Class 3:
Basic equations of cosmology II

- This class...
  - Recap of Friedmann Equation
  - Fluid Equation
  - Behavior of “radiation fluids”
  - Acceleration equation

I : The Fluid Equation

- To solve Friedmann equation and determine $a(t)$, we need to know how density depends on scale factor
- Starting with the first law of thermodynamics, we can show that
  $$\dot{\rho} + 3\frac{\dot{a}}{a} \left( \rho + \frac{p}{c^2} \right) = 0$$
  - The two terms in the bracket correspond to the (1) dilution of the energy by the expansion and (2) the loss of energy due to the work done on the surroundings.
  - **Important case**: Pressure-less matter ("dust")
    $$p = 0 \Rightarrow \dot{\rho} + 3\frac{\dot{a}}{a} \rho = 0 \Rightarrow \rho \propto a^{-3}$$
    "Matter-dominated" Universe
  - This simply describes how a constant mass gets diluted as the volume increases ($pV=\text{const}$)
II : Behavior of radiation fluids

- Suppose that the Universe were dominated by radiation and not matter!
- In fact, as we’ll see later, this is precisely the situation for the first 6,000 yrs of the Universe’s history.
- For radiation:
  \[ p = \frac{\rho c^2}{3} \]
- Substituting into Fluid Equation, we get
  \[ \dot{\rho} + 4\frac{\dot{a}}{a} \rho = 0 \Rightarrow \rho \propto a^{-4} \]
- This has an interesting interpretation...

III : The acceleration equation

- Combining the Friedmann and Fluid equations, we can derive the acceleration equation:
  \[ \frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left( \rho + \frac{3p}{c^2} \right) \]
- Notes:
  - This equation does not depend upon “k”
  - Pressure forces increase the gravitational deceleration of the Universe! Pressure acts to SLOW DOWN the expansion, not drive it. WHY???