Class 26
Thermonuclear fusion in the Sun

- Conversion of mass to energy
- Conservation laws
- The actual reactions...

Notation... we need some compact way of discussing nuclei

Total number of nucleons = protons + neutrons

Atomic number = number of protons

Symbol for element (set by atomic number)
I: Conversion of mass to energy

- Essential result of the nuclear processes in the Sun is

\[ ^{1}H \rightarrow ^{2}_{3}He + 2e^{+} \]

- Let’s add up the masses...
  - Initial state...
    - Mass of \(^{1}H = 1.67262 \times 10^{-27}\)kg
    - So... mass of 4 \(^{1}H = 6.69048 \times 10^{-27}\)kg
  - Final state
    - Mass of \(^{4}He = 6.64666 \times 10^{-27}\)kg
    - Mass positron = \(9.10938 \times 10^{-31}\)kg
    - So, total mass is \(^{4}He + 2e^{+} = 6.64648 \times 10^{-27}\)kg
  - There is a mass difference of \(4.4 \times 10^{-29}\)kg

- Where has this mass gone?

- Einstein discovered an equivalence between mass and energy

\[ E = mc^{2} \]

- The “missing mass” has been converted into energy... this is how the Sun is powered!
  - About **4 million tonnes/second** is converted from mass to energy to power the Sun
  - Need to convert about **600 million tonnes/second** of hydrogen into helium
  - Define the “efficiency” of the process by...

\[
\text{efficiency} = \frac{\text{energy released}}{(\text{total mass processed})c^{2}}
\]

- For fusion of hydrogen to helium, efficiency is about 0.007 (0.7%)... about \(10^{6}\) times more efficient that chemical burning!
II : Conservation laws

- To understand nuclear reactions, we need to keep in mind the following familiar laws of physics...
  - Conservation of energy
  - Conservation of momentum
  - Conservation of electrical charge

- There’s also important and less familiar laws...
  - Conservation of “lepton number”... total number of electron-like particles is conserved
  - Conservation of “baryon number”... total number of nucleons (protons & neutrons) is conserved
  - Anti-particles count negatively towards the lepton and baryon number

A hydrogen bomb
III : Fusion reactions in the Sun

- We cannot fuse 4 hydrogen atoms into one helium atom in one step? [Why?]
- Discussion on the board...

The proton-proton (pp) chain
Without the clutter...

\[ ^1H + ^1H \rightarrow ^2H + e^+ + \nu \]

\[ ^2H + ^1H \rightarrow ^3He + \gamma \]

\[ ^3He + ^3He \rightarrow ^4He + ^1H + ^1H \]

There are other “side reactions”

\[ ^3He + ^4He \rightarrow ^7Be + \gamma \]

\[ ^7Be + e^- \rightarrow ^7Li + \bar{\nu} \]

\[ ^7Li + ^1H \rightarrow ^4He + ^4He \]
The rate that energy is produced per unit time per unit volume scales with density and temperature...

\[ \text{rate} \propto \rho^2 T^x \]

- Where \( x \approx 4 \) for the Sun
- Explains why fusion reactions only occur in the core where density and temperature is highest.

Eventually, all of the hydrogen will be used up in the core... what then?

- Will discuss next class how structure of Sun changes when H is used up... result is core compresses, heats up, and a new reaction starts...
Triple-alpha process

\[ \begin{align*}
4\text{He} + 4\text{He} & \rightarrow 8\text{Be}^* \\
8\text{Be}^* + 4\text{He} & \rightarrow \frac{12}{6}\text{C}^* \\
12\text{C}^* & \rightarrow 12\text{C} + \gamma + \gamma
\end{align*} \]

*Kicks in at about 100 million K*