GRAVITATIONAL WAVES FROM A COSMOLOGICAL DISTRIBUTION OF SOURCES

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Abstract

The advent of gravitational wave detectors such as the Laser Interferometer Space Antenna and the Advanced-Laser Interferometer Gravitational wave Observatory will make it possible to observe many gravitational wave emission phenomena that may or may not have electromagnetic counterparts. In many cases, gravitational waves serve as probes of length and time scales that range many orders of magnitude as compared to electromagnetic waves. Sources of gravitational waves can be classified into non-primordial and primordial. Non-primordial sources include binary compact object inspirals, core-collapse supernovae and Neutron stars. Primordial sources include cosmological defects, phase transitions in the early universe, and inflation. In this paper we briefly outline the signatures of gravitational waves from different types of sources. We also provide a simple theorem to calculate the gravitational wave background due to a cosmological distribution of these sources. We determine that the background is cosmology independent and is a function of the cosmological distribution, the total time integrated energy spectrum of an individual source and the present-day comoving number density of remnants.

1 Introduction

Gravitational waves are produced wherever the quadrupolar moment of a mass distribution has a non-vanishing second derivative in time. Because of this simple requirement, these waves would be produced abundantly in the universe. However, being a quadrupolar field, it is extremely weak. Therefore, regular astronomical sources, such as stars or galaxies would radiate negligibly in gravitational waves. More likely sources include compact object inspirals, core-collapse supernovae, cosmological defects, non-axisymmetric Neutron stars, and inflation itself!

Gravitational wave observations will give us a whole new perspective on the universe. They will serve to probe electromagnetically unobservable events such as black hole inspirals, as well as probe physical phenomena such as inflation over many orders of magnitude of energy and length scales. Note that the information that can be obtained from studying the gravitational spectrum from a particular source is specific to the type of the source. Therefore, it is essential to understand how gravitational waves probe each different type of source. Sources can be classified as primordial or non-primordial. In §2, we outline both primordial [1] and non-primordial gravitational wave sources and their signatures.

We go on to derive an interesting result in §3 that the amplitude and the spectrum of the gravitational wave background due to all classes of cosmologically distributed sources, is independent of the cosmology [2]. If the universe is assumed to be homogeneous and isotropic, then the spectrum can be shown to be independent of the cosmological distribution of sources as well. The gravitational wave spectrum can be shown to depend only on the cosmological distribution of sources, the time integrated energy spectrum of an individual source, and the present day comoving number density of remnants.

2 Gravitational wave sources

Gravitational wave sources can be classified based on their origin into non-primordial or primordial sources. Non-primordial sources include compact object binaries, core-collapse supernovae, non-axisymmetric neutron stars and non-axisymmetric accretion. Primordial sources include inflation, phase transitions and decoupling,extra dimensions in string theory, and cosmic superstrings. We briefly discuss the two types below.

2.1 Non-primordial sources

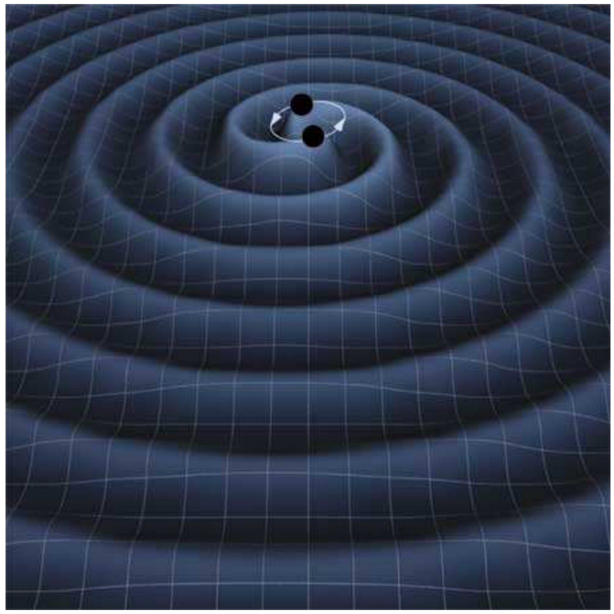

Figure 1: Merging black hole binary

Merging binaries are ideal gravitational wave sources due to a large variation in their quadrupolar moment with time. Ideal examples would include supermassive black hole binaries which emit strongly in gravitational waves when they are tens of gravitational radii ($2GM/c^2$) apart. Fig.1 shows an artistic impression of such a merger. Black hole binaries are ideal sources for LISA which is sensitive to gravitational wave frequencies between 10^{-4}Hz and 10^{-1}Hz (See Fig.2).

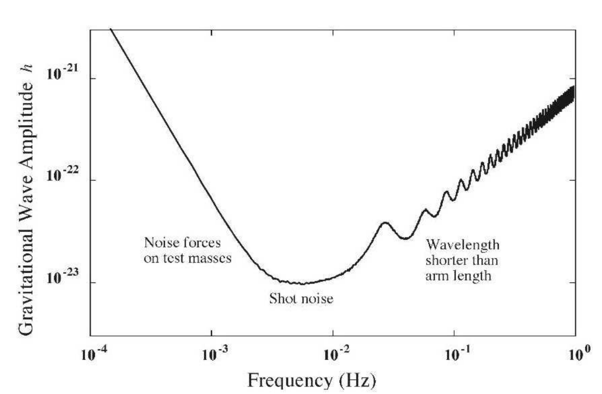

Figure 2: LISA sensitivity curve shows that it is most sensitive to gravitational wave frequencies between 10^{-3}Hz-10^{-1}Hz, from *http://www.srl.caltech.edu*

Gravitational waves from core-collapse supernovae and Neutron stars result from the non-axisymmetric parts of a supernova or a neutron star that contribute to a non-zero net quadrupolar moment. The amplitude of gravitational waves from supernovae and neutron stars can be roughly given as

$$h \approx (G/c^4)(1/r)\frac{\partial^2 I}{\partial t^2} \quad (1)$$

where I is the quadrupolar moment of the supernova or neutron star mass distribution. The amplitude of gravitational waves from both these sources would be orders of magnitude smaller than those due to black hole binaries and typically these would fall in the frequency band of LIGO or advanced-LIGO.

2.2 Primordial sources

Primordial sources include cosmological defects due to inflation, primordial phase transitions, and inflation itself. Some exotic sources such as extra dimensions in string theory may also contribute to the gravitational wave background.

Gravitational waves from inflation would have a characteristic amplitude that depends on the rate of expansion of the horizon during inflation and consequently the energy at which inflation occurred. Fig.3 shows the expected gravitational wave spectrum and the sensitivity limits for the various detectors. Therefore, gravitational waves from inflation serve as a wonderful probe of the energy scale E at which inflation occurred. The frequency of these gravitational waves is given by

$$f = 10^{-4}(E/1\text{Tev})\text{Hz} \quad (2)$$

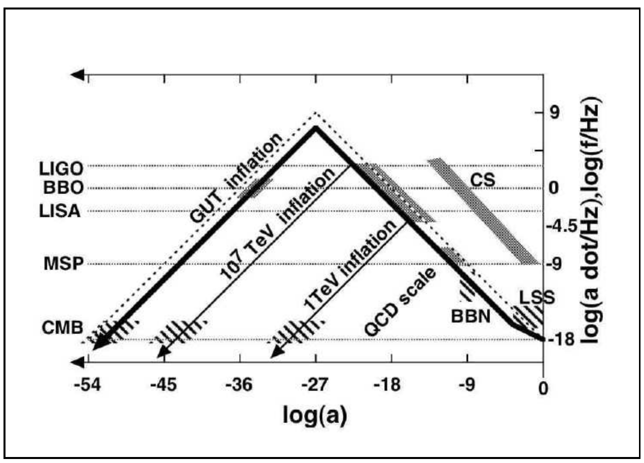

Figure 3: Gravitational wave frequencies for different inflation scenarios

As a consequence of inflation, structures known as superstrings may form by a process known as Kibble quenching. Superstrings continually intersect with themselves, inducing oscillation in small loops that ultimately radiate away all the energy in gravitational waves. The strength of the gravitational waves is a function of the dimensionless mass per length of these strings in Planck units. Current limits on gravitational wave backgrounds due to pulsar timing suggest that these superstrings must be extremely light to have no observable astrophysical effects other than their gravitational radiation.

Other gravitational wave sources include extra dimensions in string theory. The internal energy from these extra dimensions can be released on a macroscopic scale generating gravitational waves. One more mechanism that can generate gravitational waves would be the creation of spatially inhomogeneous modes in the extra dimensions by the

Kibble mechanism. However, these mechanisms are not as clearly understood at present and hence it has not been possible to even theoretically determine the background from these sources.

Having outlined both primordial and non-primordial sources, we proceed to calculate the overall spectrum of gravitational wave from these sources.

3 Calculating the gravitational wave background

The gravitational wave background from all the classes of sources quoted above can be calculated by a simple theorem that we derive below. Let ϵ_{GW} be the energy density of gravitational waves in the universe. Then

$$\epsilon_{GW} = \int_0^\infty \rho_c c^2 \Omega_{GW}(f) df/f \tag{3}$$

where

$$\rho_c = \frac{3{H_\circ}^2}{8\pi G} \tag{4}$$

is the critical density. Therefore,

$$\Rightarrow \epsilon_{GW} = \int_0^\infty \frac{\pi c^2 f^2 {h_c}^2(f)}{4G} \frac{df}{f} \tag{5}$$

We can also write derive ϵ_{GW} as an integral over all sources in frequency and redshift, i.e.,

$$\epsilon_{GW} = \int_0^\infty \int_0^\infty \left(\frac{dE_{GW}}{df_r} \right) f_r \frac{df}{f} \frac{N(z)dz}{1+z} \tag{6}$$

where $f_r = f(1+z)$ is the rest frame frequency of the gravitational waves, E_{GW} is the total energy produced by a source over a log frequency interval df_r/f_r, $N(z)$ is the number of sources at a redshift z. Eq.6 will hold provided the cosmological distribution of sources is homogeneous and isotropic. From Eqs.3,5,6 we see that,

$$\rho_c c^2 \Omega_{GW}(f) = \frac{\pi}{4} \frac{c^2}{G} f^2 h_c^2(f) = \int_0^\infty \frac{N(z)}{1+z} \left(f_r \frac{dE_{GW}}{df_r} \right) dz \tag{7}$$

When we have multiple sources

$$N(z) f_r \frac{dE_{GW}}{df_r} = \sum_i N_i(z) \left(f_r \frac{dE_{GW,i}}{df_r} \right) \tag{8}$$

From Eq.7 we see that the spectrum of gravitational waves depends only on the cosmological distribution of sources and the time integrated energy spectrum of the particular class of gravitational wave source. If the sources do not occur at any preferred redshift, then the spectrum can be shown to be independent of the source distribution $N(z)$ as well. A more mathematically rigorous formalism is given in [2]. In the next section we determine the gravitational wave background due to merging compact object binaries.

3.1 Application to merging binaries

Merging compact object binaries are expected to have the highest signal to noise in gravitational waves amongst all sources. Therefore, let us consider a class of such sources. The total energy radiated by a binary in a logarithmic frequency interval df_r/f_r is given by,

$$\frac{dE_{GW}}{df_r} = \frac{\pi}{3G}\frac{(GM)^{5/3}}{(\pi f_r)^{1/3}} \tag{9}$$

where $M = M_1M_2/(M_1+M_2)^{1/3}$ is the chirp mass. The chirp mass is representative of how fast a given binary sweeps through a given gravitational wave frequency band. Simplistically, if we consider the background to be dominated by inspirals of small objects into non-rotating massive black holes of mass M, then the maximum frequency would occur at the innermost stable circular orbit location and correspond to $f_{max} \approx c^3/6^{3/2}\pi GM$. The minimum frequency of such sources would be determined by the constraint that they must merge within a Hubble time in order to produce gravitational waves. The merger time due to gravitational wave emission is given by

$$T_{insp} = 6*10^{17}\frac{{M_\odot}^3}{M_0M_1(M_0+M_1)}\left(\frac{a}{1AU}\right)^4(1-e^2)^{7/2}yrs \tag{10}$$

By equating T_{insp} to the Hubble time we can get a as a function of the masses involved and hence the minimum frequency of the binary at formation. Substituting Eq.9 in Eqs.7 we get the strain amplitude of gravitational waves from such sources to be,

$${h_c}^2 = \frac{4}{3\pi^{1/3}}\frac{1}{c^2}\frac{(GM)^{5/3}}{f^{4/3}}N_0 < (1+z)^{-1/3} > \tag{11}$$

where N_0 is the total number of merged remnants integrated over all redshifts, and $< (1+z)^{-1/3} >$ represents the expectation value of $(1+z)^{-1/3}$ integrated over all sources and all relevant redshifts that give an observable gravitational wave frequency of f.

Substituting for typical values in Eq.11 we get

$$h_c(f) = 3\times10^{-24}\left(\frac{M}{M_\odot}\right)^{5/6}\left(\frac{f}{10^{-3}Hz}\right)^{-2/3}\left(\frac{N_0}{Mpc^{-3}}\right)^{1/2}\left(\frac{<(1+z)^{-1/3}>}{0.74}\right)^{1/2} \tag{12}$$

From Fig.2, we see that the amplitude of gravitational waves will be well within LISA's detection limit, conservatively assuming $M \approx 1000M_\odot$, $f \approx 10^{-3}Hz$, $N_0 = 1Mpc^{-3}$ and $< (1+z)^{-1/3} > \approx 1$.

Thus we have proved a splendid theorem for one type of source. To get the overall response of a detector such as LISA, we must integrate over all classes of sources as given by Eq.8.

4 Conclusions

In this paper, we briefly outlined astrophysical gravitational wave sources and their gravitational wave signatures. We also showed that the overall spectrum of the gravitational wave background is independent of cosmology. For an isotropic and homogeneous universe the background is only dependent on the time integrated energy spectrum of the source and the present day comoving number density of remnants. This theorem enables us to calculate the spectrum from any and all classes of gravitational wave sources provided they are distributed homogeneously and isotropically in the universe.

References

[1] Craig J. Hogan. Gravitational wave sources from new physics. 2006.

[2] E. S. Phinney. *MNRAS*, 000:1–7, 2001.