Class 5
Newtonian Gravity and orbits

- Newton’s Law of Gravitation
- Weight, weightlessness and the equivalence principle
- Kepler’s laws explained...
Newton’s law of Gravitation

Newton’s law of gravitation: Consider two particles with masses \( m_1 \) and \( m_2 \) separated by a distance \( r \). Then each particle experiences a (gravitational) force directed towards the other particle of magnitude \( F \) where

\[
F = \frac{Gm_1 m_2}{r^2}
\]

Notes:
- Gravitational Constant, \( G \approx 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2} \)
- Every particle in the Universe attracts every other particle
- The expression for \( F \) is symmetric in \( m_1 \) and \( m_2 \)...
  must be the case in order to satisfy Newton’s 3rd law

Using integral calculus, it can be shown that a spherical object with mass \( M \) acts gravitationally on external objects like a particle of mass \( m \) at the sphere’s center.

\[
F = \frac{GMm}{r^2}
\]
An object is in "orbit" when it is eternally falling...

II : The Weak Equivalence Principle

- Consider object (mass m) accelerating in the gravitational field of the Earth
  - Newton’s 2nd law says...
    \[ F = m_i a \]
    - \( m_i \) is the “inertial mass” of the object
  - Newton’s law of gravitation says...
    \[ F = \frac{GMm_G}{r^2} \]
    - \( m_G \) is the “gravitational mass” of the object
  - Acceleration is then...
    \[ a = \left( \frac{m_G}{m_i} \right) \frac{GM}{r^2} \]
When all other forces are eliminated, we find that **the acceleration of an object in a gravitational field does not depend on ANY property of the body** (Galilieo).

- Has been verified to 1 part in $10^{13}$
- Known as the **Weak Equivalence Principle**
III : Weightlessness

- Space Station orbits about 500km above Earth’s surface. Radius of Earth is 6300km.
- Newton’s inverse square law:
  - Gravitational acceleration at location of space station is 85% of what it is on the Earth’s surface!
  - So, why do the astronauts feel weightless?
  - The astronauts fall at the same rate as the space station – another example of the equivalence principle.
**Strong equivalence principle**: Two ways of stating the same thing...

1. No experiment can distinguish between a reference frame that is free-falling in a gravitational field, and a frame that is non-accelerating in the absence of a gravitational field
2. No experiment can tell you whether you are in a gravitational field or an accelerating reference frame
IV : Kepler’s laws explained

- Kepler’s laws are derived from Newton’s laws
- Imagine case of a particle (planet) moving in the gravitational field of a much more massive particle (Sun)...
- **Kepler’s first law**... using calculus (or very involved geometry), it can be shown that orbits have three possible shapes...
  - Ellipse
  - Parabola
  - Hyperbola

- **Kepler’s second law** : This is related to the conservation of angular momentum...
  - Suppose that (at some particular instant in time) the planet of mass \( m \) is a distance \( r \) from the Sun, and that the component of velocity in a direction perpendicular to the direction of the Sun is \( V_\perp \)
  - Then, the **angular momentum** of the planet is
    \[
    L = mV_\perp r
    \]
  - The angular momentum remains unchanged (is conserved) as the planet orbits around the Sun
Area $A$ swept out by line in a short time interval $\Delta t$ is

$$A \approx \frac{1}{2}rV_{\perp} \Delta t = \frac{L}{2m} \Delta t$$

- **Kepler’s third law**: Let’s apply Newton’s second law to a planet in a circular orbit with period $P$ about the Sun...

  
  \[ F = ma \]

  \[ \Rightarrow \frac{GM_\odot m}{r^2} = \frac{mV^2}{r} \]

  \[ \Rightarrow \frac{GM_\odot}{r^2} = \left(\frac{2\pi r}{P}\right)^2 \frac{1}{r} \]

  \[ \Rightarrow p^2 = \frac{4\pi^2}{GM_\odot} r^3 \]

  \[ \text{Remember that...} \]

  \[ V = \text{circumference/period} \]

- In fact, same formula also holds for elliptical orbits (proof is longer)
- Can see that constant of proportionality depends on mass of the central object