ASTR 601 - Radiative Processes

Final (1:30pm-3:30am Friday, Dec 18th 2009)

1 Synchrotron spectrum [25 pts]

Answer the following theory questions. If needed use plots and carry out the calculations.

a) Plot a generic spectrum of synchrotron radiation emitted by a power law distribution, $n_\gamma = n_0\gamma^{-p}$, of cosmic rays (CRs). Use scaling arguments (do not carry our the full calculation) to derive the dependence of the flux in the optically thin part of the spectrum on the magnetic field strength B_0 and n_0.

b) Explain why synchrotron self-absorption is important at long wavelengths and give a qualitative explanation of the spectral shape and flux in the optically thick regime. Derive I_ν as a function of ν and B_0 in the optically thick regime. How can you use synchrotron self-absorption to determine the strength of the magnetic field, B_0 ?

Carry out the calculations in the next two questions to elucidate why observations show that the brightness temperature of synchrotron radiation never exceeds $T_b \sim 10^{12}$ K.

c) Use the result in b) to express ν as a function of I_ν, B_0 and other constants (valid in the optically thick regime). Next, express the specific intensity in terms of the brightness temperature T_b, replace ν with the expression you derived in the first step, and solve for I_ν. Finally, derive the expression for the flux F_ν emitted from an optically thick sphere of radius R at a distance d as a function of T_b.

d) Provide a qualitative explanation on the effect that Compton losses have on the spectrum of synchrotron radiation. Using the expression derived in c) for F_ν, calculate the value of T_b at which inverse Compton losses become important. You should find that $T_b^{max} \sim 10^{12}$ K, independently on the values of n_0 and B_0. The maximum brightness temperature, T_b^{max}, has a weak dependence on ν_{max}. Assume $\nu_{max} \sim 1$ GHz only at the end of the calculation of T_b^{max}.

2 The epoch of temperature decoupling of the universe [10 pts]

After redshift $z \sim 1000$ the universe becomes neutral (recombination epoch). However, the residual ionization fraction of hydrogen is $x_e \equiv n_e/n \sim 2 \times 10^{-4}$. Compton scattering of CMB photons with the residual electrons in the universe maintains the gas temperature near the CMB temperature, until redshift z_{dec}. After z_{dec}, due to the expansion of the universe, the gas temperature decreases more quickly than the CMB temperature. The following questions will allow you to determine the redshift z_{dec} at which the temperature of gas and CMB radiation decouple.

a) The CMB temperature is $T = T_0(1 + z)$ with $T_0 = 3$ K. What is the energy density of the CMB photons as a function of redshift?

b) Derive the power per unit volume lost or gained by a non-relativistic thermal gas with density n and ionization fraction x_e due to Compton scattering with CMB photons. *Hint:*

find $\langle \beta^2 \rangle$ for a distribution of thermal electrons.

c) Derive the cooling/heating time scale t_{compt} due to Compton scattering of the CMB photons with the residual electrons in the Universe. Determine the redshift of temperature decoupling z_{dec} defined as the redshift at which $t_H = t_{compt}$, where $t_H = (15\ Gyr)(1+z)^{-1.5}$ is the Hubble time.

d) Comment on the results and explain qualitatively why Compton scattering is able to lock the gas temperature to the CMB temperature and why the coupling eventually ceases to be effective *Hint: the Hubble time is the typical time scale for the expansion rate of the universe.*

3 Short theory questions [15 pts]

(a) Derive the relationships between Einstein coefficient A_{21}, B_{21} and B_{12}.

(b) Write down the emissivity and the opacity (per unit volume) in terms of the Einstein coefficient.

(c) What are "forbidden" lines? Provide an explanation in relation to the Einstein A_{21} coefficient and clarify what the term "forbidden" refers to.

(d) Using scaling arguments and/or a toy model derive the plasma frequency and the Larmor frequency.

(e) In the derivation of the radiative transition rate of atoms discussed in class, at which point the semi-classic approach fails? At which point during the derivation the perturbative approach fails?

Physical and astronomical constants

1 yr = 3.16×10^7 s
1 pc = 3.086×10^{18} cm
1 $M_\odot = 1.99 \times 10^{33}$ g
1 $L_\odot = 3.85 \times 10^{33}$ erg s^{-1}

G = 6.673×10^{-8}dyn cm^{2}g^{-2}
c = 2.998×10^{10} cm s^{-1}
h = 6.626×10^{-27} erg s
k = 1.38×10^{-16} erg K^{-1}
σ = ac/4 = 5.67×10^{-5} dyn cm^{-2} K^{-4}
$a = 7 \times 10^{-15}$ erg cm^{-3} k^{-4}
$r_e = e^2/m_e c^2 = 2.82 \times 10^{-13}$ cm
$\sigma_T = (8\pi/3) r_e^2 = 6.65 \times 10^{-25}$ cm^2
1 eV = 1.602×10^{-12} erg
e = 4.803×10^{-10} esu
$m_e = 0.9109 \times 10^{-27}$ g
$m_p = 1.673 \times 10^{-24}$ g