DO NOT OPEN THIS EXAM UNTIL TOLD TO DO SO

ASTR615 — MIDTERM EXAM — Thursday, March 13, 2014

WHILE YOU ARE WAITING

Information About the Exam

This exam consists of 5 questions, all of which have multiple parts. The point value of each question is listed (they add up to 75 for this exam; since the exam lasts 75 minutes, you should spend no more than the point value in minutes on each question!). Write your answers in the exam booklets provided. PLEASE be sure to READ THE ENTIRE EXAM BEFORE STARTING, as some questions are easier than others. The exam booklets will be collected at 1:15 pm.

WHEN YOU ARE FINISHED, PLEASE WRITE "I do so pledge" ON YOUR EXAM BOOKLET AND SIGN IT. This is in lieu of the following statement: "I pledge on my honor that I have not given or received any unauthorized assistance on this examination."

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1. Architecture and Memory Layout

   (a) [5 points] What physical processes are the main limitations to keep increasing the CPU clock speed? Does Moore's law continue to be valid in today's personal computers and why ?

   (b) [5 points] Assume you have a CPU that works at 4.5 GHz. Given that the RAM memory latency is $t_{ram} = 4$ ns and the cache memory latency is $t_{cache} = 0.1$ ns, determine the minimum hit rate needed to feed data to the CPU without slowing down the calculations.

   (c) [5 points] Explain the difference between static and automatic variables, and the meaning of heap memory, stack memory and frames.

2. Data Representation

   (a) [5 points] Express -103 in base 2 using the two's complement convention.

   (b) [5 points] What is 1011.1101 in base 10?

   (c) [5 points] Suppose you need to use an architecture that provides a floating-point type "real," but you don't know how it is stored internally. Write a short piece of pseudo-code to estimate the machine accuracy of this data type, to within a factor of 10.

3. Linear Algebra

   (a) [5 points] What is meant by LU decomposition, and why is it useful? Be specific.

   (b) [5 points] Explain what is the row and column "pivoting" and the motivation for its use in solving linear systems.

   (c) [5 points] Consider a square 5x5 matrix A with rank 3. What is the nullity? If b is out of the range of A can you find a solution x? How many solutions exist if b is in the range of A? Explain the method to find the minimum of the residual $r = |Ax - b|$, even if b is out of the range.

4. Root Finding

   (a) [5 points] Show that the bisection method converges linearly. What does it mean that it converges linearly in terms of accuracy of the solution ?

   (b) [5 points] Show that the Newton-Raphson method converges superlinearly (recall we have done this in class!).

   (c) [5 points] Consider the following system of non-linear equations:

$$x^2 + y^2 - 25 = 0 \tag{1}$$
$$x^3 + x + 1 = 0 \tag{2}$$

   Write the Jacobian matrix and show two steps to find a root starting from x=0, y=5.

5. Statistics

   (a) [5 points] Suppose you have rotation frequency data for a collection of pulsars. Describe *briefly* a statistical test to determine whether the frequencies are drawn from a normal (Gaussian) distribution.

   (b) [10 points] Show that the following equation for the variance is more accurate than the usual one in presence of round-off error on the mean.

   $$\mathrm{Var} = \frac{1}{N-1} \left\{ \sum_{i=1}^{N} (x_i - \overline{x})^2 - \frac{1}{N} \left[\sum_{i=1}^{N} (x_i - \overline{x}) \right]^2 \right\}.$$

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