ASTR120 Fall 2009

Homework 6 : The Moon, Earth and the Terrestrial planets

(Due 24th November 2009)

1. Chapter 10, Question 27
2. Chapter 10, Question 38
3. Chapter 11, Question 47
4. Chapter 12, Question 45

5. The Earth’s exosphere: The uppermost layers of the Earth’s atmosphere are called the exosphere. They are characterized by the fact that, due to the extremely low density, atoms and molecules do not collide with each other and hence follow “ballistic” trajectories (i.e. each atom/molecule follows trajectories that conserve total energy). The base of the exosphere is at a height of approximately $H_{\text{base}} = 500 \text{ km}$.

   (a) The average kinetic energy of a molecule at the base of the exosphere is given by $\langle E_{\text{kin}} \rangle = \frac{3k_B T}{2}$, where $T$ is the temperature and $k_B$ is the Boltzmann constant. Using this fact, derive a formula for the maximum height (above the Earth’s surface) to which a molecule that is initially at the base of the exosphere carrying the average kinetic energy can travel. You should express your answer in terms of the temperature at the base of the exosphere $T$, the mass of the Earth $M_\oplus$, the radius of the Earth $R_\oplus$, the height of the base of the exosphere $H_{\text{base}}$, and the various physical constants. [Hint: Recall that the gravitational potential energy of a mass $m$ in the gravitational field of a mass $M$ at a radius $R$ is given by $E_{\text{pot}} = -\frac{GMm}{R}$.]

   (b) The base of the exosphere has a temperature of about $T = 1000 \text{ K}$. Looking up any data/constants that we need to know, compute the height to which (a) an oxygen molecule (O$_2$), (b) a carbon dioxide molecule (CO$_2$) and (c) a nitrogen molecule (N$_2$) can rise. In all cases assume that they start at the base of the exosphere with the average kinetic energy.

   (c) Using these calculations, explain why there is a systematic change in the chemical composition of the exosphere as one considers higher altitudes.

6. Detecting planetary rotation with radar

   (a) A radar signal with initial wavelength $\lambda_0$ is emitted from the Earth and bounced off a target that is moving away with a (line of sight) speed $V$ (you may assume that $V \ll c$). Show that the returned signal has a wavelength given by

   $$\lambda \approx \lambda_0 \left(1 + \frac{2V}{c}\right)$$

   Explain all of the steps of your derivation.

   (b) Using data given in the textbook, calculate the speed of the equatorial region of Venus that corresponds to the planet’s rotation.

   (c) If a radar signal with wavelength $\lambda_0 = 12.5 \text{ cm}$ is bounced off Venus, what will be the spread in the returned wavelengths.