ASTR 601 - Radiative Processes

Midterm (Oct. 20, 2005)

1 Short answers (in 1h :15 min)

Write short answers to 8 of the following 10 questions (5 points each question). The maximum score is 40/40 that can be obtained by answering correctly 8 questions or by answering partially/incorrectly more questions. When necessary draw a sketch or write formulas and equations. Hints: be concise and first answer the questions that look easy to you.

1) Define the monochromatic specific intensity including the units in cgs (cm-g-sec). Show that it is a conserved quantity along each ray.

2) How are the flux and momentum of radiation defined (include their units in cgs)? Calculate the flux from a uniformly bright sphere of radius R as a function of the distance, d, from the center of the sphere. *Hint: integration limits from* μ_{min} *to 1.* What is the flux at the surface ($d = R$)?

3) Write down the formal solution to the equation of radiative transfer in terms of the source function. How is the source function defined? and the optical depth?

4) Write the formal solution to the equation of radiative transfer for a constant source function and derive the optically thin ($\tau \ll 1$) and optically thick ($\tau \gg 1$) limits.

5) Describe the phenomenon of limb darkening. *Hint: Assume that the star interior temperature increases with increasing depth and that the emerging intensity comes from a region at depth* $\tau \sim 1$. What would happen if the temperature of the star increases approaching the stellar surface?

6) Define LTE. What is the source function in LTE? At which frequency and wavelength does a black body spectrum have maximum intensity (dimensional estimate is sufficient)? What is the dependence of the pressure and energy density of black body radiation on the temperature T?

7) Define qualitatively or with formulas the effective temperature, color temperature and brightness temperature.

8) Write down the equations of detailed balance for the ionization equilibrium of a gas of hydrogen atoms. Solve for the ionization fraction $x_e = n_e/n$ in two extreme cases: a) the photo-ionization rate Γ is balanced out by the recombination rate $\alpha(T)$ (photo-ionization); b) the collisional ionization rate, $C(T)$, is balanced out by the recombination rate (coronal approximation). *Hint: charge and mass conservation imply* $n_e = n_p$ *and* $n = n_H + n_p$.

9) What is the general expression for the grand partition function?. How is it related to the grand potential Ω?. How can you get the number of particles, pressure and entropy from the grand potential? *Hint:* $d\Omega = -pdV - SdT - Nd\mu$.

10) What are the statistical properties that differentiate a quantum system that obeys either the Bose-Einstein or Fermi-Dirac statistics from a classic system? Do photons obey the Bose-Einstein or Fermi-Dirac statistic? What is their chemical potential μ ?