Lecture 10 : Special Theory of Relativity III

- A new velocity addition law
- Mass and energy

I : A NEW VELOCITY ADDITION LAW

- Once we've taken into account the way that time and distances change, what's the new law for adding velocities? Result is…

$$V_{add} = \frac{V_1 + V_2}{1 + V_1 V_2 / c^2}$$

- Notes
  - If $V_1 < c$ and $V_2 < c$, then $V_{add} < c$
  - If either $V_1 = c$ or $V_2 = c$, then $V_{add} = c$

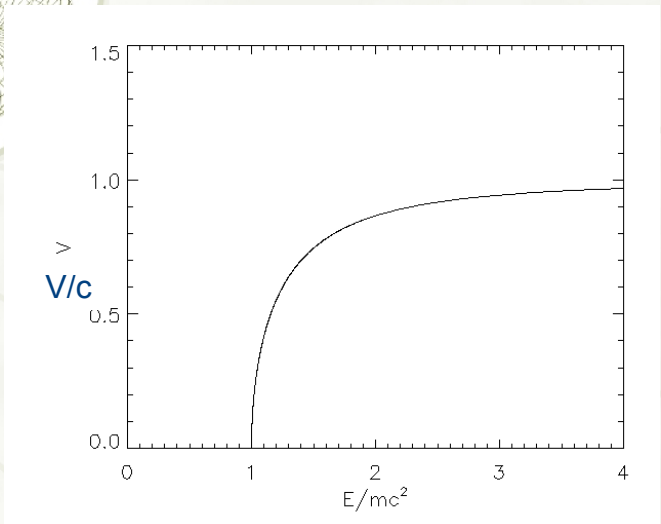
II : MASS AND ENERGY

- Einstein reworked Newton's laws of mechanics using his new relativistic formulae.
- He found a formula for the energy of a moving object with mass *m* and speed *V* –

$$E = \gamma mc^2 = \frac{mc^2}{\sqrt{1 - V^2/c^2}}$$

- Thus, energy increases as the speed increases, and energy would become infinite if *V* approaches *c*

Energy vs. V/c



Was Newton just plain wrong?

- What about objects moving at "small velocity"?
- It can be shown that:

$$E \approx mc^2 + \frac{1}{2}mV^2$$

  - The ½mv² is the Newtonian expression for the **kinetic energy** of a moving object.
- What counts as "small velocity"?
  - For car going at 30mph, approximate formula is wrong by 1 part in 10^{30}
  - For rocket going at 30,000mph, this approximate formula is wrong by 1 part in 10^{18}
  - So, approximation is fine for all velocities experienced in everyday life.

Rest mass energy

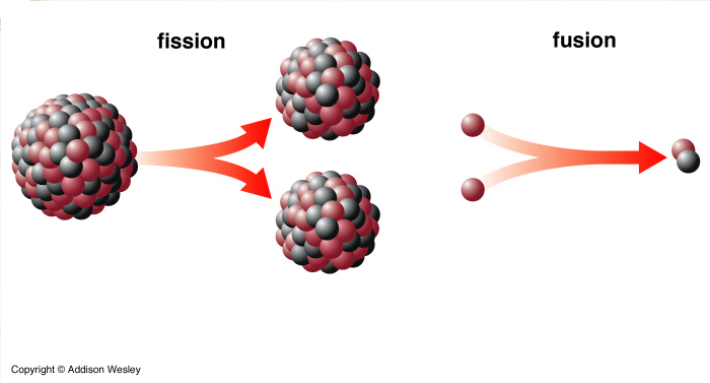
- If we put V=0 in Einstein's energy formula, we get…

$$E = mc^2$$

- What does this mean?
  - Maybe it is some fundamental "irreducible" (i.e., inaccessible) energy that every object possesses?
  - Or, perhaps this energy can be accessed? In other words, maybe mass can be turned into "usable" energy? **It turns out that this is correct!**
  - Also, this can go the other way – energy can be turned into mass.

III : EXAMPLES OF CONVERTING MASS TO ENERGY

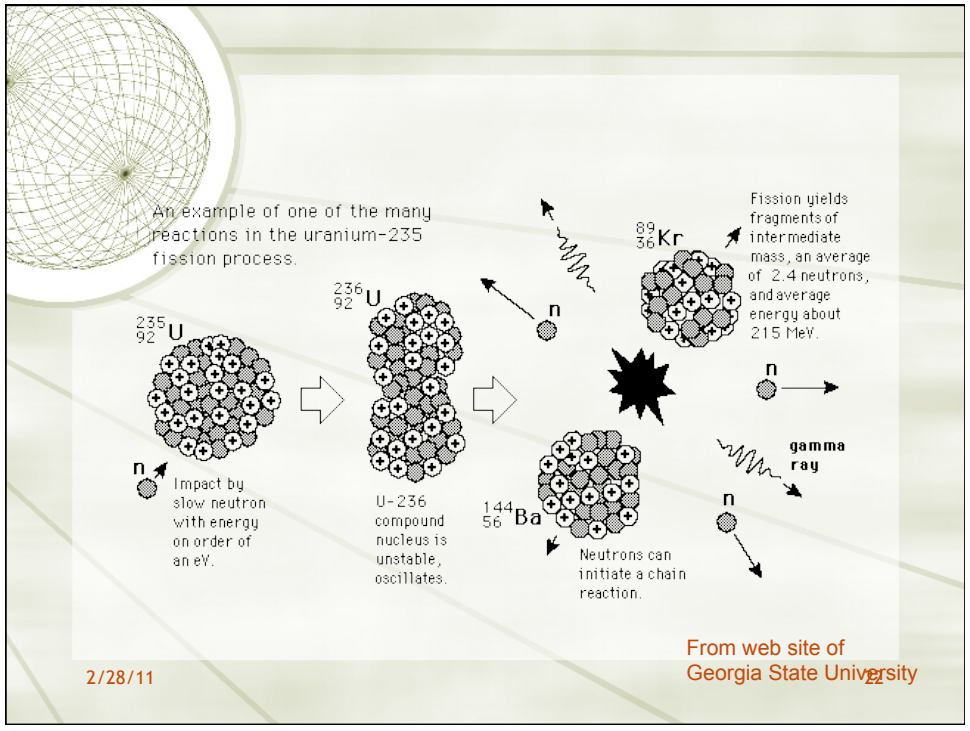
- Nuclear fission
- Nuclear fusion



Fission

- Nuclear fission (e.g., of Uranium)
  - Nuclear Fission – the splitting up of atomic nuclei
  - E.g., Uranium-235 nuclei split into fragments when smashed by a moving neutron. One possible nuclear reaction is

$$^{235}U + n \rightarrow 3n + {}^{89}Kr + {}^{144}Ba$$



From web site of Georgia State University

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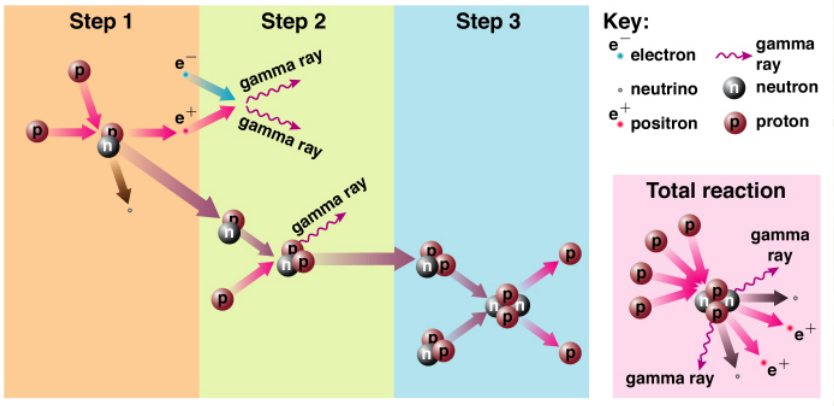
  - Mass of products of reaction (neutrons, Krypton, Barium) is slightly **less** than mass of initial Uranium nucleus + neutron
  - That mass "lost" has been converted into energy (gamma-rays and kinetic energy of fragments):
  - $E = mc^2$

Fusion

- Nuclear fusion (e.g. hydrogen)
  - Fusion – the sticking together of atomic nuclei
  - Much more important for Astronomy (and life on Earth!) than fission
    - Power source for stars, including the Sun
    - Path to making heavy elements (C, N, O, Si, Fe…)
  - Important example – hydrogen fusion.
    - Ram together 4 hydrogen nuclei to form helium nucleus
    - Spits out couple of "positrons" and "neutrinos" in process

$$4\ {}^{1}H \rightarrow {}^{4}He + 2e^{+} + 2\nu$$

The proton-proton fusion cycle



Fusion

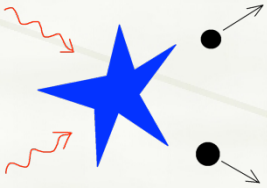
- Mass of final helium nucleus plus positrons and neutrinos is less (by about 0.7%) than original 4 hydrogen nuclei
  - $E = mc^2$
  - Mass has been converted into energy (gamma-rays and kinetic energy of final particles)
- This nuclear reaction (and similar ones) is the energy source for…
  - Hydrogen Bombs (about 1 kg of mass converted into energy gives equivalent of 20 Megatons of TNT)
  - The Sun (about 4×10^9 kg of matter per second is converted into energy, ultimately yielding sunlight)

Antimatter

- Anti-matter
  - For every kind of particle, there is an antiparticle…
    - Electron ↔ anti-electron (also called positron)
    - Proton ↔ anti-proton
    - Neutron anti-neutron
  - Anti-particles have opposite properties from the corresponding particles (e.g., opposite charge)… but exactly the same mass.
  - When a particle and its antiparticle meet, they can completely annihilate each other… **all of their mass is turned into energy (gamma-rays)!**

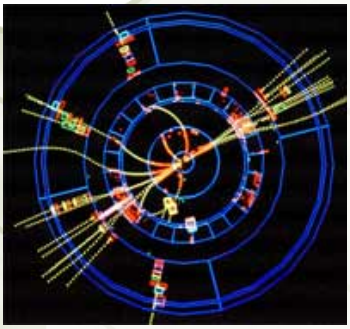
III: EXAMPLES OF CONVERTING ENERGY TO MASS

- Particle/anti-particle production
  - Opposite process to that just discussed!
  - Energy (e.g., gamma-rays) can produce particle/ anti-particle pairs



  - **Very fundamental process in Nature**… we'll see that this process, operating in early universe, is responsible for all of the mass that we see today!

- Particle production in a particle accelerator
  - Can reproduce conditions similar to early universe in modern particle accelerators…



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