Lecture 9: Special Relativity III & General Relativity I

- Einstein’s formula for energy
- Equivalence of mass and energy
- Mass turning into energy
- Energy turning into mass
- Redshifting of light
- Need for General Relativity

"I love hearing that lonesome wail of the train whistle as the magnitude of the frequency of the wave changes due to the Doppler effect."
Last time...

- We discussed further aspects of special relativity, including:
  - Simultaneity and causality
  - Space-time diagrams
  - Invariant intervals and proper time
  - Reciprocity and the twins paradox
I : MASS AND ENERGY

- Einstein reworked Newton’s laws of mechanics using his new relativistic formulae.
- He found a formula for the energy of a moving object with mass $m$ and speed $V$ -

$$E = \gamma mc^2 = \frac{mc^2}{\sqrt{1 - V^2/c^2}}$$

- Thus, energy increases as the speed increases, and energy would become infinite if $V$ approaches $c$
Energy vs. V/c
Was Newton just plain wrong?

- What about objects moving at “small velocity”?
- It can be shown that:

\[ E \approx mc^2 + \frac{1}{2}mV^2 \]

- The \( \frac{1}{2}mv^2 \) is the Newtonian expression for the kinetic energy of a moving object.
- What counts as “small velocity”?
  - For car going at 30mph, approximate formula is wrong by 1 part in \(10^{30}\)
  - For rocket going at 30,000mph, this approximate formula is wrong by 1 part in \(10^{18}\)
  - So, approximation is fine for all velocities experienced in everyday life.
Rest mass energy

- If we put $V=0$ in Einstein’s energy formula, we get...

$$E = mc^2$$

- What does this mean?
  - Maybe it is some fundamental “irreducible” (i.e., inaccessible) energy that every object possesses?
  - Or, perhaps this energy can be accessed? In other words, maybe mass can be turned into “usable” energy? It turns out that this is correct!
  - Also, this can go the other way - energy can be turned into mass.
II : EXAMPLES OF CONVERTING MASS TO ENERGY

- Nuclear fission
- Nuclear fusion
Fission

Nuclear fission (e.g., of Uranium)

- Nuclear Fission - the splitting up of atomic nuclei
- E.g., Uranium-235 nuclei split into fragments when smashed by a moving neutron. One possible nuclear reaction is

$$^{235}\text{U} + n \rightarrow 3n + ^{89}\text{Kr} + ^{144}\text{Ba}$$
An example of one of the many reactions in the uranium-235 fission process.

Impact by slow neutron with energy on order of an eV.

Fission yields fragments of intermediate mass, an average of 2.4 neutrons, and average energy about 215 MeV.

U-236 compound nucleus is unstable, oscillates.

Neutrons can initiate a chain reaction.
Nuclear fission (e.g., of Uranium)

- Nuclear Fission - the splitting up of atomic nuclei
- E.g., Uranium-235 nuclei split into fragments when capturing a moving neutron. One possible nuclear reaction is

\[ ^{235}U + n \rightarrow 3n + ^{89}Kr + ^{144}Ba \]

- Mass of products of reaction (neutrons, Krypton, Barium) is slightly less than mass of initial Uranium nucleus + neutron
- That mass "lost" has been converted into energy (gamma-rays and kinetic energy of fragments):
- \[ E = mc^2 \]
Fusion

- Nuclear fusion (e.g. hydrogen)
  - Fusion - the sticking together of atomic nuclei
  - Much more important for Astronomy (and life on Earth!) than fission
    - Power source for stars, including the Sun
    - Path to making heavy elements (C, N, O, Si, Fe...)
- Important example - hydrogen fusion.
  - Ram together 4 hydrogen nuclei to form helium nucleus
  - Spits out couple of “positrons” and “neutrinos” in process

\[ 4 \, ^1H \rightarrow ^4He + 2e^+ + 2\nu \]
The proton-proton fusion cycle
Fusion

Mass of final helium nucleus plus positrons and neutrinos is less (by about 1%) than original 4 hydrogen nuclei

\[ E = mc^2 \]

Mass has been converted into energy (gamma-rays and kinetic energy of final particles)

This nuclear reaction (and similar ones) is the energy source for...

- Hydrogen Bombs (about 1 kg of mass converted into energy gives equivalent of 20 Megatons of TNT)
- The Sun (about \(4 \times 10^9\) kg of matter per second is converted into energy, ultimately yielding sunlight)
Antimatter

Anti-matter

For every kind of particle, there is an antiparticle...

- Electron ↔ anti-electron (also called positron)
- Proton ↔ anti-proton
- Neutron ↔ anti-neutron

Anti-particles have opposite properties from the corresponding particles (e.g., opposite charge)... but exactly the same mass.

When a particle and its antiparticle meet, they can completely annihilate each other... all of their mass is turned into energy (gamma-rays)!
III: EXAMPLES OF CONVERTING ENERGY TO MASS

- Particle/anti-particle production
- Opposite process to that just discussed!
- Energy (e.g., gamma-rays) can produce particle/anti-particle pairs

Very fundamental process in Nature... we’ll see that this process, operating in early universe, is responsible for all of the mass that we see today!
Particle production in a particle accelerator

- Can reproduce conditions similar to early universe in modern particle accelerators...
The **proton-proton fusion cycle**

**Step 1**

- $p + p \rightarrow d + e^-$
- $e^- \rightarrow \gamma$

**Step 2**

- $d + d \rightarrow \alpha + 2\gamma$

**Step 3**

- $\alpha + p \rightarrow \gamma + 2p$

**Key:**
- $e^-$: electron
- $\nu$: neutrino
- $e^+$: positron
- $p$: proton
- $n$: neutron
- $\gamma$: gamma ray

**Total reaction**

- $4p \rightarrow 2\alpha + 2\gamma$
Doppler effect

http://carma.astro.umd.edu/AWE/Doppler/redshift.html
Fire Engine Siren
Redshifting of light

- Photons (light particles) are massless, but their energy also changes when observer’s frame changes
  - Recall (see Chapter 4 for review!) light has a wave/particle dual nature
  - Energy of a photon is proportional to the frequency ν of the corresponding wave: \( E = h \nu \)
    - \( h = 6.63 \times 10^{-34} \text{ Joule-s} \)
  - When changing frames with a velocity \( V \), the frequency of the light waves and energy of the photons changes by a factor

\[
\sqrt{\frac{1 + \frac{V}{c}}{1 - \frac{V}{c}}} = \left(1 + \frac{V}{c}\right) \times \gamma
\]

- Moving towards a light source, the frequency and energy increase by this factor = blueshift (bluer, not necessarily blue)
- Moving away from a light source, the frequency and energy decrease by this factor = redshift (redder, not necessarily red)
Motivation for General Relativity: Einstein’s tower

- So far, we have ignored the effects of gravity. Is this really okay??
- Consider another thought experiment, to test whether light can be affected by gravity.
- Consider a tower on Earth
  - Shine a light ray from bottom to top
  - When light gets to top, turn its energy into mass.
  - Then drop mass to bottom of tower, in Earth’s gravity field
  - Then turn it back into energy
Perpetual motion?

- If we could do this, then we could get energy from nothing!
  - Original energy in light beam = \( E_{\text{start}} \)
  - Thus, mass created at top is \( m = \frac{E_{\text{start}}}{c^2} \)
  - Then drop mass... at bottom of tower it has picked up speed (and energy) due to the effects of gravitational field.
  - When we turn it back into energy, we have \( E_{\text{end}} = E_{\text{start}} + E_{\text{grav}} \)
  - But, we started off with only \( E_{\text{start}} \) - we have made energy! We’re rich!
Maxwell and gravity

- Clearly, our assumption must be wrong...
  - light must be affected by gravity.
  - But gravity does not appear in Maxwell’s equations, which govern light
  - Thus, Maxwell’s equations are not exactly valid in the reference frame of Earth’s surface, where there is gravity.
  - The Earth’s surface must not be an inertial frame of reference!
Resolving the tower problem

- Now consider light ray aimed from top to bottom of tower
- Free-falling (FF) observer sees light ray travel unaffected by gravity, since free fall yields a state of apparent weightlessness (inertial frame)
- From “Earth’s” frame...
  - Free-falling (FF) observer is traveling faster and faster
  - Falling observer would see an increasing redshift of light source according to special relativity
  - If FF observer is supposed to see an constant frequency light beam, then light must get relatively blueshifted as it falls in gravitational field, to compensate
  - Light beam aimed upward must conversely be increasingly redshifted with height
  - Gravitational redshifting removes just the right amount of energy to solve the tower paradox!
The frequency of the photon looks like this as it moves down. The free-fall observer sees this.
The frequency of the photon looks like this as it moves down.

The free-fall observer sees this.
Resolving the tower problem

- Now consider light ray aimed from top to bottom of tower
- Free-falling (FF) observer sees light ray travel *unaffected* by gravity, since free fall yields a state of apparent weightlessness (inertial frame)
- From “Earth’s” frame...
  - Free-falling (FF) observer is traveling faster and faster
  - Falling observer would see an increasing *redshift* of light source according to special relativity
  - If FF observer is *supposed to* see an constant frequency light beam, then light must get relatively *blueshifted* as it falls in gravitational field, to compensate
  - Light beam aimed upward must conversely be increasingly *redshifted* with height
  - Gravitational redshifting removes just the right amount of energy to solve the tower paradox!
Next time...

- Light is affected by gravity
- Therefore special relativity is not enough!
- Einstein developed the General Theory of Relativity to deal with this

Read Chapter 8 of the book