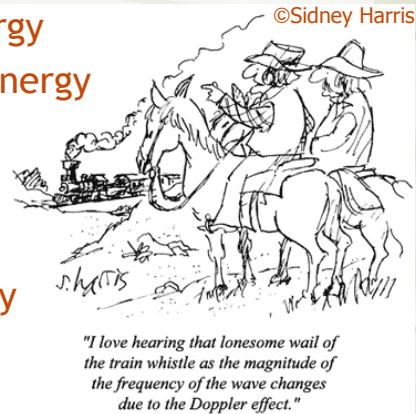




## Lecture 9: Special Relativity III & General Relativity I

- ★ Einstein's formula for energy
- ★ Equivalence of mass and energy
- ★ Mass turning into energy
- ★ Energy turning into mass
- ★ Redshifting of light
- ★ Need for General Relativity



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## Last time...

- ★ We discussed further aspects of special relativity, including:
  - ★ Simultaneity and causality
  - ★ Space-time diagrams
  - ★ Invariant intervals and proper time
  - ★ Reciprocity and the twins paradox

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## I : MASS AND ENERGY

- ★ Einstein reworked Newton's laws of mechanics using his new relativistic formulae.
- ★ He found a formula for the energy of a moving object with mass  $m$  and speed  $V$  -

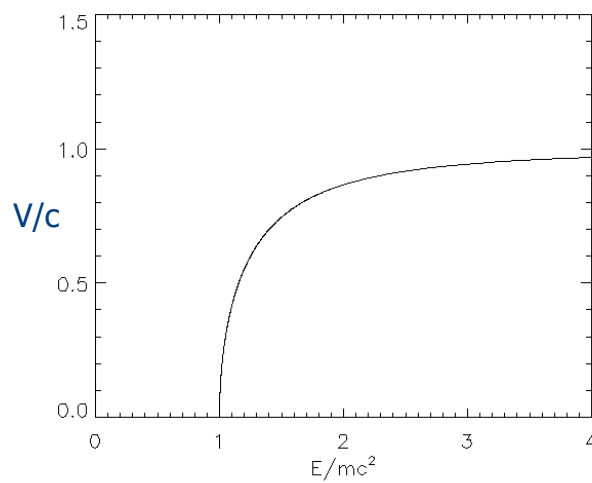
$$E = \gamma mc^2 = \frac{mc^2}{\sqrt{1 - V^2/c^2}}$$

- ★ Thus, energy increases as the speed increases, and energy would become infinite if  $V$  approaches  $c$

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## Energy vs. $V/c$



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## Was Newton just plain wrong?

- ✦ What about objects moving at “small velocity”?
- ✦ It can be shown that:

$$E \approx mc^2 + \frac{1}{2}mV^2$$

- ✦ The  $\frac{1}{2}mv^2$  is the Newtonian expression for the kinetic energy of a moving object.
- ✦ What counts as “small velocity”?
  - ✦ For car going at 30mph, approximate formula is wrong by 1 part in  $10^{30}$
  - ✦ For rocket going at 30,000mph, this approximate formula is wrong by 1 part in  $10^{18}$
  - ✦ So, approximation is fine for all velocities experienced in everyday life.

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## Rest mass energy

- ✦ If we put  $V=0$  in Einstein’s energy formula, we get...

$$E = mc^2$$

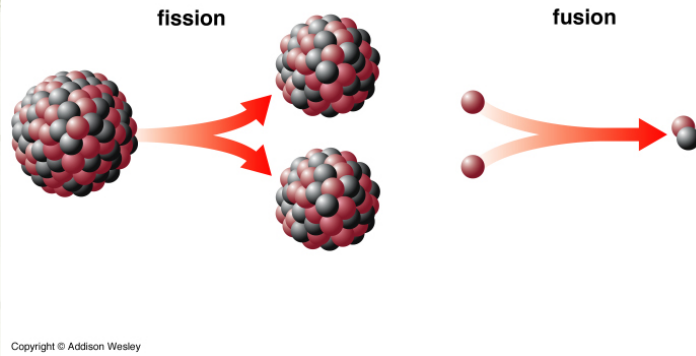
- ✦ What does this mean?
  - ✦ Maybe it is some fundamental “irreducible” (i.e., inaccessible) energy that every object possesses?
  - ✦ Or, perhaps this energy can be accessed? In other words, maybe mass can be turned into “usable” energy? **It turns out that this is correct!**
  - ✦ Also, this can go the other way - energy can be turned into mass.

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## II : EXAMPLES OF CONVERTING MASS TO ENERGY

- ★ Nuclear fission
- ★ Nuclear fusion

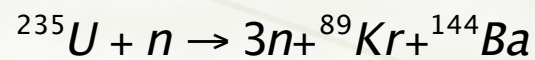


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## Fission

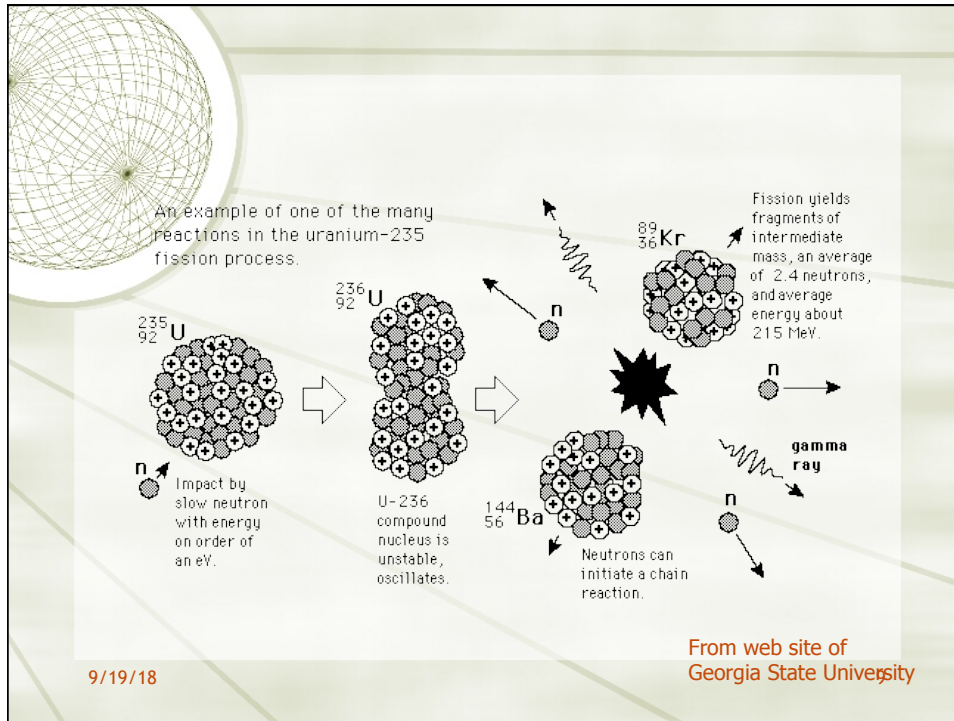
- ★ Nuclear fission (e.g., of Uranium)
  - ★ Nuclear Fission - the splitting up of atomic nuclei
  - ★ E.g., Uranium-235 nuclei split into fragments when smashed by a moving neutron. One possible nuclear reaction is



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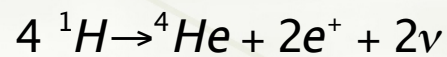
## Fission

- ★ Nuclear fission (e.g., of Uranium)
  - ★ Nuclear Fission - the splitting up of atomic nuclei
  - ★ E.g., Uranium-235 nuclei split into fragments when capturing a moving neutron. One possible nuclear reaction is
 
$$^{235}\text{U} + n \rightarrow 3n + ^{89}\text{Kr} + ^{144}\text{Ba}$$
  - ★ Mass of products of reaction (neutrons, Krypton, Barium) is slightly less than mass of initial Uranium nucleus + neutron
  - ★ That mass “lost” has been converted into energy (gamma-rays and kinetic energy of fragments):
  - ★  $E = mc^2$

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## Fusion

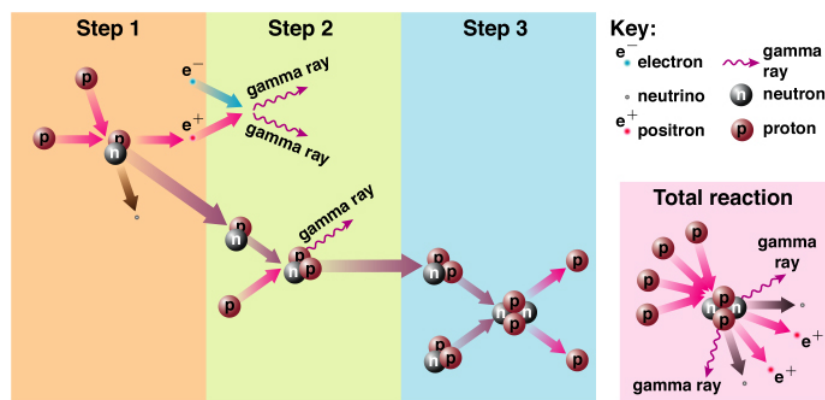
- ✦ Nuclear fusion (e.g. hydrogen)
  - ✦ Fusion - the sticking together of atomic nuclei
  - ✦ Much more important for Astronomy (and life on Earth!) than fission
    - ✦ Power source for stars, including the Sun
    - ✦ Path to making heavy elements (C, N, O, Si, Fe...)
  - ✦ Important example - hydrogen fusion.
    - ✦ Ram together 4 hydrogen nuclei to form helium nucleus
    - ✦ Spits out couple of “positrons” and “neutrinos” in process



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## The proton-proton fusion cycle



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## Hans Bethe's Nobel Prize

### The Nobel Prize in Physics 1967



Hans Albrecht  
Bethe  
Prize share: 1/1

The Nobel Prize in Physics 1967 was awarded to Hans Bethe "for his contributions to the theory of nuclear reactions, especially his discoveries concerning the energy production in stars".

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## Fusion

- ✦ Mass of final helium nucleus plus positrons and neutrinos is less (by about 1%) than original 4 hydrogen nuclei
- ✦  $E = mc^2$
- ✦ Mass has been converted into energy (gamma-rays and kinetic energy of final particles)
- ✦ This nuclear reaction (and similar ones) is the energy source for...
  - ✦ Hydrogen Bombs (about 1 kg of mass converted into energy gives equivalent of 20 Megatons of TNT)
  - ✦ The Sun (about  $4 \times 10^9$  kg of matter per second is converted into energy, ultimately yielding sunlight)

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## Antimatter

### ★ Anti-matter

- ★ For every kind of particle, there is an antiparticle...
  - ★ Electron  $\leftrightarrow$  anti-electron (also called positron)
  - ★ Proton  $\leftrightarrow$  anti-proton
  - ★ Neutron  $\leftrightarrow$  anti-neutron
- ★ Anti-particles have opposite properties from the corresponding particles (e.g., opposite charge)... but exactly the same mass.
- ★ When a particle and its antiparticle meet, they can completely annihilate each other... **all of their mass is turned into energy (gamma-rays)!**

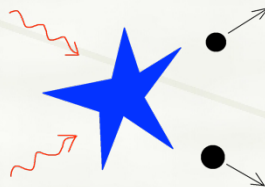
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## III: EXAMPLES OF CONVERTING ENERGY TO MASS

### ★ Particle/anti-particle production

- ★ Opposite process to that just discussed!
- ★ Energy (e.g., gamma-rays) can produce particle/anti-particle pairs



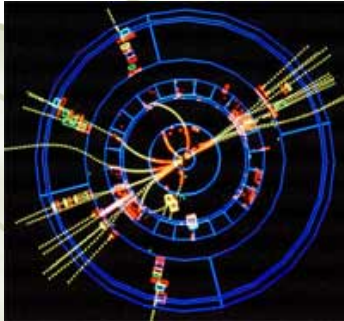
- ★ **Very fundamental process in Nature... we'll see that this process, operating in early universe, is responsible for all of the mass that we see today!**

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- ★ Particle production in a particle accelerator
- ★ Can reproduce conditions similar to early universe in modern particle accelerators...

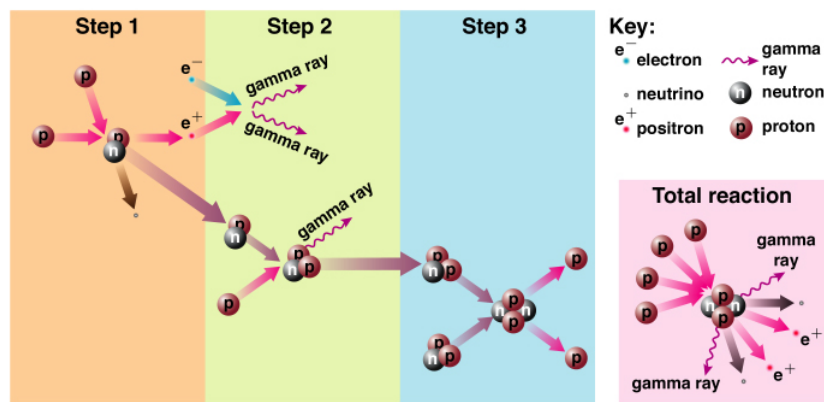


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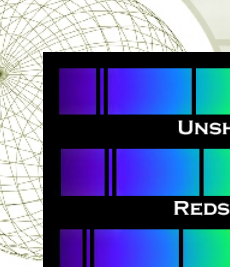
## The proton-proton fusion cycle



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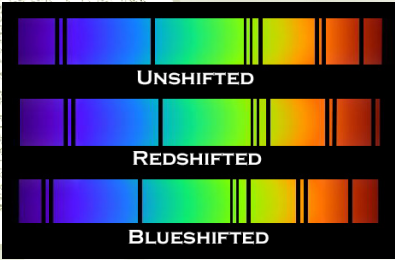
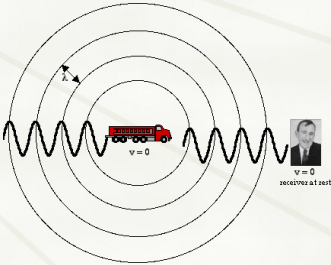
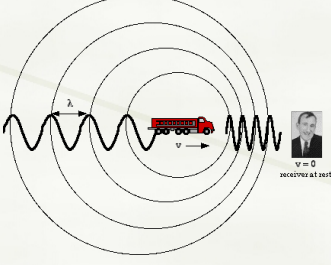
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# Doppler effect

<http://carma.astro.umd.edu/AWE/Doppler/redshift.html>

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# Fire Engine Siren



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## Redshifting of light

- ★ Photons (light particles) are massless, but their energy also changes when observer's frame changes
  - ★ Recall (see Chapter 4 for review!) light has a wave/particle dual nature
  - ★ Energy of a photon is proportional to the frequency  $\nu$  of the corresponding wave:  $E=h\nu$ 
    - ★  $h = 6.63 \times 10^{-34}$  Joule-s
  - ★ When changing frames with a velocity  $V$ , the frequency of the light waves and energy of the photons changes by a factor

$$\sqrt{\frac{1 + \frac{V}{c}}{1 - \frac{V}{c}}} = \left(1 + \frac{V}{c}\right) \times \gamma$$

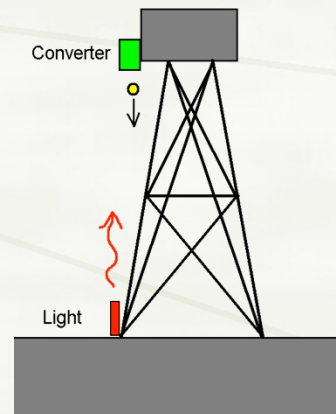
- ★ Moving towards a light source, the frequency and energy increase by this factor = **blueshift** (bluer, not necessarily blue)
- ★ Moving away from a light source, the the frequency and energy decrease by this factor = **redshift** (redder, not necessarily red)

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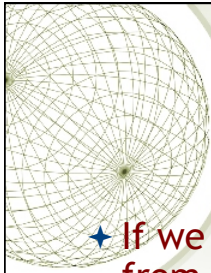
## Motivation for General Relativity: Einstein's tower

- ★ So far, we have ignored the effects of gravity. Is this really okay??
- ★ Consider another thought experiment, to test whether **light can be affected by gravity.**
- ★ Consider a tower on Earth
  - ★ Shine a light ray from bottom to top
  - ★ When light gets to top, turn its energy into mass.
  - ★ Then drop mass to bottom of tower, in Earth's gravity field
  - ★ Then turn it back into energy



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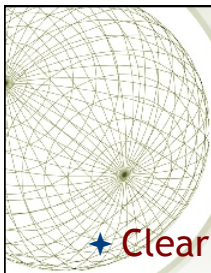


## Perpetual motion?

- ✦ If we could do this, then we could get energy from nothing!
  - ✦ Original energy in light beam =  $E_{\text{start}}$
  - ✦ Thus, mass created at top is  $m = E_{\text{start}}/c^2$
  - ✦ Then drop mass... at bottom of tower it has picked up speed (and energy) due to the effects of gravitational field.
  - ✦ When we turn it back into energy, we have  $E_{\text{end}} = E_{\text{start}} + E_{\text{grav}}$
  - ✦ But, we started off with only  $E_{\text{start}}$  - we have made energy! We're rich!

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## Maxwell and gravity

- ✦ Clearly, our assumption must be wrong...
  - ✦ light **must** be affected by gravity.
  - ✦ But gravity does not appear in Maxwell's equations, which govern light
  - ✦ Thus, Maxwell's equations are not exactly valid in the reference frame of Earth's surface, where there is gravity.
  - ✦ The Earth's surface must not be an inertial frame of reference!

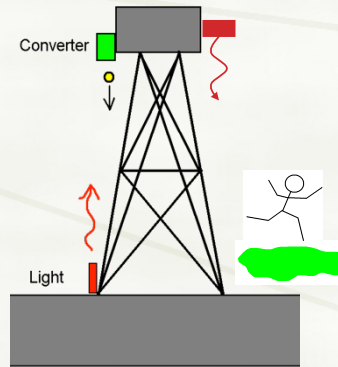
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# Resolving the tower problem

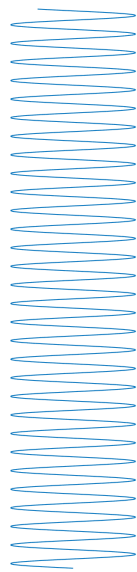
- Now consider light ray aimed from top to bottom of tower
- Free-falling (FF) observer sees light ray travel **unaffected** by gravity, since free fall yields a state of apparent weightlessness (inertial frame)
- From "Earth's" frame...
  - Free-falling (FF) observer is traveling faster and faster
  - Falling observer would see an increasing **redshift** of light source according to special relativity
  - If FF observer is **supposed** to see a constant frequency light beam, then light must get relatively **blueshifted** as it falls in gravitational field, to compensate
  - Light beam aimed upward must conversely be increasingly **redshifted** with height
  - Gravitational redshifting removes just the right amount of energy to solve the tower paradox!



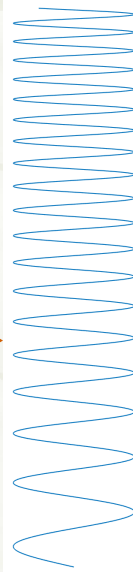
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The frequency of the photon looks like this as it moves down

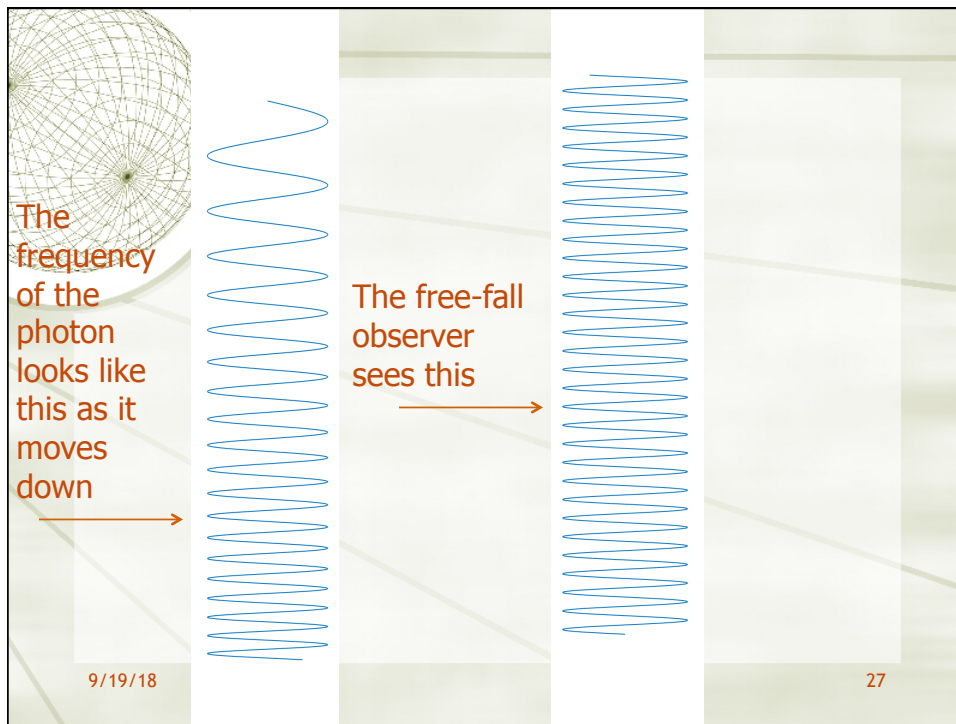


The free-fall observer sees this



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## Resolving the tower problem

- ✦ Now consider light ray aimed from top to bottom of tower
- ✦ Free-falling (FF) observer sees light ray travel **unaffected** by gravity, since free fall yields a state of apparent weightlessness (inertial frame)
- ✦ From "Earth's" frame...
  - ✦ Free-falling (FF) observer is traveling faster and faster
  - ✦ Falling observer would see an increasing *redshift* of light source according to special relativity
  - ✦ If FF observer is **supposed** to see an constant frequency light beam, then light must get relatively *blueshifted* as it falls in gravitational field, to compensate
  - ✦ Light beam aimed upward must conversely be increasingly *redshifted* with height
  - ✦ Gravitational redshifting removes just the right amount of energy to solve the tower paradox!

Converter

Light

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## *Next time...*

- ★ Light is affected by gravity
- ★ Therefore special relativity is not enough!
- ★ Einstein developed the General Theory of Relativity to deal with this

Read Chapter 8 of the book