

DO NOT OPEN THIS EXAM UNTIL TOLD TO DO SO

ASTR615 — MIDTERM EXAM — Tuesday, October 18, 2011

WHILE YOU ARE WAITING

Information About the Exam

This exam consists of 5 questions, most of which have multiple parts. The point value of each question is listed (they add up to 70 for this exam; since the exam lasts 75 minutes, you should spend about the point value in minutes on each question! ... this will give you 5 extra minutes of relax). Write your answers in the exam booklets provided. PLEASE be sure to READ THE ENTIRE EXAM BEFORE STARTING, as some questions are easier than others. The exam booklets will be collected at 3:15 pm.

WHEN YOU ARE FINISHED, PLEASE WRITE “I do so pledge” ON YOUR EXAM BOOKLET AND SIGN IT. This is in lieu of the following statement: “I pledge on my honor that I have not given or received any unauthorized assistance on this examination.”

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1. Architecture and Parallelization

- (a) [5 points] Explain what is the cache memory. How fast has to be the latency of the cache in order to not be a bottleneck for the execution speed of a processor operating at 8.4 GHz frequency? (this is the record speed of an AMD processor as of October 2011!). Comment on the result in light of the typical speed of RAM.
- (b) [3 points] What is the difference between 32 and 64-bit architectures? Why is one preferable to the other?
- (c) [5 points] If a supercomputer cluster has nominal peak computational power of 1 PetaFlop, and is made of Quad-core processors with clock speed of 4 GHz, give a rough estimate of the number of processors in the cluster.

2. Data Representation

- (a) [5 points] Consider an 8-bit signed integer. Write down the binary expressions for 9 and -9 using two's complement. Using the binary representation show that indeed $9 - 9 = 0$.
- (b) [5 points] Double precision (64-bit) floating-point convention reserves 1 bit for the sign, 11 bits for the exponent E , and 52 bits for the mantissa. Write down an expression for the smallest representable number in this convention. Assume the base is 2, the bias is 1023, and that two values $E = 0$ and $E = 2047$ are reserved. (show all the steps)
- (c) [5 points] What is the round-off error? What is the typical round-off error of a double precision (64-bit) floating point? (Show your work)

3. Linear Algebra

- (a) [5 points] What is a pivot in the Gauss-Jordan elimination (GJE) method and why is it necessary to use pivoting?
- (b) [5 points] Explain how you can use the singular value decomposition (SVD) method as a diagnostic of a linear system and its application to image compression.

4. Root Finding

- (a) [5 points] Describe the Newton-Raphson root-finding strategy in 1-D. Under what circumstances can it go wrong?
- (b) [7 points] Demonstrate that the Newton-Raphson method converges superlinearly.

5. Statistics and Least-squares Fitting

- (a) [5 points] Show that using the expression for calculating the variance

$$\text{Var} = \frac{1}{N-1} \left\{ \sum_{i=1}^N (x_i - \bar{x})^2 - \frac{1}{N} \left[\sum_{i=1}^N (x_i - \bar{x}) \right]^2 \right\}.$$

is more precise than using the standard formula. Why is that?

- (b) [5 points] Explain the method to fit a model to data when the free parameters appear linearly in the model equations.
- (c) [10 points] What is the Hessian matrix? Show how and why it is used to determine the minimum of a multi-dimensional function. (Show all your steps)

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