ASTR 220 Example Problem

An exercise in problem solving (stating assumptions, checking units, etc.), as well as an example of how astronomical numbers may not be immediately intuitive.

Question: How many times do you need to fold a piece of paper in order to make it thick enough to reach from the Earth to the Moon?

What do we need to know? *State known quantities and assumptions*

1. Distance from the Earth to the moon: $D_{moon} = 3.84 \times 10^5$ km
2. Thickness of a piece of paper: assume $t_p = 0.1$ mm

So for one sheet of paper, the thickness is t_p.

Fold it once and it doubles the thickness $t_t = t_p \times 2$

Fold it again (2 folds), and it doubles again $t_t = t_p \times 2 \times 2 = t_p \times 2^2$

Fold it again (3 folds), and it doubles again $t_t = t_p \times 2 \times 2 \times 2 = t_p \times 2^3$

For n folds, we can generalize this to be $t_t = t_p \times 2^n$
where n is the number of folds

So we need to set the Moon's distance equal to the thickness times the number of folds

$$D_{moon} = t_p \times 2^n$$

$$3.84\times10^5 \text{ km} = 0.1 \text{ mm} \times 2^n$$

But wait – Need to make units the same -- lets work with meters to be consistent

$$(3.84\times10^5 \text{ km} \times 10^3 \text{ m / km}) = (0.1 \text{ mm} \times 10^{-3} \text{ m / mm}) \times 2^n$$

$$3.84\times10^8 \text{ m} = 10^{-4} \text{ m} \times 2^n$$

$$(3.84\times10^8 \text{ m}) / (10^{-4} \text{ m}) = 2^n$$

$$3.84 \times10^{12} = 2^n$$

Solving for n:

n	2^n	
10	1.02×10^3	
20	1.05×10^6	
30	1.07×10^9	
40	1.10×10^{12}	
41	2.20×10^{12}	
42	4.40×10^{12}	$> D_{moon}$

So it takes 42 folds to reach the moon! Try it.