accelerations

- Velocities are rates of change of vector positions
- Accelerations are rates of change of vector velocities
- For motion in a *given* direction, the velocity is equal to the change in position $\Delta x = x_2 - x_1$ divided by the corresponding change in time $\Delta t = t_2 - t_1$: $v = \Delta x / \Delta t$
- Similarly, $a = \Delta v / \Delta t$



Frames of reference, again

- The frame of reference in which a measurement is made consists of the spatial coordinates (the grid) and time coordinate (the clock) that are used to make the measurement
- Note that in general, we use a "clock" that is attached to the spatial coordinate system we are using (why this matters will become apparent soon!)
- The reference frame may potentially have any arbitrary motion and/or acceleration. However, reference frames that have $a \neq 0$ are fundamentally different from those with $a = 0$
- "Inertial frame" = unaccelerated frame
- "Non-inertial frame" = accelerated frame
- *How can an observer inside the frame tell the difference?*
  - In an inertial frame, a free particle (no forces acting) has constant velocity (including v=0 special case)
  - In a non-inertial frame, a free particle's velocity (speed and/or direction) varies
  - Note that for humans, even if we don't have a free particle handy for experiments, we can sense accelerations physiologically

Real and fictitious forces

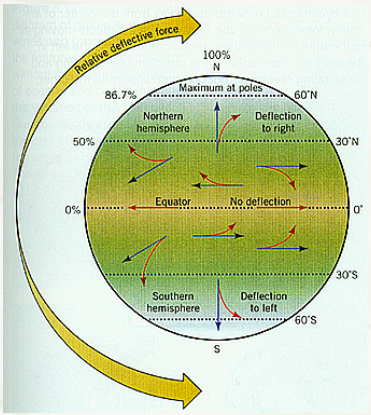
- In non-inertial frames you might be fooled into thinking that there were forces acting on free bodies, because velocities change.
- Such forces are call "fictitious forces". Examples –
  - G-forces in an accelerating vehicle.
  - Centrifugal forces in amusement park rides.
  - The Coriolis force on the Earth.
- Fictitious forces point opposite to the direction of acceleration measured in an inertial frame
- Fictitious forces are always proportional to the inertial mass of the body.

Coriolis force: a fictitious force

Inertial frame

Noninertial frame

Coriolis at work



…does this seem familiar?

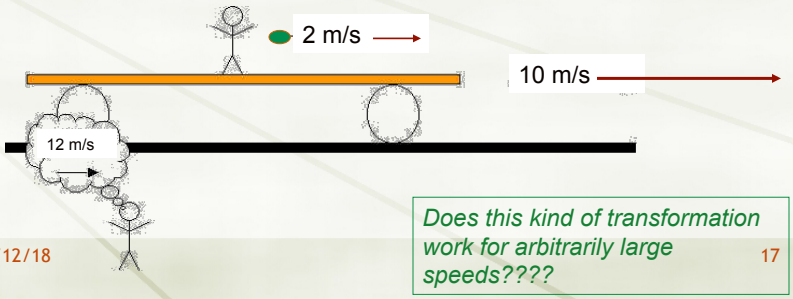
Recall that from weak equivalence principle, inertial mass=gravitational mass ⇒ *gravitational force is proportional to inertial mass.*

Maybe gravity is a fictitious force…

… and we live in an accelerating frame of reference???

Relativity

- "Relativity" refers in general to the way physical measurements made in a given inertial frame are related to measurements in another frame.
- An inertial observer is one whose rest frame is inertial
- A quantity is invariant if all inertial observers obtain the same value
- Under Galilean relativity, measurements are transformed simply by adding or subtracting the velocity difference between frames:
- v_{ball}(measured on ground)=v_{train} (measured on ground)+v_{ball}(measured on train)
  12 m/s = 10m/s + 2 m/s
- V_{ball}(measured on train)=v_{ground}(measured on train)+ v_{ball}(measured on ground)
  2 m/s = -10m/s + 12 m/s



Does this kind of transformation work for arbitrarily large speeds????

NEWTON'S WORRIES

- Newton himself had concerns about his theory
  - Gravity is "action at a distance" - i.e. gravitation force somehow (mysteriously) reaches across large distances.
    - Newton was very suspicious of this.
    - How is information *communicated* from one gravitating body to another?
  - Problems with a static universe
    - Newton imagined that the Universe was infinite and full of stationary stars, each exerting a gravitational force on the others.
    - this configuration (and any other that Newton could think of) is unstable… the smallest disturbance and it will collapse in on itself!
    - What prevents this collapse?

Subsequent developments: electromagnetic waves

- James Clerk Maxwell (1831-1879)
  - Developed theory of electromagnetic fields in the 1860's (Maxwell's equations)
  - These unify the electric and magnetic forces in a single theory

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \cdot \mathbf{E} = \rho$$

$$\nabla \times \mathbf{E} = -\partial \mathbf{B} / \partial t$$

$$\nabla \times \mathbf{B} = 4\pi \mathbf{J} / c + (1/c)\partial \mathbf{E} / \partial t$$

THE SPEED OF LIGHT PROBLEM

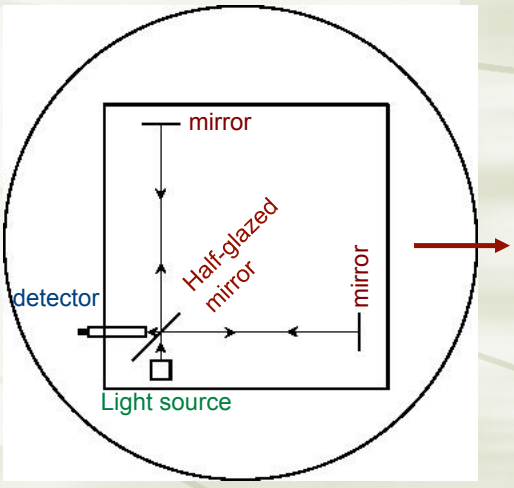
- Maxwell's equations:
  - Predict "waves" of electromagnetic energy – and it was quickly realized that these are light waves!
  - The speed of light "c" appears as a fundamental constant in the equations.
  - c = 299,792.458 km/s (3×10^5 km/s is close)
  - **BUT, what frame of reference is this measured relative to???**

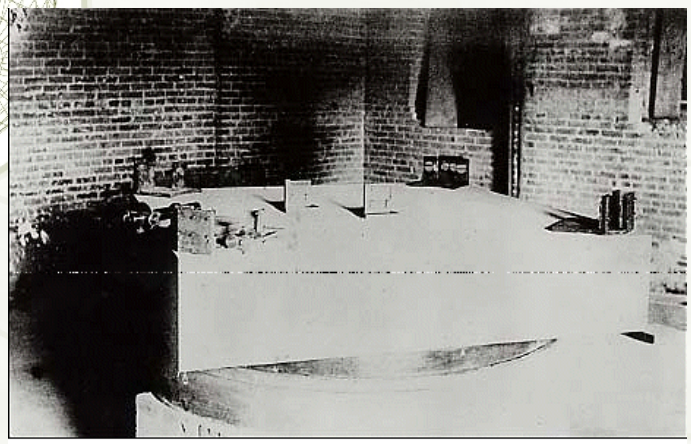
Ether and light waves

- Luminiferous Ether (19th century)
  - Hypothetical substance that fills space - provides a "medium" through which light can travel.
  - Idea was that Maxwell's equations, as written, would apply only in frame of ether
  - This would explain why the speed of wave propagation "c" is a constant in the equations
  - If speed of light in ether is "c" , and *if Galilean relativity holds*, then speed of light measured in other frames would be different from "c"
  - Albert Michelson & Edward Morley attempted (1887) to measure motion of Earth through ether…

Michelson-Morley Experiment

- Light leaves source, and is partly reflected 45°/partly transmitted at half-glazed mirror
- Light returning from both paths is collected at detector
- Path length of light along either "arm" of apparatus is the same
- If one arm is along Earth's motion through ether, and the other arm is perpendicular to motion through ether, then light travel time was expected to be shorter for perpendicular arm





Michelson & Morley's 1887 interferometer
built in the basement of Western Reserve
Photo: Case Western Reserve Archive

Michelson-Morley results

- Travel time difference would be measured using interference fringes of light from two paths
- Apparatus could be rotated to make sure no effects from set-up
- Repeated at different times of year, when Earth's motion differs; Earth's speed around the Sun is ~30 km/s
- Experiment performed in 1887
- Results
  - M-M showed that speed of light was same in any direction to within 5 km/s
  - Modern versions of the experiment show constancy to better than 1 micron/s
- So, what's going on??

Attempts to deal with M-M results

- Maybe the ether "sticks" to the Earth?
  - Gets "dragged" as Earth spins and orbits Sun…
  - Possibility at the time, but no-longer viable.
- Maybe the ether squeezes the arms of the M-M experiment and distorts the result? "Fitzgerald contraction" (1889)?
  - A contraction (in the direction parallel to motion through ether) would change the light travel time to compensate for the difference expected due to different speed of light

$$L = L_0\sqrt{1 - V^2/c^2}$$

- Major mystery ("crisis") in 19th century physics – two highly successful theories seemed incompatible!
  - Mechanics – Galilean Relativity and Newton's laws
  - Electromagnetism – Maxwell's equations

Next time…

Einstein to the rescue!