1. Suppose that we measure the spectrum of a distant galaxy. We see absorption lines in the spectrum with a wavelength of 484 nm but we know from laboratory measurements that the true intrinsic wavelength of the absorption lines should be 440 nm.

(a) Calculate the redshift of the galaxy.
(b) Assuming the non-relativistic formula for the redshift-velocity conversion, calculate the apparent velocity of recession for the galaxy.
(c) Assuming that Hubble’s constant is 71 km/s/Mpc, calculate the distance to the galaxy.

2. In vector terms, Hubble’s law states that the velocity \( \mathbf{v} \) at which a galaxy with position vector \( \mathbf{r} \) is receding from us is given by \( \mathbf{v} = H_0 \mathbf{r} \). Prove that an observer in another galaxy would measure the same law (i.e., galaxies would be moving away from them in the same manner). What would go wrong with your proof if Hubble’s law had some different mathematical form?

[Extra credit: Let’s turn this around... start with the assumption that all observers in the Universe measure the same relationship between velocity and displacement, \( \mathbf{v} = f(\mathbf{r}) \) (this assumption follows from the cosmological principals of homogeneity and isotropy). Prove as rigorously as you can that the only choice is \( f(\mathbf{r}) = H \mathbf{r} \) where \( H \) is a scalar constant.]

3. The special relativistic formula for the Doppler shift is

\[
\lambda_0 = \lambda_e \left( \frac{c + V}{c - V} \right)^{1/2}.
\]  

(1)

By assuming that Hubble’s law (\( v = Hd \)) is strictly correct for all velocities, derive a formula which gives the distance of a galaxy in terms of its redshift \( z \) that works even when the redshift is large.

4. Assuming that \( H_0 = 71 \text{ km/s/Mpc} \), compute the actual value of the critical density. Express your answer in SI units (i.e., kg/m\(^3\)). How many hydrogen atoms per cubic meter does this correspond to?