General Relativity

Classical tests
Gravitational lensing
Gravitational waves
Shortest flight paths are geodesics
Geodesics on sphere and torus
Hyperbolic space

- Two-dimensional version of a hyperbola - a “saddle”

- Geodesics diverge
How does matter “warp” space?

- Use two-dimensional space as an analogy: think of how rubber sheet is affected by weights.
- Any weight causes sheet to sag locally.
- Amount that sheet sags depends on how heavy weight is.

From web site of UCSD
Effect of matter on coordinates

- Lines that would be straight become curved (to external observer) when sheet is “weighted”
How are orbits affected?

- Marble would follow straight line if weight were not there.
- Marble’s orbit becomes curved path because weight warps space.
Warping of space by Sun’s gravity

Light rays follow geodesics in warped space
General Relativity II

- The General Theory of Relativity
  - Einstein’s equations
- Consequences of GR
  - Orbit of Mercury
  - Gravitational lensing
  - Gravitational waves
Within a free-falling frame, the Special Theory of Relativity applies.

Free-falling particles/observers move on geodesics through curved space-time.

The distribution of matter and energy determines how space-time is curved.

“Space-time curvature tells matter/energy how to move. Matter/energy tells space-time how to curve.”
Mathematical description of GR

\[ G = \frac{8\pi G}{c^4} T \]

- The Einstein curvature tensor “\( G \)” is mathematical object describing curvature of 4-D space-time.
- The Stress-Energy tensor “\( T \)” is mathematical object describing distribution of mass/energy.
- Both \( G \) and \( T \) can be written in terms of components, similar to a matrix.
- Newton’s constant of gravitation “\( G \)” and the speed of light “\( c \)” appear as fundamental constants in this equation.
- This is actually a horrendous set of 10 coupled non-linear partial differential equations!!
- For weak gravitational fields, this reduces to Newton’s law of gravitation, to an excellent approximation.
GR EFFECTS IN THE SOLAR SYSTEM

- Already discussed bending of star light by the Sun (detected by Eddington).
- Orbit of Mercury:
Precession of the perihelion

Effect called “precession of perihelion”
Precession of Mercury’s perihelion

- This happens to all planets, but Mercury is closest to the Sun (a place with unusually bendy space-time in the Solar system)
- Effect is small - orbit twists by 5600 arc-seconds (1.56 degrees) per century
  - With Newtonian gravity, we can explain 5557 arc-seconds/century as due to:
    - Changes in Earth’s frame (equinox precession): 5025.6 arcsecs
    - Gravitational effect of other planets: 531.4 arcsec
    - Deformation of the Sun: 0.025 arcsec
  - But this still leaves 43 arc-seconds per century unexplained... (Vulcan?)
- Using GR, Einstein predicted (with no fiddling!) that Mercury’s orbit should precess exactly 43 extra arcseconds per century!
THE BENDING OF LIGHT (GRAVITATIONAL LENSING)

Galaxy Cluster Abell 2218
HST • WFPC2
NASA, A. Fruchter and the ERO Team (STScI) • STScI-PRC00-08
Actual and apparent light paths with gravitational lens

- Massive galaxy or cluster is foreground ‘lens’
- Light rays from distant quasar or galaxy bend around foreground object
- Two or more images appear

From Chandra telescope web page

From University of Georgia web page
“The Einstein Cross”
Image brightnesses vary as source & lens move
Smithsonian Castle
Smithsonian with black hole

E. Falco CASTLES survey, black hole of Saturn’s mass in the middle of the mall
Simulated galaxy lensing

Lensing Galaxy
Gravitational micro-lensing

- Individual stars can also make a gravitational lens... **microlensing**.
- Suppose we...
  - Look at a distant star in our galaxy
  - Another (dark) star passes in front...

Causes temporary increases in observed brightness of stellar images

Binary microlens, from web site of Ned Wright (UCLA)
Images for binary lenses with high mass ratio (i.e., a star and a planet, analogous to the Sun and Jupiter; Bolatto & Falco 1994)
So many stars...makes this really hard to do!
GRAVITATIONAL WAVES

- Accelerating masses produce continual changes in space geometry
- Periodically-moving bodies (e.g. orbiting stars) create ripples in space-time curvature
- Ripples travel at speed of light through space (how do we know this if we’ve never found one?)
- These are called gravitational waves.

From LISA2 movie
Gravitational waves

- Features of gravitational waves...

- Usually extremely weak!

- Only become strong when massive objects are orbiting close to each other.

- Gravitational waves carry energy away from orbiting objects... this causes objects to spiral toward each other

- The grand challenge - to compute the spiralling together of two black holes.

- How do we know that these waves exist?
The binary pulsar (PSR1913+16)

- Discovered remarkable double star system
- Nobel prize in 1993

From Nobel Prize website
Two neutron stars orbiting each other
- One neutron star is a pulsar -
  - Neutron star is spinning on its axis (period of 59ms)
  - Emits pulse of radio towards Earth with each revolution
  - Acts as a very accurate clock!
- Interesting place to study GR
  - Orbit precesses by 4 degree per year!
  - Orbit is shrinking due to gravitational waves
Precise test of certain aspects of GR

- When pulsar is approaching Earth, pulse frequency increases (Doppler shift); when pulsar is receding, pulse frequency decreases -- orbit of pulsar can therefore be “mapped”
- Orbit is observed to be precessing and shrinking at exactly the rate predicted by Einstein’s theory

![Graph showing the shift in periastron time from 1974 to 1982.](image)

![Diagram showing orbits and the center of mass.](image)
Direct detection of gravitational waves

- How do you search for gravitational waves?
- Look for tidal forces as gravitational wave passes: local compression or expansion of space
- Pioneered by Joseph Weber (UMD Professor)
  - Estimated wave frequency (10000Hz)
  - Looked for “ringing” in a metal bar caused by passage of gravitational wave
  - Insufficient technology in the 1970’s for detection
Binary black hole merger

Movie: NASA GSFC
Modern experiments: LIGO

- **Laser Interferometer Gravitational Wave Observatory**
- Two L-shaped 4km components: Hanford, Washington, & Livingston, Louisiana
- Can detect gravitational waves with frequencies of about 10-1000Hz.
- Operating since mid 2015 with advanced detectors
- VERY sensitive... need to account for
  - Earthquakes and Geological movement
  - Traffic and people!
- What will it detect?
  - Stellar mass black holes spiraling together
  - Neutron stars spiraling together
Binary black hole merger
Laser Interferometer Space Antenna “pathfinder”
Space-based version of LIGO (planned launch date November 2015, used to be 2011)
Sensitive to lower-frequency waves (0.0001 - 0.1Hz)
Will be able to see
- Normal binary stars in the Galaxy
- Stars spiraling into large black holes in the nearby Universe.
- Massive black holes spiraling together anywhere in the universe!
Black holes merging and gravity waves

Caltech-Cornell numerical GR group, http://www.black-holes.org
Black holes merging and gravity waves

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