ASTR 601 - Radiative Processes

Midterm (Oct. 19, 2006)

1 The H^- Ion (10pts)

The H$^-$ ion is a weakly bound species consisting of two electrons bound to a single proton. This ion occurs in trace amounts in warm plasmas, including the Sun's photosphere where it is an important source of optical opacity. It can also be the catalyst for the formation of H$_2$ molecules in absence of dust grains.

Assume a plasma typical of solar-type star's photosphere, with total hydrogen density $n_H = 10^{18}$ cm^{-3} and $T = 5000$ K. To simplify the problem assume that the plasma is pure hydrogen consisting solely of H I , H II and H$^-$. Then, use the Saha-type equations to find the ratios of n(H I)$/n_H$, n(H II)$/n_H$, and $n(H^-)/n_H$ in LTE. Assume that $n(H^-) \ll n_H$ and use the ground-state approximation for the relevant partition functions. The ionization energies are $I(HI) = 13.6$ eV and $I(H^-) = 0.754$ eV.

Hints: Write down two set of reactions: one for the formation/ionization of H^- and the other for the formation/ionization of H I . The statistical weights are $g_{H^-} = 1$, $g_{HI} = 2$ and $g_{HII} = 1$. The thermal wavelength of the electron is $h/(2\pi m_e kT)^{1/2}$.

2 Short answers(10pts)

1) Write down the 0th, 1st and 2nd moment equations of the specific intensity. Which physical properties of the radiation field do they describe? For an isotropic radiation field, what is the relationship between energy density and pressure and what is the net flux ?

2) What is the Roseland opacity? How is it related to the random walk problem?

3) Write down the formal solution of the radiative transfer equation neglecting scattering processes. Define the source function and opacity.

4) Write down the equation for the ionization fraction of a hydrogen gas in LTE as a function its density and temperature. Why the thermal energy requited to ionize an atom is typically less than the ionization potential of that atom ?

5) Which quantum properties are used to derive an expression for the grand partition function of a fermion or boson gas ? How can we derive the mean number density and pressure of the gas given an expression for the grand potential? What is the "classic" limit for the occupation number of an energy state E and when can we use this limit ?

Some (possibly) useful numbers:

Astronomical constants

1 yr = 3.16×10^7 s
1 pc = 3.086×10^{18} cm
1 AU = 1.50×10^{13} cm
1 $M_\odot$ = 1.99×10^{33} g
1 $L_\odot$ = 3.85×10^{33} erg s^{-1}
1 $R_\odot$ = 6.96×10^{10} cm
G = 1.33×10^{11} km^3 s^{-2} M$_\odot^{-1}$

Physical constants

G = 6.673×10^{-8} dyn cm^{2}g^{-2}
c = 2.998×10^{10} cm s^{-1}
h = 6.626×10^{-27} erg s
k = 1.38×10^{-16} erg K^{-1}
σ = ac/4 = 5.67×10^{-5} dyn cm^{-2} K^{-4}
N_0 = 6.02×10^{23} mol^{-1}
1 eV = 1.602×10^{-12} erg
e = 4.803×10^{-10} esu
m_e = 9.109×10^{-28} g
m_p = 1.673×10^{-24} g

Units

1 arcsec (1″) = 4.84814×10^{-6} radian
1 Angstrom (Å) = 10^{-8} cm
1 Micron (μ) = 10^{-4} cm
1 Jansky (Jy) = 10^{-26} W m^{-2} Hz^{-1} = 10^{-23} erg cm^{-2} s^{-1} Hz^{-1}