1. Liddle Q12.2

2. **The structure of dark matter halos**: In a simple model for a spherically symmetric dark matter halo, the dark matter density is given by

   \[ \rho_{DM} = \frac{M_v}{4\pi r_v^2 r^2} \]  

   where \( M_v \) is enclosed mass at the virial radius \( r_v \).

   (a) Let \( \sigma_c(r) \) be the velocity of a dark matter particle that is executing a circular orbit in the halo with radius \( r \) centered on \( r = 0 \). Show that \( \sigma_c \) is independent of radius, and obtain its value in terms of \( M_v \) and \( r_v \).

   (b) Assuming that \( \sigma_c \) typifies the velocity of a dark matter particle, what is the average gravitational potential energy of a dark matter particle. Explain how you got your answer. [Hint — there is a fast route to this!]

   (c) Suppose that this dark matter halo has formed recently, and that the average density of the halo (measured out to the virial radius) is \( \Delta \approx 200 \) times greater than the average matter density today. Derive an expression for the mass of the halo (in units of solar masses) as a function of \( \sigma_c \), scaled to a fiducial \( \sigma_c \) of 100 km/s.

3. **Reionization and the smallest dark matter halos**: At a redshift of \( z \approx 10 \), the intergalactic baryons underwent a process called *reionization*: the radiation from the first stars and black holes was sufficient to ionized hydrogen and heat the gas up to temperatures of \( 10^4 \) K. This question explores some implications of this for galaxy formation.

   (a) Assume that the reionized gas consists of protons and electrons such that the average particle mass is \( \mu m_p \) with \( \mu \approx 0.5 \). Using the expression given in class, calculate the sound speed of the reionized gas.

   (b) Noting that the sound speed is also approximately the average speed of a particle in the gas, explain why the process of reionization will drive baryons out of the smallest dark matter halos thereby preventing any star formation in those halos.

   (c) Consider the first population of dark matter halos which where formed at high redshift \( (z \sim 20 \) or greater). For these halos, it can be shown that \( M_v \approx 3 \times 10^6 (\sigma/10 \text{ km s}^{-1})^3 \). Estimate the smallest halo that can still retain its baryons following reionization.

4. **L-T relationship in clusters of galaxies**: In clusters of galaxies, we directly see the shocked baryons using X-ray observations. At such temperatures, the cooling function (discussed in class) is given by \( \Lambda(T) \propto T^{1/2} \). Using expressions from class and ignoring any feedback effects, show that the total X-ray luminosity of the baryons in a cluster should be proportional to the square of the temperature (i.e. \( L_X \propto T^2 \)). We actually observe \( L_X \propto T^3 \). Conjecture how feedback may be responsible for this difference.