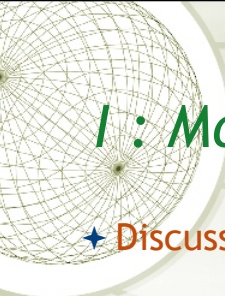


## Lecture 9 : Special Theory of Relativity II

- ◆ More about time dilation...
  - ◆ Twin's paradox
  - ◆ The Muon Experiment
- ◆ Length (Fitzgerald) contraction
- ◆ Simultaneity and causality

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## I : More about time dilation...

- ◆ Discussion of the Twin's paradox [Board]

9/27/10 2



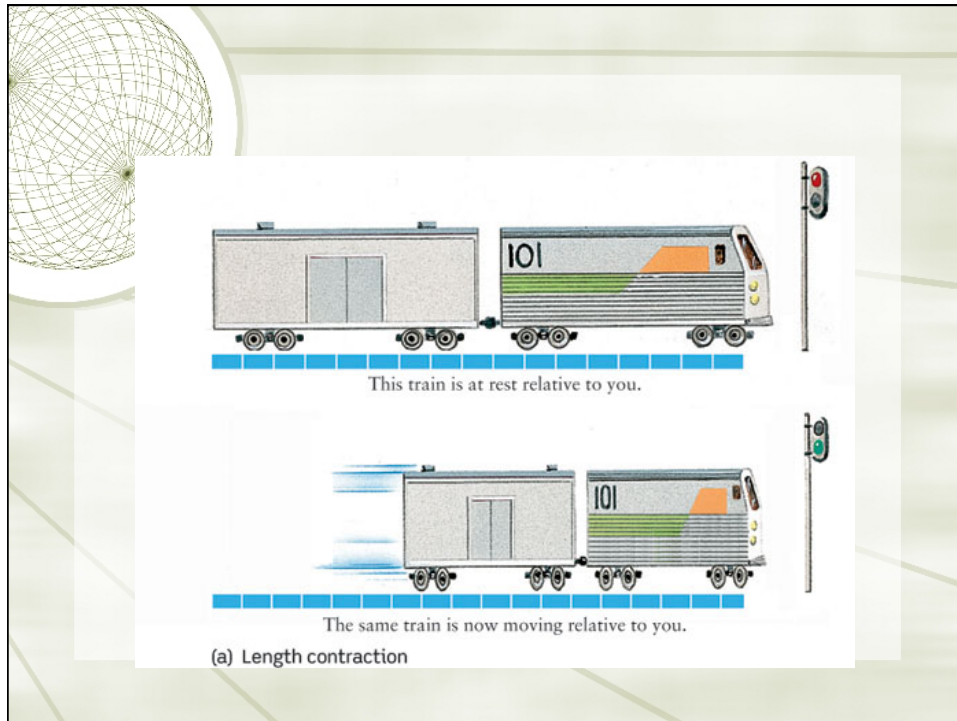
## Rossi & Hall Muon Experiment

- ★ Classic experiment verifying time dilation was performed by Rossi & Hall in 1941...
  - ★ Muons are “electron-like” particles... when at rest, they decay with a half-life of about  $1.56\mu\text{s}$
  - ★ Muons are produced when cosmic rays slam into upper atmosphere, then rain down to Earth
  - ★ Rossi & Hall measured the number of muons detected at the top of a 2000m mountain, and compared it to the number at sea-level...
    - ★ Find 560 muons/hour at top of mountain
    - ★ Even at  $v=c$ , will take  $6.5\mu\text{s}$  for muon to travel 2000m
    - ★ More than 4 half lives... less than  $1/16$ th of particles should be left by the time they reach the bottom
    - ★ BUT, they measured 422 muons/hour at bottom
    - ★ It seems like only  $0.64\mu\text{s}$  have passed in the muon’s frame of reference... so they are moving with  $\gamma \approx 10$



## II : Length (Fitzgerald) contraction

- ★ [Discussion on the board : Think again about the Rossi & Hall muon experiment... but now from a muon’s perspective!]
- ★ Fitzgerald contraction...
  - ★ A moving object **contracts** by a factor  $\gamma$  (the same Lorentz factor) in the direction of motion
  - ★ This is really a contraction of space itself... the object does not experience forces or stresses that make it contract
  - ★ Again, everything is relative... if someone watches you travel past them at high speed, you will appear to be contracted in the direction of motion



### III: RELATIVITY OF SIMULTANEITY

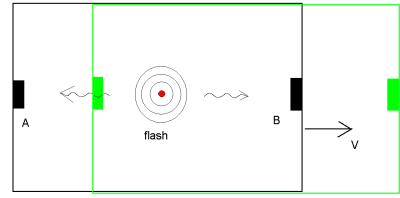
- Consider an observer in a room. Suppose there is a flash bulb exactly in the middle of the room.
- Suppose sensors on the walls record when the light rays hit the walls.

Since speed of light is constant, light rays will hit opposite walls at precisely the same time. Call these events A and B.

9/27/10 6

## Change frames...

- ★ Imagine performing same experiment aboard a moving spacecraft, and observing it from the ground.
- ★ For the observer on the ground, the light rays will not strike the walls at the same time (since the walls are moving!). Event A will happen before event B.




The diagram shows a rectangular frame representing a spacecraft moving to the right with velocity  $v$ . A central point is labeled "flash". Two light rays, represented by wavy arrows, emanate from the flash towards two vertical walls labeled A and B. Wall A is on the left and wall B is on the right. The spacecraft is shown in a slightly tilted position to indicate its motion.

- ★ But astronaut in spacecraft thinks events are simultaneous.
- ★ Concept of “events being simultaneous” (i.e. simultaneity) is different for different observers (**Relativity of simultaneity**).

9/27/10 7

## Change frames...

- ★ Imagine performing same experiment aboard a moving spacecraft, and observing it from the ground.
- ★ For the observer on the ground, the light rays will not strike the walls at the same time (since the walls are moving!). Event A will happen before event B.



The diagram shows two vertical brown lines representing walls. A yellow circle representing a flash is positioned between the two walls. The walls are slightly tilted, suggesting they are part of a moving frame.

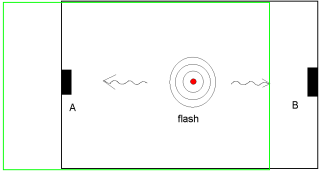
- ★ But astronaut in spacecraft thinks events are simultaneous.
- ★ Concept of “events being simultaneous” (i.e. simultaneity) is different for different observers (**Relativity of simultaneity**).

9/27/10 8



## Change frames again!

- ★ What about perception of a 3<sup>rd</sup> observer who is moving faster than spacecraft?



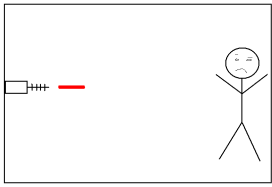
The diagram shows a rectangular spacecraft with two vertical lines representing the ends, labeled 'A' on the left and 'B' on the right. In the center, there is a circular 'flash' with concentric circles radiating outwards. Wavy lines representing light waves are shown emanating from the flash towards both ends of the spacecraft.

- ★ 3<sup>rd</sup> observer sees event B before event A
- ★ So, order in which events happen can depend on the frame of reference.

9/27/10 9

## The laser gun experiment


- ★ Suppose there is a laser gun at one end of spacecraft, targeted at a victim at the other end.



The diagram shows a rectangular spacecraft. On the left side, there is a laser gun represented by a small rectangle with a red line extending to the right. On the right side, there is a stick figure representing a victim with a sad face.

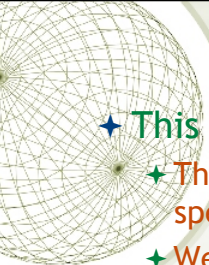
- ★ Laser gun fires (event A) and then victim gets hit (event B).
- ★ Can we change the order of these events by changing the frame of reference? i.e., can the victim get hit **before** the gun fires?

9/27/10 10



- ✦ This is a question of **causality**.
- ✦ The events described are **causally-connected** (i.e. one event can, and does, affect the other event).
- ✦ In fact, it is **not possible** to change the order of these events by changing frames, according to Special Relativity theory.

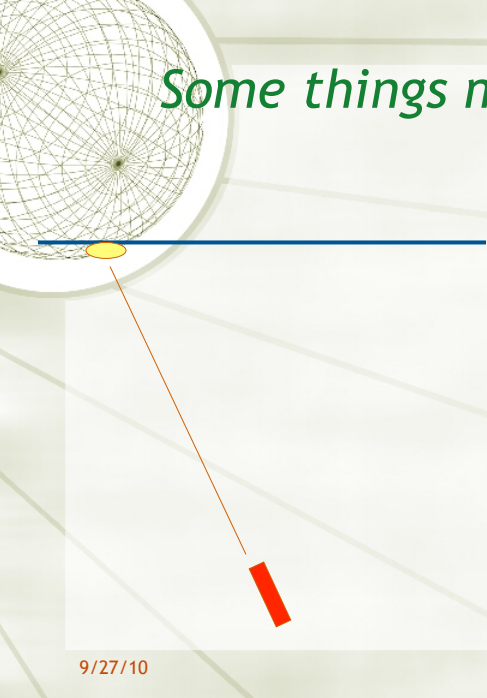
9/27/10 11



- ✦ This is true provided that
  - ✦ The laser blast does not travel faster than the speed of light
  - ✦ We do not change to a frame of reference that is going faster than the speed of light
- ✦ To preserve the **Principle of Causality** (cause precedes effect, never vice versa), the speed of light must set the upper limit to the speed of anything in the Universe. Anything? Well, **anything that transmits any information**.

9/27/10 12

## Some things move faster than light

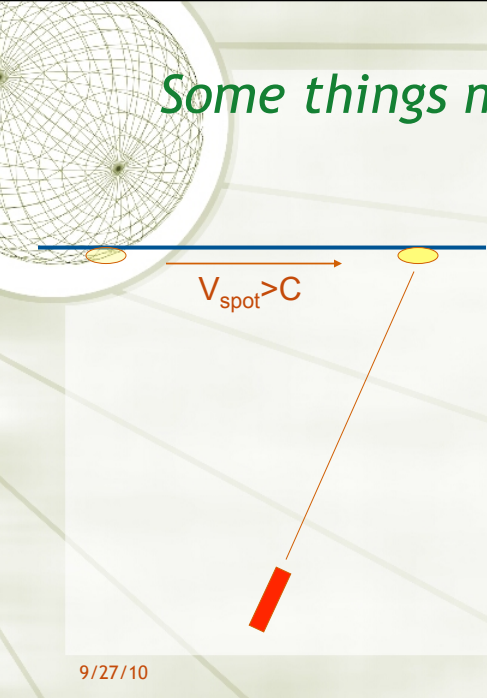


The diagram shows a wireframe sphere on the left. A horizontal blue line represents a screen. A yellow oval on the left end of the screen is connected by a red line to a red rectangular object below it, representing a laser. A red line also extends from the yellow oval to the right, representing a light beam. The screen is tilted upwards from left to right.

- ✦ But they transmit no information
- ✦ E.g., light spot on a distant screen

9/27/10 13

## Some things move faster than light



The diagram is similar to the one above, but with an orange arrow pointing from the left yellow oval to the right yellow oval on the screen. Below the arrow is the text  $V_{\text{spot}} > C$ .

- ✦ But they transmit no information
- ✦ E.g., light spot on a distant screen

9/27/10 14

## Causality

- ★ Can causality be proved?
  - ✦ No, it is an axiom of physics
- ★ What if causality doesn't hold?
  - ✦ Then the Universe returns to being random, unconnected events that can't be understood or predicted.
  - ✦ This would be a true "end of science."
- ★ So we will *insist* on causality as we continue to explore relativity.

9/27/10
15

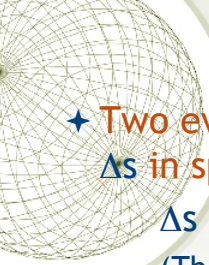
## Space-time diagrams

- ★ Because space and time are "mixed up" in relativity, it is often useful to make a diagram of events that includes both their space and time coordinates.
- ★ This is simplest to do for events that take place along a line in space (one-dimensional space)
  - ✦ Plot as a 2D graph
  - ✦ use two coordinates:  $x$  and  $ct$
- ★ Can be generalized to events taking place in a plane (two-dimensional space) using a 3D graph (volume rendered image):  $x$ ,  $y$  and  $ct$
- ★ Can also be generalized to events taking place in 3D space using a 4D graph, but this is difficult to visualize

$x$   
World lines of events

9/27/10
16



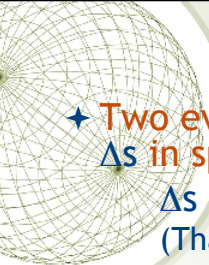


## Distances in time and space

- ★ Two events A and B separated by distance  $\Delta s$  in space (x, y, z):
 
$$\Delta s = [(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2]^{1/2}$$
 (Thanks, Pythagoras!)
 

where  $\Delta x = x_A - x_B$ ,  $\Delta y = y_A - y_B$ ,  $\Delta z = z_A - z_B$

9/27/10 17



## Distances in time and space

- ★ Two events A and B separated by distance  $\Delta s$  in space (x, y, z):
 
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 (Thanks, Pythagoras!)
 

where  $\Delta x = x_A - x_B$ ,  $\Delta y = y_A - y_B$ ,  $\Delta z = z_A - z_B$
- ★ Two events A and B separated by distance  $\Delta s$  in time (t):
 
$$\Delta s = [(c\Delta t)^2]^{1/2}$$

where  $\Delta t = t_A - t_B$ , and we've multiplied by c to make the units of  $\Delta s$  come out as a distance
- ★ Two events A and B separated in x and t:
 
$$\Delta s = [(c\Delta t)^2 - (\Delta x)^2]^{1/2}$$

9/27/10 18

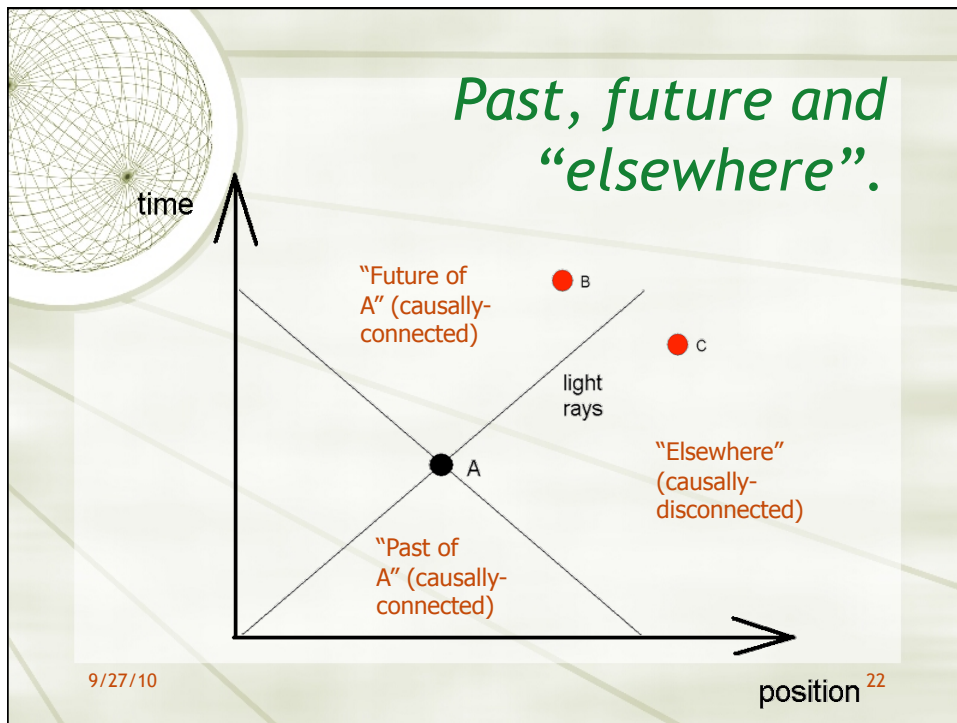
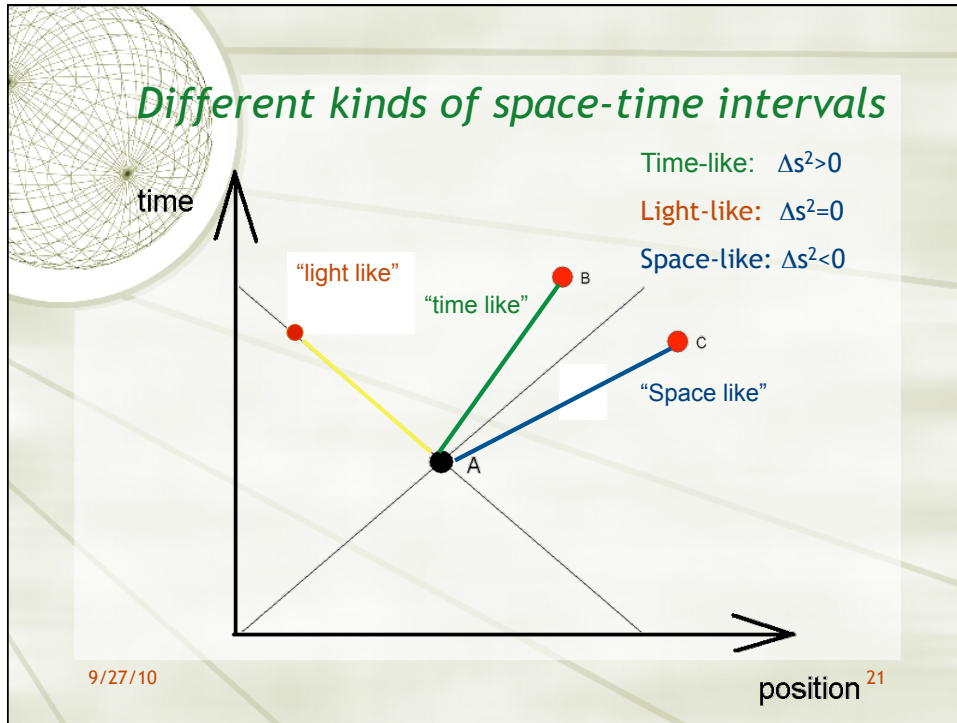
## Space-time intervals

- + Two events A and B in space-time are separated by an **invariant interval**, given by
 
$$\Delta s = [(c\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2]^{1/2}$$
 where  $\Delta t = t_A - t_B$ ,  $\Delta x = x_A - x_B$ ,  $\Delta y = y_A - y_B$ ,  $\Delta z = z_A - z_B$ ,
- + The formula is analogous to Pythagorean equation, but modified to account for the difference between space (x) and time (ct)
- + The invariant space-time interval is an important quantity because it is independent of the frame in which it is measured; *all* observers agree on it!
  - + This is true even though the  $\Delta t$ ,  $\Delta x$ , etc *individually* are different for different observers (due to time dilation, space contraction)
  - + The invariant interval is equal in value to (proper time of event)  $\times c$
- + Space-time interval is zero for any two points on light ray world line
- + Proper time between two events connected by a curved world line is computed by adding up results for small straight intervals along curve
  - + Even if two curved world lines start and end at the same place, they may result in different proper time intervals

9/27/10
19

## Light cone for event "A"

9/27/10
position<sup>20</sup>



## Spacetime diagrams in different frames

- ✦ Changing from one reference frame to another...
  - ✦ Affects time coordinate (time dilation)
  - ✦ Affects space coordinate (length contraction)
  - ✦ Leads to a distortion of the space-time diagram as shown in figure.
- ✦ Events that are simultaneous in one frame are **not** simultaneous in another frame

23

## Causality

- ✦ Events A and B...
  - ✦ **Cannot** change order of A and B by changing frames of reference.
  - ✦ A can also communicate information to B by sending a signal at, or less than, the speed of light.
  - ✦ This means that A and B are causally-connected.
- ✦ Events A and C...
  - ✦ **Can** change the order of A and C by changing frame of reference.
  - ✦ If there were any communication between A and C, it would have to happen at a speed faster than the speed of light.
- ✦ If idea of **cause and effect** is to have any meaning, we must conclude that no communication can occur at a speed faster than the speed of light.

24