Class 18 : The hadron and lepton eras

This class

- Study a particularly important period of the Universe... \( t = 10^{-4} - 10 \text{s} \) after big bang
- Formation of the neutrino background
- Ratio of \( p \) to \( n \) (important for H/He ratio!!)

I : Adiabatic behavior of matter in an expanding Universe

On board, we will prove two important facts...

- For non-relativistic matter \((E_{\text{kin}} < mc^2)\), we have
  \[ T_{\text{mat}} \propto a^{-2} \]
- For relativistic matter \((E_{\text{kin}} > mc^2)\), we have
  \[ T \propto 1/a \]
- i.e., relativistic matter behaves very much like radiation (or, viewed another way, photons are the ultimate in relativistic matter!)
II : Physics in the hadron era

- For $t<10^{-6}$s ($T>10^{13}$K)
  - Universe consists of soup of elementary particles, most noteworthy are:
    - Photons $\leftarrow$ BOSONS
    - Protons + anti-protons
    - Neutrons + anti-neutrons
    - Electrons + positrons
    - Neutrinos (all flavors) $\Rightarrow$ FERMIONS
  - All of these species are relativistic.
  - All of these species are in thermal equilibrium
    - Energy density in a given fermionic species = $(7/8) \times$ energy density in a given bosonic species (Not obvious!)
    - Need to include spin-states in this assessment

- This can’t go on forever...
  - Define threshold temperature of a species as the point where the average particle energy equals the rest-mass energy of the species
    \[
    \frac{3}{2} kT_{thres} = mc^2 \Rightarrow T_{thres} = \frac{2mc^2}{3k}
    \]
  - For protons, $T_{thres} = 7 \times 10^{12}$K
    - Once $T$ falls below this temperature ($t \sim 10^{-5}$s), p/n and anti-p/n cease to be relativistic, and it becomes difficult to make them (need higher energy than average photons to interact)
    - At $t \sim 10^{-4}$s, we have $T \sim 10^{12}$K and there’s no more p/n or anti-p/n productions.
    - Vast majority of protons/anti-protons and neutrons/anti-neutrons annihilate... but there is a slight excess of matter over anti-matter and so 1 in every $10^9$ survive!
III: Physics in the lepton era

- Zoom forward to \( t \sim 10^{-2} \) s (\( T \sim 10^{11} \) K)
  - Universe consists of soup of....
    - Photons
    - Electrons and positrons
    - Neutrinos (all species)
    - Small number of \( p \) and \( n \)
  - Other important physical considerations...
    - \( p \) and \( n \) are non-relativistic; all other species relativistic
    - Energy density completely dominated by relativistic species
    - All of these ingredients are in thermal equilibrium
    - Again... energy density in a given fermionic species = \( (7/8) \times \) energy density in a given bosonic species (i.e. photons)
  - At \( t \sim 10^{-2} \) s (\( T \sim 10^{11} \) K), average photon energy much greater than mass-energy difference between \( p \) and \( n \)...
    - equal numbers of \( p \) and \( n \)

- As Universe cools and becomes less dense, the typical time between interactions increases...
- At \( t \sim 10^{-1} \) s (\( T \sim 3 \times 10^{10} \) K), the typical time between neutrino interactions exceeds expansion timescale...
  - Neutrinos decouple and free stream
    - So we now have a decoupled neutrino background
    - Neutrinos continue to cool as the Universe expands, preserving their thermal spectrum (\( T \sim 1/a \))
    - Since radiation is also cooling in same way, neutrinos and radiation maintain same temperature even though they are no longer interacting
    - ALSO, the average photon energy now becoming comparable to the \( p-n \) mass difference... reactions start to prefer protons over neutrons (putting \( T = 3 \times 10^{10} \) K gives \( n_n : n_p = 0.38 : 0.62 \)).
At $t \sim 1 \text{s} (T \sim 10^{10}\text{K})$, time between weak-force reactions involving $p$ and $n$ exceeds expansion time...
protons and neutrons fall out of thermal equilibrium

- Known as **proton-neutron freeze-out**
- At this time, $n_p:n_n \approx 1:5$... we will see that this had a profound effect on the H:He ratio in the Universe!!
- Photons, electrons, positrons still in thermal equilibrium
  $$\gamma + \gamma \rightleftharpoons e^- + e^+$$

- Electrons and positrons still relativistic (just)
- But the Universe is still cooling...

Finally, $t \sim 10 \text{s} (T \sim 3 \times 10^9\text{K})$: Temperature falls below threshold temperature for the electrons.

- All positrons annihilate with an electron to form photons
- Thus, huge amount of energy dumped into photon field...
  temperature of the radiation increases!
- In fact, the annihilation occurs at constant entropy which, for relativistic species, is $S = gT^3$
- These considerations allow us to calculate that the temperature of the radiation increases by a factor $(11/4)^{1/3}$ (see board)

**The neutrino background**

- Up to this point, neutrino & radiation background are in lock-step
- Suddenly, radiation temperature increases by factor $(11/4)^{1/3}$
- From that point on (until today!), the radiation background evolves passively (no wholesale destruction or creation of photons)
- Can use this fact to calculate the neutrino background today.
Answer is
$$\rho_\nu = \frac{3}{8} \times \left(\frac{4}{11}\right)^{4/3} \rho_\gamma$$