1. Liddle 5.3
2. Liddle 5.5
3. Liddle 9.1
4. Liddle 9.2
5. Fun with 3-spheres: The metric of a 3-dimensional spherical surface with radius of curvature $a$ is

$$ds^2 = \frac{dr^2}{1 - r^2/a^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2).$$

(a) Derive an expression for the physical volume contained within the region defined by $r \leq r_0$, where $r_0$ is some constant. (*Hint: In class, we derived an expression for the surface area of the 2-d sphere with $r = \text{constant}$. Use this and the metric above as the starting point to calculate the physical volume of a shell $r \rightarrow r + dr$.*

(b) Show that this volume reduces to the usual flat space result when $r_0/a \ll 1$.

(c) There is a maximum allowable value for $r_0$. What is this value?

(d) What is the total volume of the 3-sphere?

6. Closed radiation-dominated Universes: Solve the Friedman equation for a closed ($k > 0$) radiation-pressure dominated Universe in order to derive an algebraic expression relating $a$ and $t$, putting your answer in the simplest form possible. Show that, at early times, the answer approximates the flat space result, $a \propto t^{1/2}$.

7. Time evolution of the mass-to-light ratio: Consider the following two pieces of information... (1) In star forming regions, the number of stars formed in the mass range $M \rightarrow M + dM$ is given by $n(M)dM \propto M^{-2.35}$, with a minimum mass of $M = 0.1M_\odot$ and maximum mass $100M_\odot$ (this is known as the Salpeter initial mass function). (2) The luminosity of a star of mass $M$ scales as $L \propto M^{3.5}$ unless one is considering stars with mass greater than $20M_\odot$, in which case the dependence becomes $L \propto M$. Using this information, derive the mass-to-light ratio as a function of time for a galaxy that has a short but very intense burst of star formation. If you need to make any further assumptions, state them clearly.