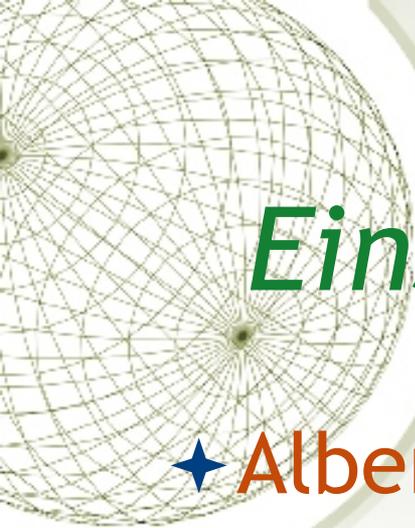


# Lecture 7: Special Relativity I

- ★ Einstein's postulates
- ★ Time dilation
- ★ Length contraction
- ★ New velocity addition law



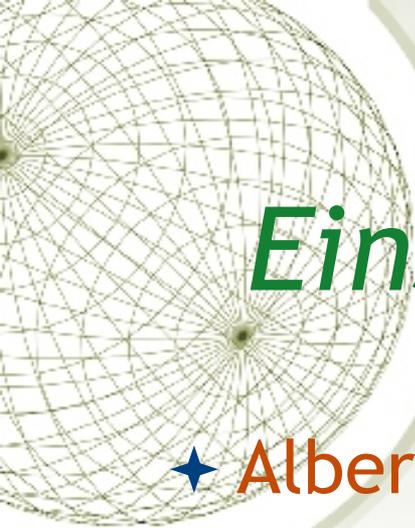
*Please start reading Chapter 7 of the text*



# *Einstein enters the picture...*

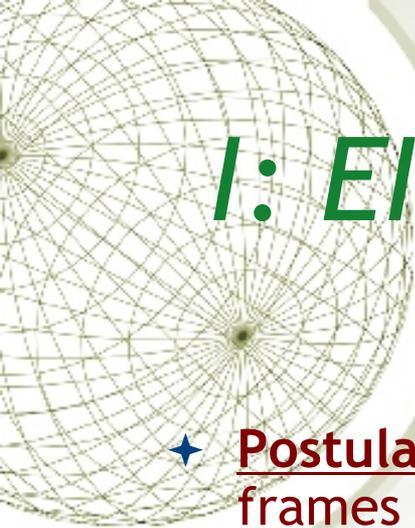
## ★ Albert Einstein (1879-1955)

- ★ Three papers in 1905: Brownian Motion, Photoelectric Effect (showing that light is quantized in energy), Special Theory of Relativity.
- ★ Didn't like idea of a luminiferous ether
- ★ Knew that Maxwell's equations were invariant under "Lorentz transformation" of space and time



# *Einstein enters the picture...*

- ★ Albert Einstein (1879-1955)
  - ★ How to resolve conflict between mechanics and electromagnetism?
    - ★ Throw away the idea of Galilean Relativity for mechanics!
    - ★ Galilean transformation between frames does not hold: velocities do not simply add/subtract (although the effects are small when the speeds are much less than the speed of light).
  - ★ Came up with the two “Postulates of Relativity”

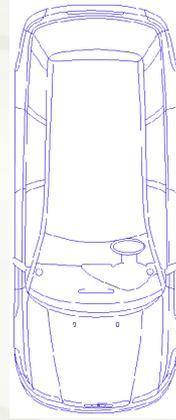


# I: EINSTEIN'S POSTULATES OF RELATIVITY

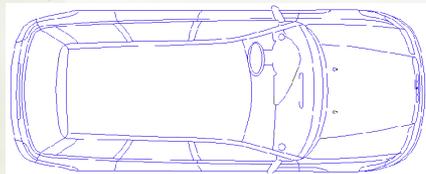
- ✦ **Postulate 1** - The laws of nature are the same in all *inertial* frames of reference
- ✦ **Postulate 2** - The speed of light in a vacuum is the same in all *inertial* frames of reference.
- ✦ Let's start to think about the consequences of these postulates.
- ✦ We will perform “thought experiments” (Gedankenexperimenten) to think of what observers moving at different speeds will think
- ✦ For now, we will ignore effect of gravity - we suppose we are performing these experiments in the middle of deep space (or in free fall)

# What if the speed of light weren't the same in all inertial frames?

Collision or not? If the speed of light were not the same in all inertial frames, you would see one car reach the collision point earlier than the other. But there either is or isn't a collision!



$$\Delta\text{time} = \frac{\text{distance}}{\text{speed}}$$



100 km/hr



$c + 100 \text{ km/hr?}$

# Quiz



★ What was the Michelson-Morley experiment about?

- A. Measuring the speed of light
- B. Measuring the weak equivalence principle
- C. Finding the ether
- D. Measuring the difference in the speed of light between two orthogonal directions



# Quiz

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# Quiz

★ Why were Galileo/Newton in contradiction with Maxwell?

- A. Maxwell says light is a wave, Newton says it is a particle
- B.  $c$  is in the equations of Maxwell
- C. The Coriolis force is not in Maxwell
- D. Two velocities can add up to more than  $c$

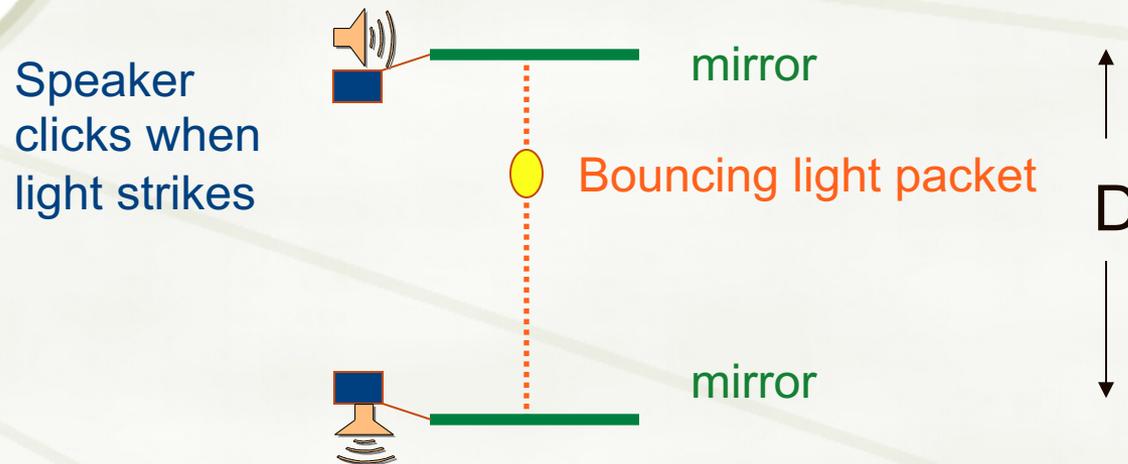
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## II: TIME DILATION

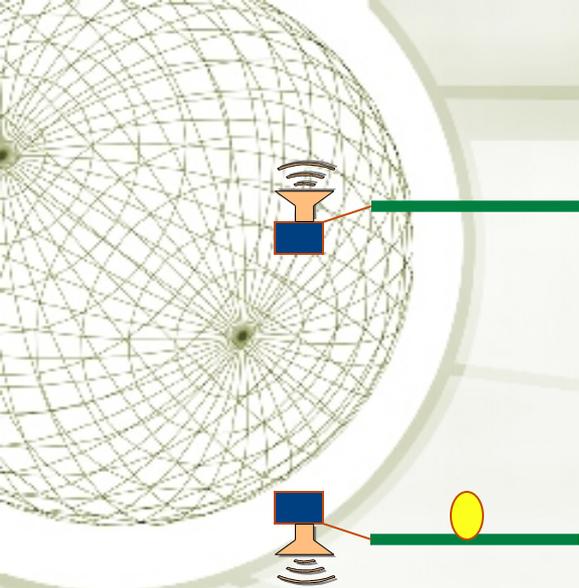
- Imagine building a clock using mirrors and a light beam.



- One “tick” of the clock is the time it takes for light to travel from one mirror to the other mirror.

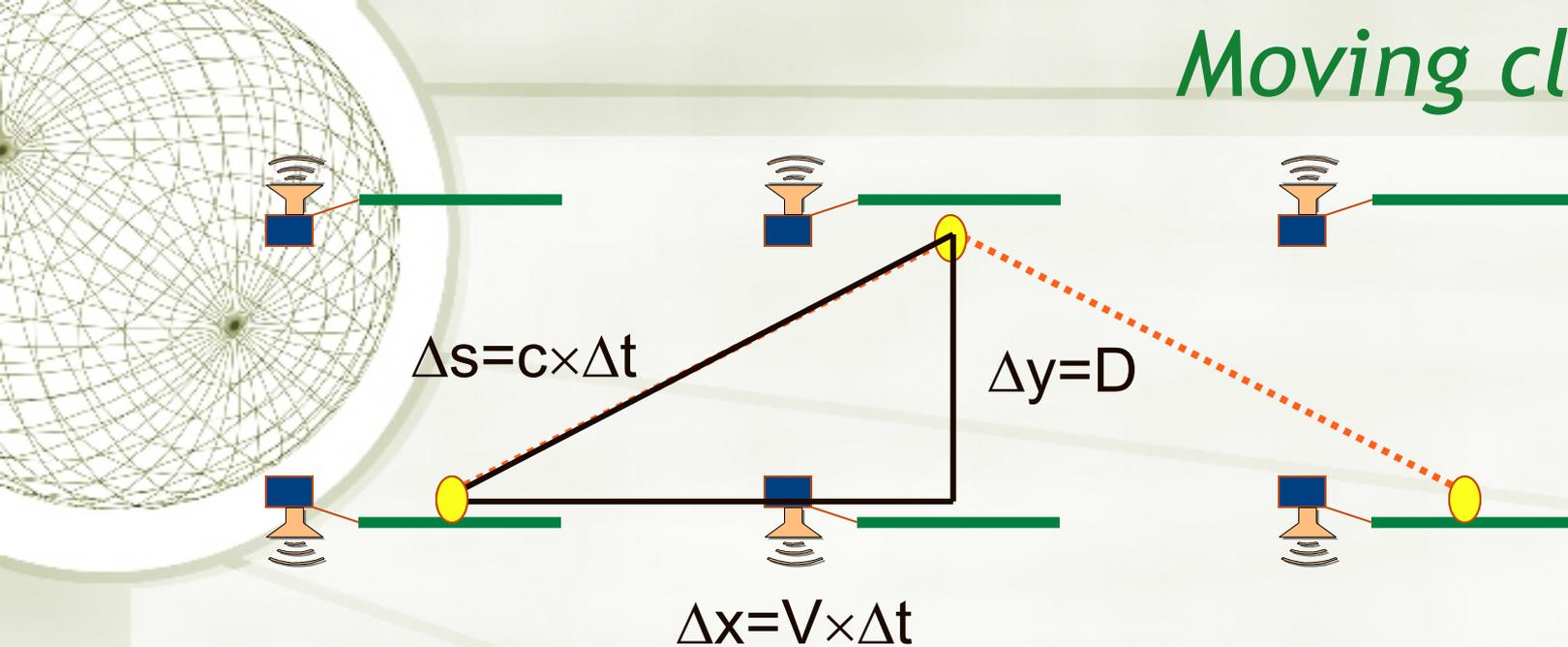
$$\Delta T = \frac{D}{c}$$

# Moving clock

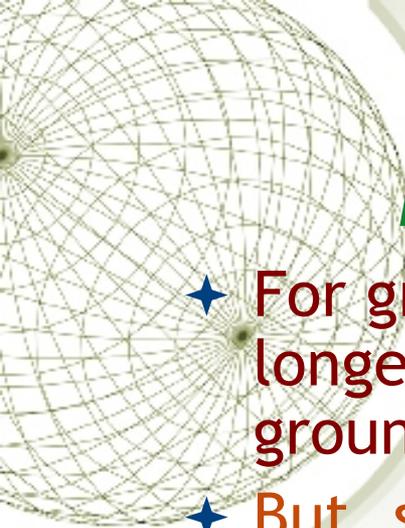


- ★ Now suppose we put the same “clock” on a spaceship that is cruising (at constant velocity,  $V$ ) past us.
- ★ How long will it take the clock to “tick” when we observe it in the moving spacecraft? Use Einstein’s postulates...
- ★ Total distance travelled by light beam is  $\Delta s = c \times \Delta t$
- ★ Therefore time  $\Delta t = \Delta s / c$
- ★ By Pythagorean theorem,  $\Delta s = c \Delta t = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(V \Delta t)^2 + D^2}$
- ★ Can solve to obtain  $\Delta t = (D/c) \div (1 - V^2/c^2)^{1/2} > D/c$
- ★ Clock appears to run more slowly!!

# Moving clock



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## *Now change the point of view...*

- ✦ For ground-based observer, clock on spaceship takes longer to “tick” than it would if it were on the ground
- ✦ But, suppose there’s an astronaut in the spacecraft
  - ✦ the inside of the spacecraft is also an inertial frame of reference - Einstein’s postulates apply...
  - ✦ So, the astronaut will measure a “tick” that lasts

$$\Delta T = \frac{D}{c}$$

- ✦ This is just the same time as the “ground” observers measured for the clock their own rest frame
- ✦ So, different observers see the clock going at different speeds!

## **So time is not absolute!!**

It depends on your point of view...

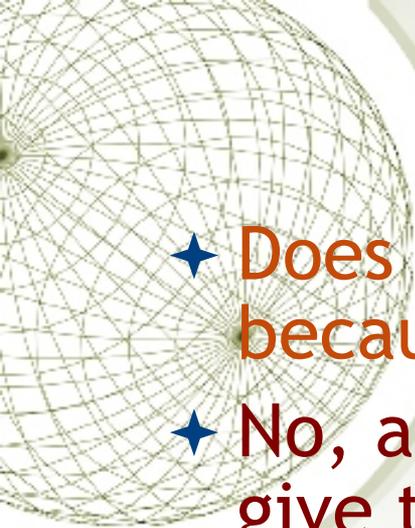
# Time dilation

- ★ This effect called **Time Dilation**.
- ★ Clock always ticks most rapidly when measured by observer in its own rest frame
- ★ Clock slows (ticks take longer) from perspective of other observers
- ★ When clock is moving at **V** with respect to an observer, ticks are longer by a factor of

$$\Delta t \div \Delta T = \frac{D/c}{\sqrt{1 - V^2/c^2}} \div \frac{D}{c} = \frac{1}{\sqrt{1 - V^2/c^2}}$$

- ★ This slowing factor is called the **Lorentz factor,  $\gamma$**

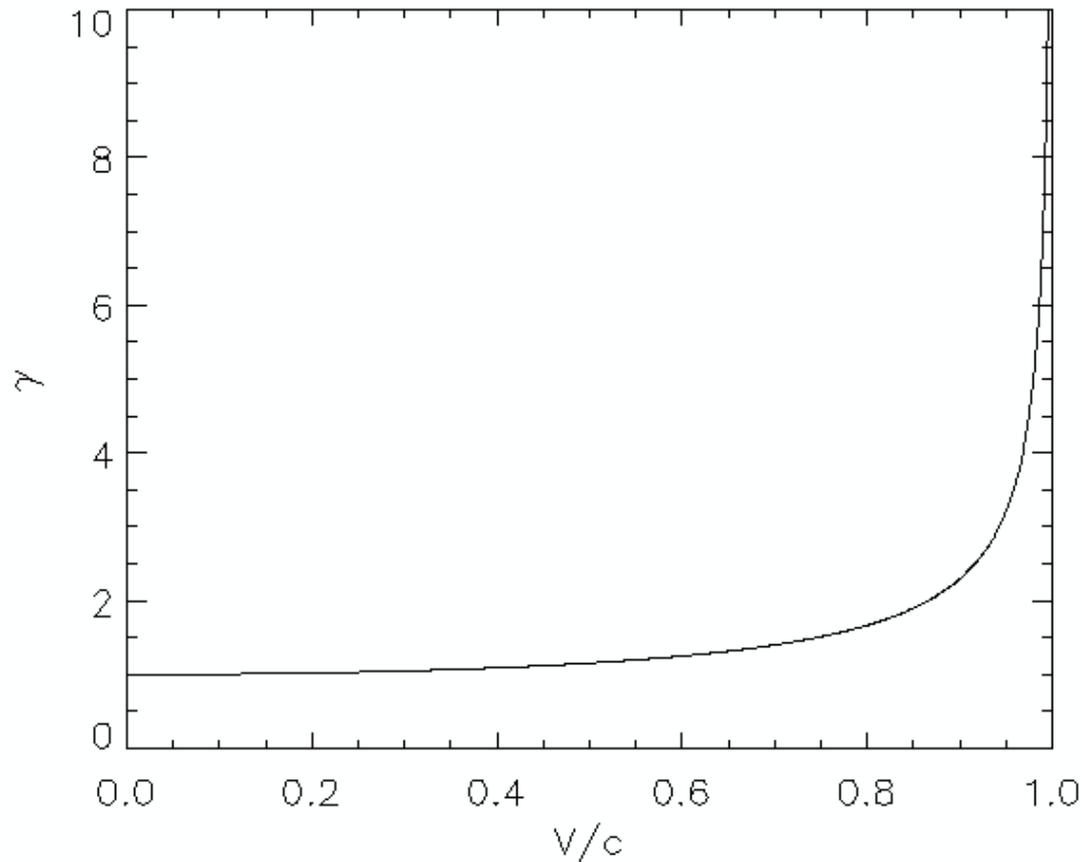
$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$



# Clocks and time

- ★ Does this “time dilation” effect come about because we used a funny clock?
- ★ No, any device that measures time would give the same effect!
- ★ The time interval of an event as measured in its own rest frame is called the *proper time*
- ★ Note that if the astronaut observed the same “light clock” (or any clock) that was at rest on Earth, it would appear to run slow by the same factor  $\gamma$ , because the dilation factor depends on *relative speed*
- ★ This is called the *principle of reciprocity*

# Lorentz factor



A 1% effect at  
 $v = 0.14 c$ , or  
about  
42,000,000 m/s

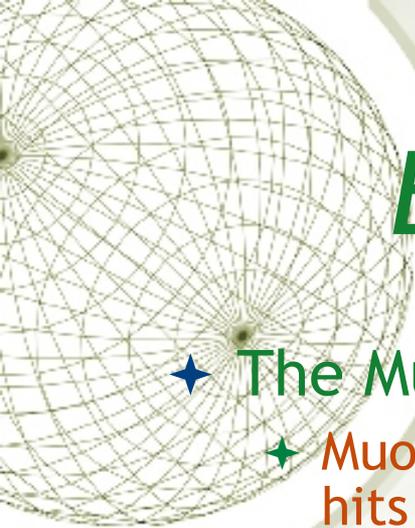
Lorentz factor goes to infinity when  $V \rightarrow c!$

But it is very close to 1 for  $V/c$  small

# Why don't we ordinarily notice time dilation?

## Some examples of speeds in m/s

- ★ 1 m/s a typical human walking speed
- ★ 12.4 m/s for the fastest 20 meters of the 100 m sprint by Usain Bolt
- ★ 28 m/s a car travelling at 60 miles per hour (mi/h or mph) or 100 kilometres per hour (km/h); also the speed a cheetah can maintain
- ★ 341 m/s the current land speed record, which was set by ThrustSSC in 1997.
- ★ 343 m/s the approximate speed of sound under standard conditions, which varies according to air temperature
- ★ 464 m/s Earth's rotation at the equator.
- ★ 559 m/s the average speed of Concorde's record Atlantic crossing (1996)
- ★ 1000 m/s the speed of a typical rifle bullet
- ★ 1400 m/s the speed of the Space Shuttle when the solid rocket boosters separate.
- ★ 8000 m/s the speed of the Space Shuttle just before it enters orbit.
- ★ 11,082 m/s High speed record for manned vehicle, set by Apollo 10
- ★ 29,800 m/s Speed of the Earth in orbit around the Sun (about 30 km/s)
- ★ 36,666 m/s Speed of Pioneer 10, the fastest probe (36.6 km/s)
- ★ 299,792,458 m/s the speed of light (about 300,000 km/s)

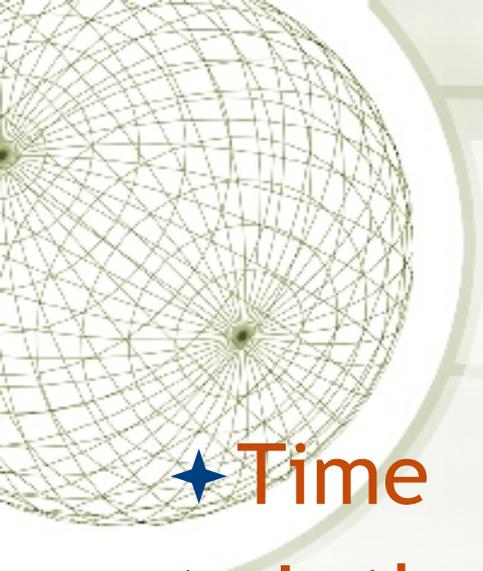


# Examples of time dilation

## ✦ The Muon Experiment

- ✦ Muons are created in upper atmosphere from cosmic ray hits
- ✦ Typical muon travel speeds are  $0.99995 \times c$ , giving  $\gamma=100$
- ✦ Half-life of muons in their own rest frame (measured in lab) is  $t_h = 2$  microseconds  $= 0.000002s$
- ✦ Traveling at  $0.99995 \times c$  for  $t_h = 0.000002s$ , the muons would go only 600 m
- ✦ But traveling for  $\gamma \times t_h = 0.0002s$ , the muons can go 60 km
- ✦ *They easily reach the Earth's surface, and are detected!*
- ✦ *Half-life can be measured by comparing muon flux on a mountain and at sea level; result agrees with  $\gamma \times t_h$*

# Quiz



## ★ Time

- A. Is the same for all frames of reference
- B. Always runs faster in the observer rest frame
- C. Always runs slower in the observer rest frame
- D. Can be reversed if we go fast enough

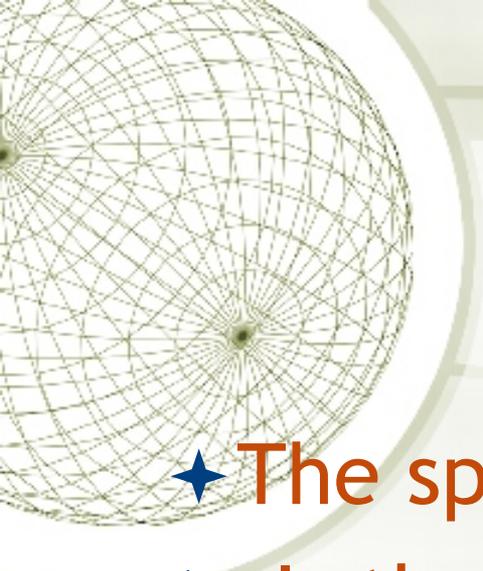
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# Quiz



★ The speed of light

- A. Is the same for all frames of reference
- B. Is not constant because of time dilation
- C. Is the same for all inertial frames of reference
- D. Is different in different directions

# Quiz

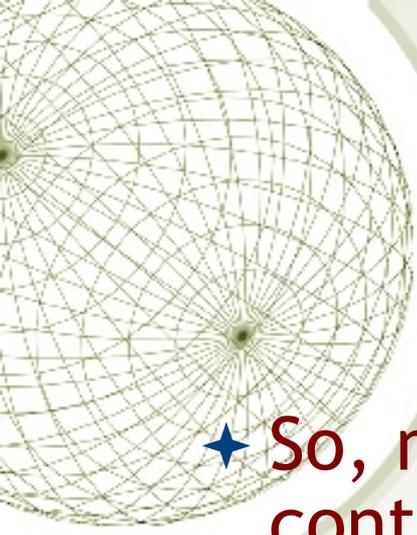


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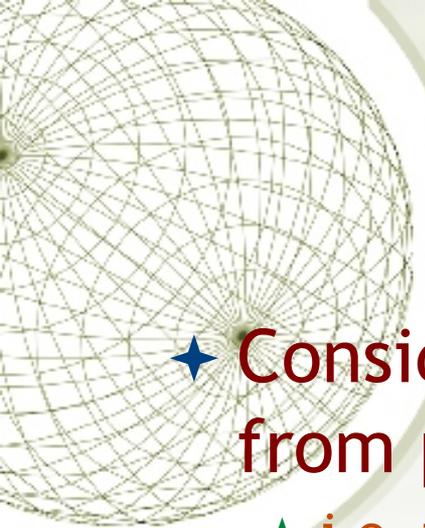
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# III: LENGTH CONTRACTION

- ✦ Consider two “markers” in space.
- ✦ Suppose spacecraft flies between two markers at velocity  $V$ .
- ✦ A flash goes off when front of spacecraft passes each marker, so that anyone can record it
- ✦ Compare what would be seen by observer at rest with respect to (w.r.t.) the markers, and an astronaut in the spacecraft...
- ✦ Observer at rest w.r.t. markers says:
  - ✦ Time interval is  $t_R$ ; distance is  $L_R = V \times t_R$
- ✦ Observer in spacecraft says:
  - ✦ Time interval is  $t_S$ ; distance is  $L_S = V \times t_S$
- ✦ We know from before that  $t_R = t_S \gamma$
- ✦ Therefore,  $L_S = V \times t_S = V \times t_R \times (t_S / t_R) = L_R / \gamma$
- ✦ *The length of any object is contracted in any frame moving with respect to the rest frame of that object, by a factor  $\gamma$*



- ★ So, moving observers see that objects contract *along the direction of motion*.
- ★ **Length contraction...** also called
  - ★ Lorentz contraction
  - ★ FitzGerald contraction
- ★ Note that there is no contraction of lengths that are perpendicular to the direction of motion
  - ★ Recall M-M experiment: results consistent with *one* arm contracting



# *Muon experiment, again*

- ★ Consider atmospheric muons again, this time from point of view of the muons
  - ★ i.e. think in frame of reference in which muon is at rest
  - ★ Decay time in this frame is  $2 \mu\text{s}$  ( $2/1,000,000 \text{ s}$ )
  - ★ How do they get from top of the atmosphere to sea level before decaying?
- ★ From point of view of muon, the atmosphere's height *contracts by factor of  $\gamma$* 
  - ★ Muons can then travel reduced distance (at almost speed of light) before decaying.

# IV: NEW VELOCITY ADDITION LAW

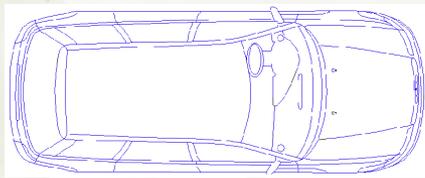
- ★ Einstein's theory of special relativity was partly motivated by the fact that Galilean velocity transformations (simple adding/subtracting frame velocity) gives incorrect results for electromagnetism
- ★ Once we've taken into account the way that time and distances change in Einstein's theory, there is a new law for adding velocities
- ★ For a particle measured to have velocity  $V_p$  by an observer in a spaceship moving at velocity  $V_s$  with respect to Earth, the particle's velocity as measured by observer on Earth is

$$V = \frac{V_p + V_s}{1 + V_p V_s / c^2}$$

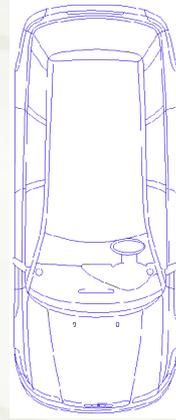
- ★ Notice that if  $V_p$  and  $V_s$  are much less than  $c$ , the extra term in the denominator  $\ll 1$  and therefore  $V \sim V_p + V_s$
- ★ Thus, the Galilean transformation law is *approximately correct* when the speeds involved are small compared with the speed of light
- ★ This is consistent with everyday experience
- ★ Also notice that if the particle has  $V_p = c$  in the spaceship frame, then it has  $V_p = c$  in the Earth frame. The speed of light is frame-independent!

# What if the speed of light weren't the same in all inertial frames?

Collision or not? If the speed of light were not the same in all inertial frames, you would see one car reach the collision point earlier than the other. But there either is or isn't a collision!



100 km/hr



100 km/hr

$$\Delta t = \frac{\Delta D}{c}$$



c

$c + 100 \text{ km/hr?}$

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*Next time...*

★ *Special Relativity II:*

- ★ Simultaneity and causality
- ★ Space-time diagrams
- ★ Reciprocity and the twins paradox