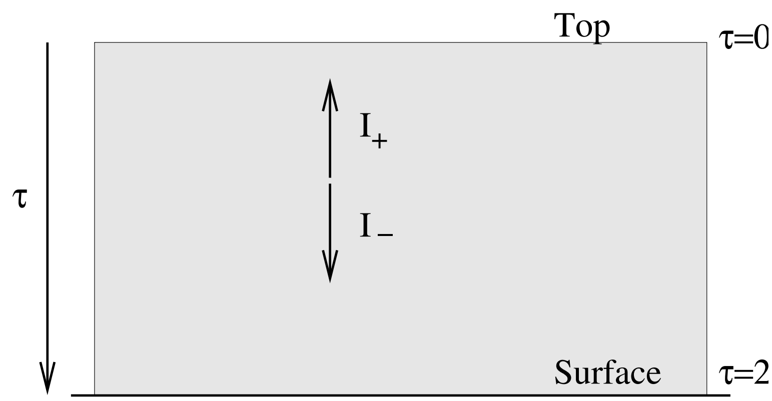
ASTR 601 - Radiative Processes

FINAL (Dec. 19, 2005)

1 The Greenhouse Effect(18pts)

Consider a plane-parallel (one-dimensional) atmosphere subject to the "greenhouse effect". As shown in the sketch below, the planet has an effective temperature $T_{eff} = 250$ K, and the greenhouse gas is mixed homogeneously with height. The infrared lines of this gas are optically thick, with $\tau = 2$. Assume a grey atmosphere and use the two-stream approximation with I_+ and I_- representing the outgoing and ingoing intensities, respectively.



Other assumptions: (1) Assume no incoming IR radiation at the upper boundary; (2) Assume that the mean intensity at the base, $J(\tau = 2)$, is the Planck function at temperature T_b.

(a) [3pts]In the two-stream approximation, derive the moments (J, H, K, where $J = (c/4\pi)\mathcal{E}_{rad}$, $F_{ast} = F/\pi = 4H$ and $K = (c/4\pi)P_{rad}$) of the specific intensity $I(\mu)$. Note that they obey the Eddington approximation ($J = 3K$).

(b) [5pts]Take the $n = 0, 1$ moments of transfer equation (with μ^n) and derive equations for $H(\tau)$, and $K(\tau)$. Recall that the radiative flux $F_{ast} = 4H$. Then, evaluate the specific intensities, $I_+(\tau)$ and $I_-(\tau)$. in terms of F_{ast} and τ.

(c) [5pts]Using the fact that the *effective* temperature of the atmosphere is 250 K, derive the surface temperature, $T(\tau = 2)$, of the atmosphere.

(d) [5pts]At what optical depth τ is the atmosphere temperature equal to T_{eff}?

2 Cooling of Radio Lobes(15pts)

The radio lobes observed in radio galaxies contain highly relativistic electrons which cool passively. The cooling mechanisms of the electrons include emission of radiation from inverse Compton scattering with photons from the microwave background and synchrotron radiation in the ambient magnetic field. The expression for energy loss rate of an electron via synchrotron radiation is

$$\frac{dW}{dt} = \frac{4}{3}c\sigma_T\gamma^2\beta^2U_{mag},$$

where the symbols have their usual meaning. Assume that the energy spectrum of the electrons is $n(\gamma) = n_0\gamma^{-p}$ for $0 \leq \gamma \leq \gamma_{max}$ and $n(\gamma) = 0$ otherwise (also assume $n_0 = const$ and $\gamma \leq 2$).

(a) [3pts]Assume that the magnetic field strength is $B = 10\mu G$ and the microwave background radiation is a black body at temperature $T_{cmb} = 2.7$ K. Which radiative process is dominant, synchrotron or Compton scattering?

(b) [3pts]Show that the synchrotron power per unit volume is

$$\frac{dP}{dV} = \frac{4}{3} c\sigma_T U_{mag} \left(\frac{n_0}{3-p}\right) \gamma_{max}^{3-p}.$$

(c) [3pts]Show that the energy density (energy per unit volume) of the electrons is given by

$$U_e = m_e c^2 \left(\frac{n_0}{2-p}\right) \gamma_{max}^{2-p}.$$

(d) [6pts]Find the expression for the evolution of $U_e(t)$ as a function of time. Assume that p remain constant as the electrons cool. Express your answer as a function of γ_{max} at t=0.

3 Short answers(18pts)

(a)[3pts] Sketch the spectrum of EM radiation for a Gaussian EM impulse, a sinusoidal impulse and damped sinusoidal impulse. How is the width of the spectrum related to the impulse time-scale?

(b)[3pts] How the polarization of a monochromatic wave can be described? How many parameters are needed? How the polarization of a non-monochromatic wave is described? How many parameters are needed and how do you measure them?

(c)[3pts] What are the scalar and vector potentials? and the retarded times? What is the wave zone? Describe the electric dipole approximation. When do you need to consider higher order multipole terms?

(d)[3pts] Describe physically what is the plasma frequency and what are its implications for the propagation of radiation. What is Faraday rotation and what are its astronomical applications?

(e)[3pts] Explain the origin of the "natural" line width in terms of the quantum description of the transition probability. What is the Voigt line profile and how do the Doppler width of a line arise?

(f)[3pts] Are the assumptions $v/c \ll 1$ and $\lambda \gg a_0$, where a_0 is the Bohr radius justified for the hydrogen atom. Are they always justified? The non-relativistic quasi-classical Hamiltonian of the atom in the EM radiation field is $H = (1/2m)|\vec{p} + (e/c)\vec{A}(x,t)|^2 - e\Phi(x,t)$. Write the interaction Hamiltonian $H_{int} = H_1 + H_2$ and the static Hamiltonian H_0 in the Coulomb gauge. Estimate H_1/H_0 and H_2/H_1. Write H_1 in terms of the operator $\vec{p}$ showing that $H_1 = (e/m_e c)(\vec{A}\vec{p})$.