1. The apple and the Moon [30 pts.]

Legend is that Isaac Newton conceived the theory of gravity while contemplating an apple falling to the ground. In any case, Newton reasoned that the force accelerating a falling apple and the force accelerating the Moon in its orbit around the Earth were one and the same. The radius of the Earth is 6350 km, and the acceleration of gravity on its surface is $g=9.81 \text{ m/s}^2$. The radius of the Moon’s orbit is 384,350 km, and it moves with an average speed of 1,000 m/s.

In any physics problem is important to be consistent in the use of the units. A handy thing to do is to convert all the quantities to the same unit system. We will use here the MKS system: meters for length, kilograms for mass, and seconds for time. Make sure that you convert all quantities to MKS units before using them in a formula.

A. What is the acceleration experienced by the Moon in its orbit? (Hint: recall the formula for the acceleration for a body moving with uniform speed on a circular trajectory that we have shown in class).

B. A force that gets weaker as the square of the distance implies that if a distance is doubled, the accelerations generated by the force become $2^2=4$ times smaller. Show that the force of gravity must decrease as the square of the distance, by comparing the acceleration computed in part A with the acceleration on the surface of the Earth in relation to their respective distances to the center of the Earth. (Note that a spherical body acts as if all its mass were concentrated in its center).

C. What are the mass and the mean density of the Earth? (Hint: recall the expression of the force of gravity. To compute the mean density use that the volume of a sphere is $V=4/3\pi R^3$. The value of the gravitational constant $G$ is $6.67\times10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$. The mean density of the Earth should come up to be a few times that of water, which is 1,000 kg/m$^3$).

2. The laws of Newtonian dynamics [20 pts.]

A. What are the three principles of Newtonian dynamics?

B. What are inertial and accelerated frames of reference?
C. After watching many Road Runner cartoons you decide to try out one of Wile E. Coyote’s most promising schemes. You attach a fan and a sail to yourself while wearing roller skates, hoping to move really fast in the forward direction. Surprisingly, although the sail catches well the wind from the fan, you stand perfectly still no matter how fast you make the fan blow. Why?

3. Conservation of momentum [30 pts.]

You decide to try a new propulsion system based in your knowledge of momentum conservation. At the nearest ice rink, while wearing your skates, you try throwing baseballs. Every time you throw a ball, the reaction will moves you. Assume a baseball’s mass is 0.175 kg and your mass is constant at 77 kg. You are pretty good at throwing, and you consistently reach speeds of 110 km per hour for the balls.

A. How fast do you move (in m/s) after you throw your first baseball? (Assume there is no friction, that’s why you are at the ice rink). In which direction do you move?

B. After this disappointing first result, and because you know about Galilean transformations, you decide that you can keep adding to your velocity by throwing more balls. How many baseballs do you have to throw to reach “walking speed” (5 km per hour). Again, neglect the effects of friction.

4. The Cosmological Principle [20 pts.]

A. What is the Cosmological Principle?

B. Explain what is meant by isotropy and homogeneity. How does isotropy in combination with the Cosmological principle imply homogeneity?

C. What does the Cosmological Principle imply about the physical laws that govern the Universe?