

ASTR 220 Homework #6 Solutions Spring 2005

1. *Ch. 12, Sensible Statements, #18.* This statement does not make sense. Only stars with more than 8 times the Sun's mass can explode in a supernova. The Sun will die by making a planetary nebula and becoming a white dwarf. This sequence of events will also be harmful to Earth, but not on the same scale as a supernova.
2. *Ch. 12, Sensible Statements, #22.* This statement makes sense. The Sun gives off energy because it has the proton-proton chain of fusion reactions taking place in its core. The end result of this chain is to take four hydrogen nuclei and fuse them into one helium nucleus. Over time, the Sun will have less and less hydrogen in its core as fusion changes more and more of it to helium.
3. *Ch. 12, Review Questions, #4.* Degeneracy pressure is the pressure that resists further compression when the material is compressed as small as quantum mechanics will allow. The exclusion principle of quantum mechanics says that electrons cannot be compressed so much that two electrons have the same position and velocity. When this is on the verge of happening, the electrons resist any more compression. The temperature of the electrons doesn't matter.

Thermal pressure is the pressure a group of particles has because they each individually have some kinetic energy. You can think of thermal pressure as being like a swarm of bees. In thermal pressure, the temperature of the gas **does** matter. The hotter the gas is, the more thermal pressure it has.

Degeneracy pressure can keep the core of a star from shrinking from its own gravitational force even if the core is cold, because degeneracy pressure doesn't depend on the temperature of the gas. It is the same strength at all temperatures.

4. (a) *What happens to the core of a star when its hydrogen is exhausted from nuclear fusion?* When hydrogen runs out in the core of a star, fusion stops. Some of the pressure in the core was from the constant production of energy from fusion, so the core can no longer resist the pull of gravity inward. The core starts to shrink.
(b) *Why does hydrogen shell burning begin around the inert helium core?* As the core shrinks (in the previous part of question), the layers immediately above it shrink, too. The layers above the core still have some hydrogen in them. As the core and layers above it shrink, they heat up from the compression. Eventually, the layer immediately above the core is hot enough to start the proton-proton chain of fusion reactions. However, the core itself is not yet hot enough to start fusing helium. The result is that a shell around the core has hydrogen fusion.
5. *Red giants have surface temperatures of about 3000 K.*
 - (a) *At what wavelength will the light emitted from the red giant be the brightest?* We can use Wien's law to figure this out, which is:

$$\lambda_{max} = \frac{0.0029m \cdot K}{T}$$

λ_{max} is the wavelength at the peak of the thermal emission curve, like in Fig. 5.10. T is the temperature in Kelvins. Plug in the temperature:

$$\lambda_{max} = \frac{0.0029m \cdot K}{3000K} = 9.7 \times 10^{-7}m$$

(b) *What type of light is this?* Looking at Fig. 5.2, we can estimate that this is just longer than red light, so in the infrared range.

6. *If you compare a photograph of a **nearby** planetary nebula taken 100 years ago to one taken today, in what two ways would the nebula's appearance be different?*

A planetary nebula is made up of gas from the outer layers of a low-mass star that was blown away at the end of the star's life. The gas in the nebula is glowing because it's being heated up by the UV light from the white dwarf left behind. That means that the gas is expanding outward and cooling off. For a distant planetary nebula, we probably wouldn't see much change in 100 years, but a nearby one will show some changes. The nebula will be larger in diameter (as it expands), and it will be fainter (as it cools off).

7. *Ch. 12, Review Questions, #12.* The first part of the questions asks you to describe some of the nuclear reactions that occur in high-mass stars after they finish fusing helium in their cores. The basic fusion reactions for helium fuse helium into carbon, so the question is asking about reactions that happen after this.

There are several reactions called "helium-capture" reactions because helium is fusing with heavier nuclei. These involve fusing carbon and helium to make oxygen, oxygen and helium to make neon, and neon and helium to make magnesium. At even higher temperatures, reactions fusing heavier nuclei can happen. These include fusing carbon and oxygen to make silicon, oxygen and oxygen to make sulfur, and silicon and silicon to make iron.

The second part of the question asks why high-mass stars have these reactions but low-mass stars cannot. This is because high-mass stars have more mass, so they have more gravitational force on themselves. They can crush their cores small enough that the temperature can become high enough to start fusion reactions after helium fusion. In low mass stars, the core stops compressing because of degeneracy pressure and can never get hot enough to fuse elements heavier than helium.

8. *The supernova remnant from SN 1054 is called the Crab Nebula. The latest observations show that the Crab Nebula has a radius of 5 lyr (which is $4.73 \times 10^6 m$). Observations of the emission spectrum from the part of the nebula heading directly toward us have found that the red hydrogen emission line has a wavelength of $6.524 \times 10^{-7} m$. Measured in a laboratory, the red hydrogen line has a wavelength of $6.563 \times 10^{-7} m$.*

(a) *How much is the wavelength of the emission line Doppler shifted?*

The shift in wavelength is: $\Delta\lambda = \lambda - \lambda_0$. λ is the wavelength emitted by the moving object, so that's $6.524 \times 10^{-7} m$ in this case. λ_0 is the wavelength emitted when the object is at rest, which would be $6.563 \times 10^{-7} m$ in this case. So the shift is:

$$\Delta\lambda = 6.524 \times 10^{-7} m - 6.563 \times 10^{-7} m = -3.9 \times 10^{-9} m$$

The shift is negative, which makes sense since the problem said that this was the part of the nebula that was heading directly toward us.

(b) *What is the velocity of the gas in the nebula?* We can use the rest of the Doppler effect equation to figure this out:

$$v = \frac{\Delta\lambda}{\lambda_0} c$$

Plug in the quantities.

$$v = \frac{-3.9 \times 10^{-9} m}{6.563 \times 10^{-7} m} (3 \times 10^8 m/s)$$

$$v = (-0.0059)(3 \times 10^8 m/s) = -1.8 \times 10^6 m/s$$

- (c) *Assuming that the gas has always been traveling at the same velocity, how long ago was the supernova? Give your answer both in seconds and in years. Are you in rough agreement with the accepted date of the supernova, 1054 A.D.?*

The supernova remnant is approximately spherical in shape. We've observed that its radius is 5 lyr. We can't see the depth of the remnant, but we assume that gas in the remnant is traveling toward us on the near side and away from us on the far side. We assume that the bit of gas that we have observed the Doppler effect from has been moving straight toward us at the same speed since the supernova occurred. So, we assume that the gas moving toward us has also traveled 5 lyr since when the supernova happened.

We can figure out how long this took by using the idea that distance equals velocity \times time, or $d = vt$. The distance is 5 lyr. Let's put this into meters.

$$d = (5\text{lyr})(9.46 \times 10^{15}\text{m/lyr}) = 4.73 \times 10^{16}\text{m}$$

From the previous part of the question, we found the velocity of the gas: $1.8 \times 10^6\text{m/s}$. (I have dropped the negative sign because we assume that all the gas is traveling out from the supernova at the same speed in all directions.) Now we can find the time it took for the gas to travel that far.

$$t = \frac{d}{v} = \frac{4.73 \times 10^{16}\text{m}}{1.8 \times 10^6\text{m/s}}$$

$$t = 2.6 \times 10^{10}\text{s}$$

What is this time in years?

$$t = (2.6 \times 10^{10}\text{s}) \frac{1\text{min}}{60\text{s}} \times \frac{1\text{hr}}{60\text{min}} \times \frac{1\text{d}}{24\text{hr}} \times \frac{1\text{yr}}{365\text{d}} = 820\text{yr}$$

The time is approximately 820 yrs. So when did the supernova happen, according to this calculation? Well, this is the year 2005, so $2005 - 820 = 1185$. Our estimate shows that the supernova happened in 1185 AD. That's not too far off from the actual date.

How can we explain why the actual date was longer ago? Probably the gas was initially moving faster, but has slowed down a little bit. There isn't much material in space to cause friction with the gas, but there is enough to have a small effect.