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TRAPPING OF MAGNETIC FLUX BY THE PLUNGE REGION OF A BLACK HOLE ACCRETION DISK

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ABSTRACT

The existence of the radius of marginal stability means that accretion flows around black holes invariably undergo a transition from a magnetohydrodynamic (MHD) turbulent disklike flow to an inward-plunging flow. We argue that the plunging inflow can greatly enhance the trapping of large-scale magnetic fields on the black hole, and therefore may increase the importance of the Blandford-Znajek (BZ) effect relative to previous estimates that ignore the plunge region. We support this hypothesis by constructing and analyzing a toy model of the dragging and trapping of a large-scale field by a black hole disk, revealing a strong dependence of this effect on the effective magnetic Prandtl number of the MHD turbulent disk. Furthermore, we show that the enhancement of the BZ effect depends on the geometric thickness of the accretion disk. This may be, at least in part, the physical underpinnings of the empirical relation between the inferred geometric thickness of a black hole disk and the presence of a radio jet.

Subject headings: accretion, accretion disks — black hole physics — magnetic fields — X-rays: binaries
Online material: color figures

1. INTRODUCTION

One of the most spectacular phenomena associated with accretion onto black holes is the creation of powerful, highly relativistic jets. However, despite intense observational and theoretical study, the basic energy source of these relativistic jets remains unknown. Broadly speaking, there are two possibilities. First, jets could be powered by the liberation of the gravitational potential energy of accreting matter. If this is the case, the most likely scenario is the formation and subsequent focusing and acceleration of a magnetohydrodynamic (MHD) disk wind (Blandford & Payne 1982). While this mechanism has the appealing feature of potentially being universal to all accreting systems (and therefore allowing a unified model for jets from protostellar systems, accreting white dwarfs, and accreting neutron stars, as well as accreting black holes), it is not clear that such a disk wind can be accelerated to the highly relativistic velocities seen from many black hole systems. The alternative is that jets could be powered by the magnetic extraction of the spin energy of the central black hole using the mechanism described in the seminal paper by Blandford & Znajek (1977, hereafter BZ). The power extracted from a Kerr black hole with dimensionless spin parameter a_* threaded by a magnetic field of strength B_H (in the membrane paradigm sense, see Thorne et al. 1986) is

$$L_{\rm BZ} \approx \frac{1}{32}\omega_F^2 B_H^2 r_H^2 a_*^2 c, \qquad (1)$$

where r_H is the radius of the event horizon and $\omega_F^2 = \Omega_F(\Omega_H - \Omega_F)/\Omega_H^2$, with Ω_H and Ω_F being the angular velocities of the black hole and magnetic field lines, respectively. It is often argued (e.g., see BZ) that the magnetic field structure adjusts itself such that $\Omega_F = \Omega_H/2$ (Phinney 1983), hence maximizing ω_F^2 to a value of $\frac{1}{4}$. While the initial work of BZ was based on force-free black hole magnetospheres, the basic mechanism is seen to operate in the recent generation of fully relativistic MHD accretion disk simulations (e.g., see Koide et al. 2000; Komissarov 2004; De Villiers et al. 2005; McKinney & Gammie 2004; McKinney 2005a, 2005b, 2005c).

In the past dozen years or so, several studies have cast doubt on whether nature can produce significant hole-threading magnetic fields, leading to the suggestion that the BZ mechanism is insufficient to energize powerful black hole jets. Developing on work done by van Ballegooijen (1989), Lubow et al. (1994) and Heyvaerts et al. (1996, hereafter HPB) have examined the dragging and concentration of an external field by an MHD turbulent accretion disk. Both sets of authors find that, due to the high effective magnetic diffusivity of such disks, the inward dragging and subsequent concentration of an external field is rather ineffective. In a different approach to this problem, Ghosh & Abramowicz (1997, hereafter GA97) construct force-free black hole magnetosphere models within the radius of marginal stability and show that the field threading the black hole is only of comparable strength to that threading the inner disk. Since the disk-threading field has to be rather weak (with the magnetic pressure at least an order of magnitude less than the total pressure) so as not to quench the magnetorotational instability (MRI) that drives the accretion itself (Balbus & Hawley 1991, 1998), the inferred black hole–threading field would be insufficient to energize the powerful jets in the active galactic nuclei (AGNs) that we observe. This argument was further developed by Livio et al. (1999), who pointed out that, under these circumstances, the electromagnetic power extracted from the inner regions of the disk would necessarily dominate the black hole spin-energy extraction.

In all of the studies described above, the plunge region of the black hole accretion disk has been neglected. This is the region of the disk within the radius of marginal stability in which the accretion flow is undergoing rapid inward acceleration (ultimately crossing the event horizon at the velocity of light as seen by a locally nonrotating observer). Unless the magnetic field is extremely strong, this is a region where inertial forces will dominate and the commonly employed force-free approximation will break down. For example, examination of the GA97 steady state magnetosphere solution shows a magnetic field crossing the plunge region with a strength very similar to that in the diffusive

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region of the disk. This situation seems unlikely; any dynamically unimportant magnetic field that threads the plunge region would be swept into the black hole on a dynamical timescale. This means that the actual field threading the plunge region would be very weak. However, it does not imply that the field threading the black hole horizon, which is what counts for the BZ effect, is also weak. The field swept in by the plunge region would be "cleaned" into some well-ordered configuration threading the black hole (for a full discussion of the "cleaning" of a magnetic field by a black hole, see Thorne et al. [1986]) and can be confined by the inertial forces of the plunging accretion flow even if it achieves strengths appreciably higher than the characteristic field strengths in the inner disk. Since the strength of the BZ mechanism depends on the square of the magnetic field, this enhancement could have major implications for the relative dominance of spin-energy extraction.

In this paper we extend these previous works by examining the role of the plunge region of a black hole accretion disk in enhancing the horizon-threading flux. In essence, we use the HPB formalism for flux dragging in an accretion disk and impose an inner boundary condition appropriate for the plunge region around a central black hole. This formalism is described in § 2. Although this analysis is nonrelativistic, it should provide guidance about flux enhancement by the plunge region, at least in the case of slowly rotating black holes. As reported in § 3, our analysis confirms the basic intuition discussed in the previous paragraph and uncovers a strong dependence of the equilibrium trapped flux on the disk thickness and the effective magnetic Prandtl number of the disk. Section 4 discusses the sensitivity of our results to the outer boundary condition and then places our results into a wider astrophysical context. Our conclusions are presented in § 5.

2. THE TOY MODEL

We follow the approach of HPB to study the dragging of an external magnetic field by an MHD turbulent accretion disk. As already noted, this results in a nonrelativistic model but should be able to provide quantitative insights on the behavior of slowly rotating black holes (where the radius of marginal stability is in a rather low gravitational redshift regime). We consider a thin Keplerian accretion disk with geometric thickness $h \ll r$ extending down to the radius of marginal stability at $r = r_{\rm ms}$. We suppose that the disk has an effective magnetic diffusivity (due to reconnection in the MHD turbulence) of η_*, which is comparable to the effective viscosity ν_*. In other words, the effective magnetic Prandtl number $\mathrm{Pr}_m = \nu_*/\eta_*$ is of order unity (see HPB for an explicit justification of $\mathrm{Pr}_m \sim 1$). Note that we employ the standard definition of Pr_m as the ratio of the viscosity to the magnetic diffusivity, which is the reciprocal of that used in Lubow et al. (1994) and HPB.

We now suppose that an external uniform magnetic field with strength B_0 is present in the vertical direction (i.e., aligned with the normal to the accretion disk). We are interested in the dragging of this field by the accretion flow. Assuming the system remains axisymmetric at all times and employing cylindrical polar coordinates (r, z, ϕ), the poloidal magnetic field structure is completely described by the flux function $A(r, z; t)$ via $\boldsymbol{B}_p = \nabla \times (A\hat{\boldsymbol{\phi}}/r)$. With such a definition, the magnetic flux threading a ring of radius r at height z from the disk plane is $2\pi A(r, z; t)$, and the magnetic field components are given by

$$B_r = -\frac{1}{r}\frac{\partial A}{\partial z}, \tag{2}$$

$$B_z = \frac{1}{r}\frac{\partial A}{\partial r}. \tag{3}$$

The flux function can be decomposed into three components,

$$A(r, z; t) = A_{\rm BH}(r, z; t) + a(r, z; t) + \frac{r^2 B_0}{2}, \tag{4}$$

where $A_{\rm BH}(r, z; t)$ is the flux function associated with the cleaned black hole–threading field (generated by currents in the disk), the final term on the right-hand side is just the uniform imposed flux (generated by currents at infinity), and $a(r, z; t)$ accounts for all other (disk-threading) magnetic field structures (generated, in principle, by currents either into or out of the disk plane). For definiteness we suppose that, in the region exterior to the disk ($|z| > h$), the black hole–threading field has the form of a split monopole,

$$A_{\rm BH}(r, z; t) = A_*(t)\left[1 - \mathrm{sgn}(z)\frac{z}{(z^2 + r^2)^{1/2}}\right], \tag{5}$$

where $A_*(t)$ is $1/2\pi$ times the total hole-threading flux. To reiterate, this hole-threading field is generated by toroidal currents flowing in the disk ($|z| < h$) and is a vacuum solution to Maxwell's equations elsewhere. While the precise structure of the cleaned black hole field is unclear, the choice of the split monopole has support from recent general relativistic MHD simulations (e.g., see Hirose et al. 2004; Komissarov 2005).

HPB showed that the time evolution of the flux function in the diffusive part of the disk plane, $A(r, 0; t)$ ($r > r_{\rm ms}$), is given by

$$\frac{\partial A}{\partial t} + v_r\frac{\partial A}{\partial r} - \eta_*\left[\frac{1}{h}\left(\frac{\partial A}{\partial z}\right)_{z=h} + r\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial A}{\partial r}\right)\right] = 0, \tag{6}$$

where v_r is the radial velocity of the accretion flow. The $(\partial A/\partial z)_{z=h}$ term represents the effect of magnetic tension due to the curvature of field lines across the disk plane, and thus depends on the structure of the magnetic field above and below the disk. For example, one could formulate steady state MHD wind solutions that take the instantaneous value of $A(r, 0; t)$ as a boundary condition. This would be a task of great complexity (note that the radial structure of the boundary condition would not admit self-similar wind solutions). Here we make the following simplifications. First, we assume that the magnetic field outside of the disk (hereafter referred to as the disk magnetosphere) is force-free, i.e., $(\nabla \times \boldsymbol{B}) \times \boldsymbol{B} = 0$. Second, we assume that the Alfvén speed in the disk magnetosphere is sufficiently high as to reduce the toroidal field to essentially zero (through the production of torsional Alfvén waves). Setting $B_\phi = 0$, the field in the disk magnetosphere becomes potential ($\nabla \times \boldsymbol{B} = 0$), and the flux function obeys

$$\mathcal{D}A = 0, \tag{7}$$

where $\mathcal{D}$ is the linear differential operator

$$\mathcal{D} \equiv \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}\right) + \frac{\partial}{\partial z}\left(\frac{1}{r}\frac{\partial}{\partial z}\right). \tag{8}$$

Noting that both the imposed uniform field and (exterior to the disk) the black hole–threading field $A_{\rm BH}(r, z; t)$ individually obey $\mathcal{D}A = 0$, the structure of the disk magnetosphere is determined by solving the potential problem for $a(r, z; t)$, i.e., $\mathcal{D}a = 0$.

At this point, a brief discussion of our $B_\phi = 0$ assumption (which leads to the potential field condition) is in order. In the nonrelativistic treatment here we can always assume that the Alfvén speed in the disk magnetosphere is large enough such that any twist in the magnetic field is removed via a torsional Alfvén wave. However, our intent is to produce a toy model for accretion onto a black hole, so we should be wary of an assumption that so explicitly relies on nonrelativistic physics. In a fully relativistic treatment, the force-free magnetosphere around a black hole accretion disk would be described by the Grad-Shafranov equation (BZ; MacDonald & Thorne 1982; Uzdensky 2004, 2005). It is found that the poloidal field structure depends on both the poloidal current distribution (which gives rise to toroidal fields) and the field line rotation (due to the fact that the field lines are frozen into an orbiting accretion disk, for example). In particular, the field structure is affected by the presence of an inner and outer light cylinder. Ultimately, a relativistic version of our model should study the flux dragging and the magnetization of the black hole, including these physical effects. Here we simply note that detailed studies of nonrotating (or slowly rotating) black hole magnetospheres have shown that the field line rotation associated with a Keplerian accretion disk has only a small effect on the poloidal field as compared with the equivalent nonrotating configuration (MacDonald 1984; Uzdensky 2004). In this sense, Keplerian accretion disks are "slow rotators" (Uzdensky 2004).

For the rest of this paper we explicitly consider the behavior of the magnetic field in the upper half of the z-plane, $z > 0$; we suppose the system to be symmetric in the $z = 0$ plane such that $B_z(r, z) = B_z(r, -z)$ and $B_r(r, z) = -B_r(r, -z)$. The tension term in equation (6) can be decomposed into

$$\left(\frac{\partial A}{\partial z}\right)_{z=h} = \left(\frac{\partial A_{\rm BH}}{\partial z}\right)_{z=h} + \left(\frac{\partial a}{\partial z}\right)_{z=h}. \qquad (9)$$

The contribution from the hole-threading flux can be evaluated directly from equation (5),

$$\left(\frac{\partial A_{\rm BH}}{\partial z}\right)_{z=h} \approx -\frac{A_*}{r}, \qquad (10)$$

where we have neglected a term that is smaller by a factor of $(h/r)^2$. The remaining contribution to equation (9) follows from the solution to the potential problem $\mathcal{D}a = 0$ with boundary conditions $a(r = 0, z; t) = 0$ and $a(r, 0; t)$ specified. As shown by HPB, this gives

$$\left(\frac{\partial A}{\partial z}\right)_{z=h} = \mathcal{P}\int_0^\infty dx\, \frac{a(x, 0; t) - a(r, 0; t)}{\pi(r - x)^2} - \frac{a(r, 0; t)}{\pi r}, \qquad (11)$$

where $\mathcal{P}$ signifies the principal part of the integral. We can now write an explicit integrodifferential equation for the time evolution of $a(r, 0; t)$ in the diffusive part of the disk ($r > r_{\rm ms}$):

$$\begin{aligned} \frac{\partial a}{\partial t} + \frac{\partial A_*}{\partial t} + v_r r B_0 + \left(v_r + \frac{\eta_*}{r}\right)\frac{\partial a}{\partial r} \\ = \eta_*\left[\frac{1}{h}\mathcal{P}\int_0^\infty dx\, \frac{a(x, 0; t) - a(r, 0; t)}{\pi(r - x)^2} \right. \\ \left. - \frac{a(r, 0; t)}{h\pi r} - \frac{A_*(t)}{hr} + \frac{\partial^2 a}{\partial r^2}\right]. \end{aligned} \qquad (12)$$

As part of our model, we must specify $h(r)$, $v(r)$, and $\eta_*(r)$. For definiteness, we define $h(r)$ by taking the ratio h/r as a fixed parameter of our model [in principle, one could substitute a particular form for $h(r)$ resulting from a detailed disk model]. To specify the radial velocity field, we follow Lubow et al. (1994) and split our disk into two zones, which we dub an "active" and a "dead" zone. In the active zone ($r_{\rm ms} < r < r_{\rm dead}$), we set $v_r = -\nu_*(1/r - 1/r_{\rm dead})$, where $\nu_* = \alpha h^2 (GM/r^3)^{1/2}$ (HPB) and $\eta_* = \nu_*/{\rm Pr}_m$. The magnetic Prandtl number ${\rm Pr}_m$ is a fixed and constant parameter of the active disk. Note that we have introduced the usual α of accretion disk theory (in contrast with HPB, who implicitly employ $\alpha \sim 1$). In the dead zone ($r_{\rm dead} < r < r_{\rm out}$) the diffusivity is still given by $\eta_* = \alpha h^2 (GM/r^3)^{1/2}/{\rm Pr}_m$, but the velocity is set to zero. For computational necessity we impose an outer cutoff on the system at $r = r_{\rm out}$. We assume that the disk beyond $r_{\rm out}$ is a perfect and static conductor. Hence, the total magnetic flux threading a loop ($r = r_{\rm out}, z = 0$) is constant and has the value $\pi r_{\rm out}^2 B_0$. The inclusion of the dead zone makes the evolution of the inner part of the system essentially independent of the position or exact nature of the $r = r_{\rm out}$ boundary. In particular, the dead zone acts as a reservoir of magnetic flux that can feed the actively accreting part of the disk; only in the outermost parts of the active disk does the conservation of magnetic flux lead to a nonnegligible magnetic pressure trying to "suck" magnetic flux out of the active disk. The physical nature of the dead zone is discussed in § 4.

Finally, we must specify boundary conditions on $a(r, 0; t)$. The implementation of the inner boundary condition must capture the fact that the plunge region is extremely effective at sweeping in a poloidal magnetic field that crosses within $r = r_{\rm ms}$. Consider a poloidal magnetic field line that is dragged toward the plunge region on the viscous timescale $t_{\rm visc} \approx (r_{\rm ms}/h_{\rm ms})^2 (r_{\rm ms}^3/GM)^{1/2} \alpha^{-1}$. Once in the plunge region, the radial velocity of the disk material rapidly increases, with no associated increase in the effective magnetic diffusivity (indeed, to the extent that the plunge region becomes a laminar rather than a turbulent flow, the effective magnetic diffusivity may well plummet to very small values). For the field strengths under consideration here (i.e., with an energy density much less than the kinetic energy density of the accretion flow), inward advection of the field line on a dynamical timescale $t_{\rm dyn} \approx (r_{\rm ms}^3/GM)^{1/2}$ will dominate all other processes. Since the characteristic evolution timescale of the system is $t_{\rm visc} \gg t_{\rm dyn}$, flux conservation requires that the vertical magnetic field in the $z = 0$ plane in the plunge region compared with that in the disk just outside be

$$\frac{B_z({\rm plunge})}{B_z({\rm disk})} \approx \frac{t_{\rm dyn}}{t_{\rm visc}} \approx \alpha\left(\frac{h}{r}\right)^2 \ll 1. \qquad (13)$$

To a good approximation, we can say that the magnetic flux locally crossing the plunge region is zero. Thus, the only magnetic flux passing through a loop ($r < r_{\rm ms}, z = 0$) is that which threads the black hole, i.e., $A(r \le r_{\rm ms}, 0; t) = A_*(t)$. To cancel the contribution from the externally imposed uniform field in this region, we must have

$$a(r, 0; t) = -r^2 B_0/2 \quad (r < r_{\rm ms}). \qquad (14)$$

Thus, the appropriate inner boundary condition for equation (12) is $a(r = r_{\rm ms}, 0; t) = -r_{\rm ms}^2 B_0/2$, and we must use equation (14) in the evaluation of the integral term of equation (12). The fact that $\partial a(r_{\rm ms}, 0; t)/\partial t = 0$ allows us to use equation (12) to evaluate the rate of change of black hole–threading flux,

$$\begin{aligned} \frac{\partial A_*}{\partial t} = \eta_*(r_{\rm ms})\frac{1}{h}\mathcal{P}\int_0^\infty dx \left[\frac{a(x, 0; t) - a(r, 0; t)}{\pi(r - x)^2}\right. \\ \left. - \frac{a(r, 0; t)}{h\pi r} - \frac{A_*(t)}{hr} + \frac{\partial^2 a}{\partial r^2} + B_0\right]_{r=r_{\rm ms}}, \end{aligned} \qquad (15)$$

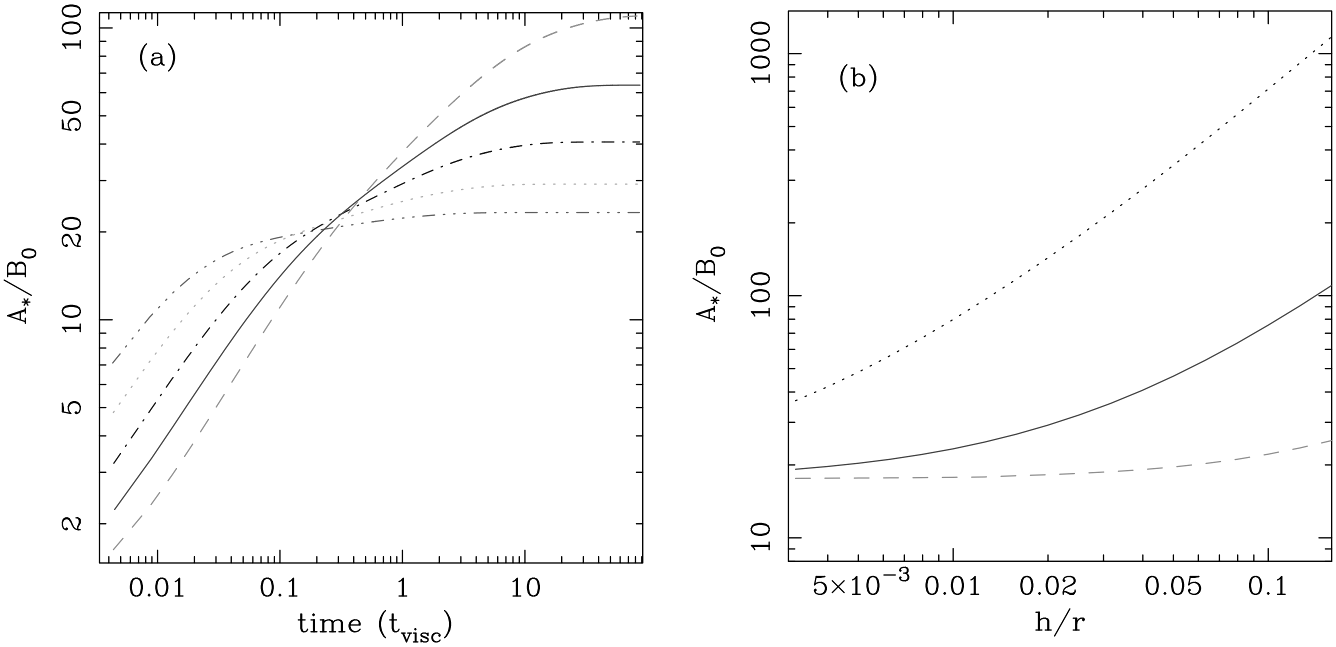

FIG. 1.—(*a*) Time dependence of the black hole–threading flux for $\mathrm{Pr}_m = 2$ and $h/r = 0.01$ (*triple-dot-dashed line*), 0.02 (*dotted line*), 0.04 (*dot-dashed line*), 0.08 (*solid line*), and 0.16 (*dashed line*). For comparison, $A_*/B_0 = 18$ corresponds to the flux of the uniform external field threading the radius of marginal stability. Time is in units of the viscous timescale at $r_{\rm ms}$, $t_{\rm visc} = r^2(R^3/GM)/\alpha h^2$. (*b*) Equilibrium value of A_*/B_0 as a function of h/r for $\mathrm{Pr}_m = 0.2$ (*dashed line*), 2.0 (*solid line*), and 20.0 (*dotted line*). [*See the electronic edition of the Journal for a color version of this figure.*]

where we have used the continuity of $\partial a/\partial r$ across $r = r_{\rm ms}$ to combine the third and fourth terms on the left-hand side of equation (12). We can justify this assumption of continuity as follows. Suppose that this derivative was *discontinuous* across $r = r_{\rm ms}$, resulting in a discontinuity in the strength of the vertical magnetic field. This would lead to a large magnetic pressure gradient and a very rapid rearrangement of material until continuity was achieved. We do note, however, that we expect a rather narrow transition zone just outside of $r = r_{\rm ms}$ where the vertical magnetic field goes from zero to the value characteristic of the disk. We must spatially resolve this transition in our numerical model.

For the outer boundary condition, we set $a(r_{\rm out}, z = 0; t) = -A_*(t)$ for some $r_{\rm out} > r_{\rm dead}$. This amounts to bounding the entire system by a perfect and static conductor in the disk plane ($z = 0$) for all $r > r_{\rm out}$, as discussed above.

With these assumptions, equations (12) and (15) completely describe the evolution of $a(r, 0; t)$ and $A_*(t)$ from some initial state once we fix the magnetic Prandtl number Pr_m, the disk thickness h/r, the characteristic radii of the problem ($r_{\rm ms}$, $r_{\rm dead}$, $r_{\rm out}$), the external field strength B_0, and the viscosity parameter α. In fact, α and B_0 are trivial parameters of the model, affecting only the scaling of the time coordinate and the absolute normalization of a, respectively. Furthermore, the inclusion of the dead zone makes the evolution of the inner disk/field essentially independent of the location of the outer boundary $r = r_{\rm out}$. Hence, the nontrivial parameters describing this system are Pr_m, h/r, and $r_{\rm dead}$. For our initial condition we take

$$a(r, z = 0, t = 0) = \begin{cases} -r^2 B_0/2, & r < r_{\rm ms}, \\ -r_{\rm ms}^2 B_0/2, & r \geq r_{\rm ms}. \end{cases} \tag{16}$$

This amounts to saying that the initial currents flowing in the disk are only those required to cancel the imposed uniform field in the plunge region.

3. SOLUTION METHOD AND RESULTS

We solve equation (12) numerically by discretizing it on a logarithmic grid with 200 zones from $r_{\rm ms} = 6$ to $r_{\rm out} = 150$, with the dead zone starting at $r_{\rm dead} = 100$. Here and for the rest of this paper, radii will be given in units of gravitational radii GM/c^2. We treat the advective ($\partial a/\partial t$) terms using the second-order van Leer (1977) method. All other terms (including the principal part integral) are also differenced to second-order spatial accuracy. The time evolution is achieved through a simple first-order explicit scheme. To ensure numerical stability, we set the time step to be $dt = (1/dt_{\rm ad}^2 + 1/dt_{\rm diff}^2 + 1/dt_{\rm field}^2)^{-1/2}$, where the advective, diffusive, and field time steps are given by $dt_{\rm ad} = 0.5\min[\Delta r/(v + \eta_*/r)]$, $dt_{\rm diff} = 0.5\min(\Delta r^2/\eta_*)$, and $dt_{\rm field} = 0.5\min(h\Delta r/\pi\eta_*)$.

Figure 1 shows the time evolution of A_* for the case of $\mathrm{Pr}_m = 2$ and various choices of h/r from 0.01 to 0.16. In all cases, the flux threading the black hole grows from zero and achieves some positive steady state. In all cases, the final equilibrium flux threading the black hole exceeds $\pi r_{\rm ms}^2 B_0$ (corresponding to $A_* = 18B_0$), thereby establishing the basic fact that the plunge region can aid in the accumulation of significant magnetic flux through the black hole. For thicker disks, the increased inward advection of the field (due to the increased radial-inflow speed of the accreting matter) coupled with the decreased effectiveness of field diffusion leads to significant enhancements of the black hole–threading flux above this baseline value. The dependence of the equilibrium value of A_* on disk thickness and magnetic Prandtl number is shown in Figure 1*b*. For small Pr_m, the enhancement of the hole-threading flux above the canonical value of $A_* = 18B_0$ is very small. However, for Pr_m of order unity or higher, there is a strong h/r-dependent enhancement.

The full magnetic field configuration can be derived by solving the potential problem for $a(r, z; t)$ using the solution method laid out in HPB. In Figure 2 we show the initial field configuration, as well as the final configuration for $h/r = 0.08$ and two

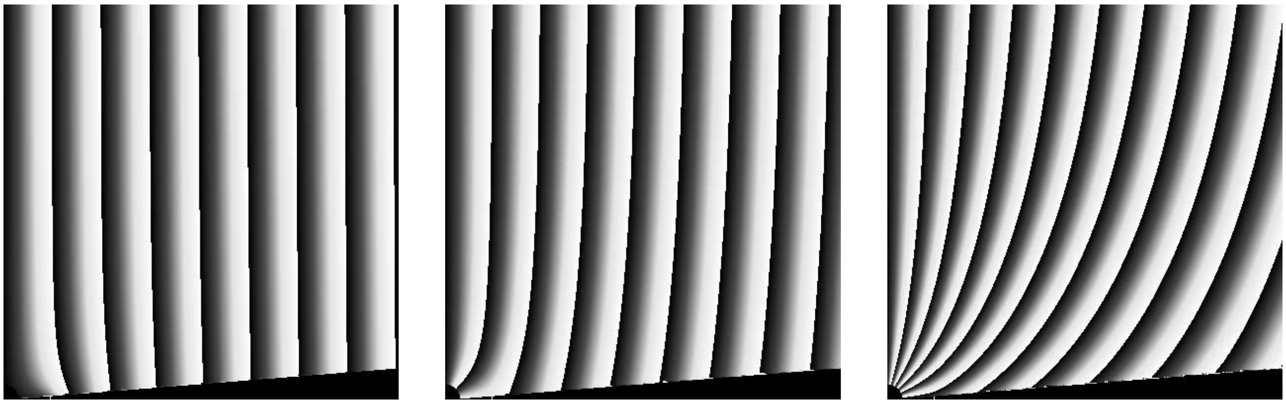

FIG. 2.—Magnetic field configuration for the initial condition (*left*), the final state of the $\mathrm{Pr}_m = 2$, $h/r = 0.08$ case (*middle*), and the final state of the $\mathrm{Pr}_m = 20$, $h/r = 0.08$ case (*right*). Note how the higher magnetic Prandtl number results in a powerful inward dragging of magnetic field and subsequent magnetization of the black hole. Each of these three panels is 50 gravitational radii ($50GM/c^2$) on a side. [*See the electronic edition of the Journal for a color version of this figure.*]

choices of magnetic Prandtl number $\mathrm{Pr}_m = 2$ and 20. The initial configuration deviates from a simple uniform field due to the fact that flux is excluded from the region $r < r_{\rm ms}$, which leads to a "bowing" of the field lines away from the radius of marginal stability. This curvature is rapidly reversed as the field is advected inward, finally achieving a steady state in which the bend angle of field lines as they enter the diffusive part of the disk is approximately constant. As pointed out by Lubow et al. (1994) and discussed below, we expect this bend angle (away from the disk normal) to be $i \sim \tan^{-1}(h\mathrm{Pr}_m/r)$. This is indeed seen in our equilibrium solutions.

The central quantity of interest in this work is the magnetic field threading the black hole event horizon. Recalling the definition of the flux function, it is straightforward to show that the magnetic field threading the event horizon is $B_H = A_*/r_H^2$, where $r_H = 2$ is the event horizon radius of the (slowly rotating) black hole considered in this work. From the results described above we conclude that the equilibrium flux threading the black hole always exceeds the flux of the external uniform field through the plunge region ($\pi r_{\rm ms}^2 B_0$ corresponding to $A_* = 18B_0$), sometimes by a large factor in the case of high effective magnetic Prandtl numbers and/or thick disks. Scaling to this fiducial flux we have $B_H = 4.5\Upsilon B_0$, where $\Upsilon = A_*/18B_0$. Using a least-squares fit to the results displayed in Figure 1, we find that a good approximation is $\Upsilon \approx 1 + 20\mathrm{Pr}_m(h/r)$. Hence, we have

$$B_H \approx 4.5\left[1 + 20\mathrm{Pr}_m\left(\frac{h}{r}\right)\right]B_0, \qquad (17)$$

which is accurate to the 20% level for $\mathrm{Pr}_m < 20$. As we discuss below, the factor multiplying the $\mathrm{Pr}_m h/r$ term in equation (17) has a dependence on the size of the dead zone; the precise form of equation (17) is strictly valid only for $r_{\rm dead} = 100$.

4. DISCUSSION

4.1. *Dependence on the Size of the Dead Zone*

At first glance, the dragging of magnetic flux by the accretion disk leads to a surprisingly large enhancement in the black hole–threading field. However, as we now explain, simple arguments can be put forward to support the results encapsulated in equation (17).

First, we note that the existence of the dead zone is crucial for setting an overall size scale to the magnetic disturbances introduced by the disk. To see this, consider the limit in which $r_{\rm dead} \to \infty$ (also requiring $r_{\rm out} \to \infty$, of course). In this case the imposed uniform magnetic field is dragged inward by the accretion flow, but a balance will never be achieved between the inward advection and the magnetic tension; without an imposed spatial scale, the field curvature through the disk and hence the magnetic tension can be made arbitrarily small. A balance is possible only when one imposes an outer truncation on the part of the disk that drags the magnetic flux. In this case, the undragged field at $r > r_{\rm dead}$ acts as an anchor and limits the vertical extent to which the magnetic field can be appreciably distorted. Indeed, our calculations show that the magnetic field at $|z| > r_{\rm dead}$ is essentially just the imposed uniform field.

Now, as already noted, we find that the magnetic field threads the active part of the diffusive accretion disk ($r_{\rm ms} < r < r_{\rm dead}$) with a bend angle (away from the disk normal) of $\tan i \equiv B_r/B_z \approx h\mathrm{Pr}_m/r$. As shown by HPB and Lubow et al. (1994), this is a direct consequence of a balance between outward magnetic diffusion due to field line tension and the inward advection of magnetic field,

$$v_r \frac{\partial A}{\partial r} \approx \eta_* \left(\frac{\partial A}{\partial z}\right)_{z=h}. \qquad (18)$$

Consider the field line that threads the inner edge of the diffusive disk at $r = r_{\rm ms}$. This field line follows a roughly parabolic path in the magnetosphere that can be described by the flux function $\Psi = \Psi_0(r^2 + 2\xi z) = \mathrm{const}$. We can determine the parameter ξ using the fact that, at the disk plane, we have $B_r/B_z \approx h\mathrm{Pr}_m/r$,

$$\frac{B_r}{B_z} = -\frac{\partial\Psi/\partial z}{\partial\Psi/\partial r} = -\frac{\xi}{r_{\rm ms}} \approx \left(\frac{h}{r}\right)\mathrm{Pr}_m, \qquad (19)$$

where we have dropped a term that is second order in h/r. At a vertical distance of $z = r_{\rm dead}$, this same field line has a cylindrical radius R given by

$$R^2 = r_{\rm ms}^2\left[1 + 2\frac{r_{\rm dead}}{r_{\rm ms}}\left(\frac{h}{r}\right)\mathrm{Pr}_m\right]. \qquad (20)$$

Using our observation above concerning the vertical extent of the field disturbances and the fact that the field is essentially uniform for $|z| > r_{\rm dead}$, we calculate the magnetic flux threading the plunge region and hence the black hole,

$$\Phi_H = \pi R^2 B_0 = \pi r_{\rm ms}^2 B_0\left[1 + 2\frac{r_{\rm dead}}{r_{\rm ms}}\left(\frac{h}{r}\right)\mathrm{Pr}_m\right]. \qquad (21)$$

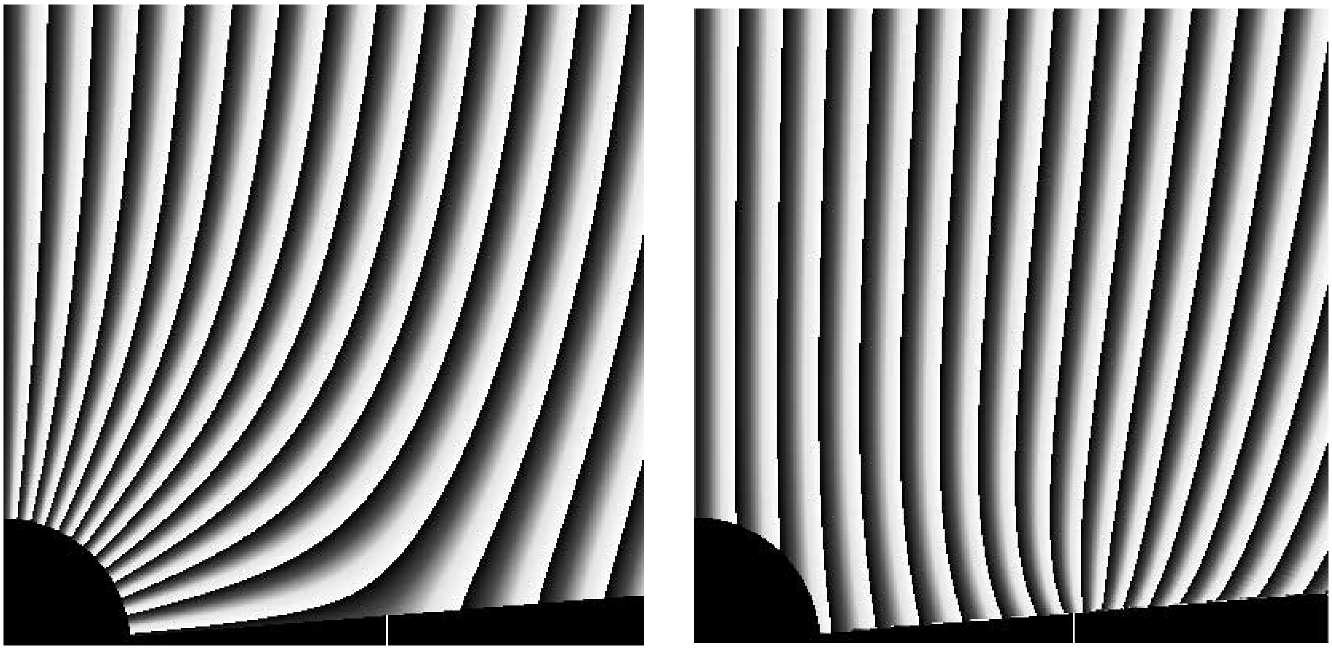

FIG. 3.—Magnetic field configurations for the plunge boundary condition (*left*) and the uniform flux bundle boundary condition (*right*). In both cases, the figure shows a zoom-in ($10GM/c^2$ on a side) of the final field structure in the $\mathrm{Pr}_m = 2$, $h/r = 0.08$ case. A white vertical line on the accretion disk denotes the radius of marginal stability. [*See the electronic edition of the Journal for a color version of this figure*.]

In terms of the field threading the hole (putting $r_{\rm ms} = 6$), we get

$$B_H = 4.5\left[1 + \frac{r_{\rm dead}}{3}\left(\frac{h}{r}\right)\mathrm{Pr}_m\right]B_0. \qquad (22)$$

Thus, we can see that the numerical factor multiplying the $(h/r)\mathrm{Pr}_m$ term in equation (17) is directly related to the value of $r_{\rm dead}$.

The above discussion helps to elucidate the role of the plunge region in enhancing the black hole–threading flux; the plunge region "shields" the diffusive part of the disk from the large bundle of magnetic flux that threads the black hole. This bundle of flux is the ultimate repository for the magnetic flux that has been scooped up by the accretion flow. The larger the region of the disk that can drag the flux inward, the larger is this repository. To illustrate this issue, we have run a modified version of our code in which the plunge region boundary condition is replaced with the assumption that the magnetic flux contained within $r = r_{\rm ms}$ has the form of a uniform field on the disk plane. We employ canonical values of the model parameters; $h/r = 0.08$, $\mathrm{Pr}_m = 2$, and $r_{\rm dead} = 100$. As expected, we get a weak (50%) enhancement in the flux contained within $r = r_{\rm ms}$, compared with over a factor of 3 for the plunge case. The magnetic field structures of the two cases are illustrated in Figure 3.

Performing a full numerical solution to equation (12) for $r_{\rm dead} = 50$, 100, and 200 reveals that the enhancement of the magnetic flux increases with $r_{\rm dead}$ slightly more slowly than the linear relationship predicted by our simple arguments in this section. Since the implementation of the dead zone is one of most artificial aspects of our toy model, we do not explore this dependence in any more detail in this paper. In real systems, the dead zone might be identified with the outer edge of the MHD turbulence–dominated accretion disk, e.g., the self-gravity region in an AGN disk or the tidal truncation radius for the disk in a galactic black hole binary (GBHB). Both of these radii are likely to be at significantly larger radius than the $r_{\rm dead} = 100$ used here. Alternatively, if the magnetosphere is treated using a full MHD wind model, the crucial length scale that determines the magnetic field enhancement is likely to be the vertical height of the Alfvénic surface. It is beyond the scope of this paper to address such models. However, our approach allows us to illustrate an essential point; the inward dragging of the magnetic field over some region of the inner disk, coupled with the existence of the plunge region, allows for a significant enhancement in the strength of the magnetic field threading the black hole.

4.2. *Limitations of Our Approach*

Before discussing the astrophysical implications of our result, we must address the three major limitations of our approach. First, we have made no attempt to include relativistic effects (beyond our simple treatment of the radius of marginal stability) on the dynamics or electrodynamics of the disk/field system. Our model is an adequate representation for slowly spinning black holes (where the radius of marginal stability is rather large and in a comparatively low-gravity region of spacetime), but we acknowledge that a full relativistic electrodynamic treatment is required to robustly treat the case of rapidly rotating black holes. While the same basic phenomenon of magnetic flux trapping by the plunge region should be at work around rapidly rotating black holes, the geometry of the system (i.e., the fact that the radius of marginal stability is much closer to the event horizon) might be unfavorable for producing dramatic enhancements in the black hole–threading magnetic field. On the other hand, an ergospheric wind (Punsly & Coroniti 1990; Punsly 1991) could aid in the production of a strong poloidal field (through the inertial effects of the outflowing plasma), as well as the inward dragging of the field (through the removal of angular momentum from the accretion flow).

Second, we assume the existence of a preexisting large-scale magnetic field. The origin of such a field depends on the system under consideration. For the accreting black hole at the heart of a gamma-ray burst (GRB) collapsar, such a field may arise naturally from the collapsed stellar envelope. In the case of AGNs, the field corresponds to that of the accreting interstellar medium.

For GBHBs, the presence of a large-scale field probably depends on the mode of accretion, with wind accretors likely possessing a much stronger and better organized large-scale field than Roche-lobe-overflow accretors.

Third, we assume axisymmetric large-scale fields with a disk magnetosphere consisting of force-free and purely poloidal fields. As mentioned in § 2, a more physical treatment would entail matching an MHD wind solution to the disk-plane flux function. With such an approach, one could capture the inertial effects of a disk outflow on the field structure, the hoop stresses resulting from any toroidal fields present, and the angular momentum losses in the disk due to the wind. These could have competing effects on the ultimate ability of the disk to drag the field into the plunge region. The inertial effects tend to bend the field lines outward, increasing the field-line curvature at the disk plane and hence increasing outward diffusion of the field. The loss of disk angular momentum to the wind, on the other hand, would lead to an increase in the radial velocity of the accretion flow but no change in the magnetic diffusivity. This, in turn, increases the inward advection of the magnetic field. Clearly, more detailed calculations of this scenario are warranted. As for the axisymmetric assumption, we note that Spruit & Uzdensky (2005) have recently examined the dragging of a large-scale magnetic field by an accretion disk under the assumption that the MHD turbulence in the disk concentrates the field into small bundles (giving rise to the accretion disk equivalent of sunspots). Through an analysis of the dynamics of these bundles, they conclude that this is a generally favorable scenario for accumulating a large amount of magnetic flux in the central regions of the disk. Thus, in at least one specific model, an extreme deviation from axisymmetry aids in the inward dragging of magnetic flux.

We reiterate that the principal result of this paper is that the existence of a plunge region together with magnetic field dragging in the accretion disk can significantly enhance the black hole–threading magnetic field and hence the BZ effect. Furthermore, the enhancement becomes increasing effective for thicker disks or higher magnetic Prandtl numbers. However, we acknowledge that the caveats given above, together with the dependence of the enhancement on the size of the dead zone, prevents us from further quantifying the enhancement.

4.3. *Astrophysical Implications*

Given the caveats discussed above, the results of § 3 have important implications for the strength of the black hole–threading field and the relevance of the BZ process. Suppose that the magnetic pressure due to the large-scale field B_0 is a fraction f of the maximum pressure in the accretion disk, $p_{\rm max}$, i.e., $B_0 = (8\pi f p_{\rm max})^{1/2}$. Using this together with equation (1) and equation (17) gives $L_{\rm BZ} \approx 5\pi\omega_F^2 f p_{\rm max} \Upsilon^2 r_H^2 a_*^2 c$. Using the expressions for $p_{\rm max}$ for radiation-pressure-dominated (RPD) and gas-pressure-dominated (GPD) disks from Moderski & Sikora (1996) and GA97, and assuming the usual BZ impedance matching criterion is obeyed, gives

$$L_{\rm BZ}({\rm ergs\ s^{-1}}) \approx \begin{cases} 1.5\times10^{45}\alpha^{-1} f M_8 \Upsilon^2 a_*^2, & {\rm RPD}, \\ 9\times10^{43}\alpha^{-9/10} f M_8^{11/10} \dot{m}_{-4}^{4/5} \Upsilon^2 a_*^2, & {\rm GPD}, \end{cases} \quad (23)$$

where we have scaled to a black hole mass of $M = 10^8 M_8\ M_\odot$ and $\dot{m} = 10^{-4}\dot{m}_{-4}$ is the mass accretion rate in Eddington units. This can be directly compared with the expressions for $L_{\rm BZ}$ in GA97 if we set $f\alpha^{-1} \approx 0.1$ (which results from their relation between α and the disk magnetic field). For $\Upsilon = 1$ (corresponding to small effective magnetic Prandtl numbers or very thin disks), we find low BZ luminosities that agree very well with those computed by GA97. However, as we have shown, large magnetic Prandtl numbers and/or thick disks can result in large enhancements of the black hole–threading fields, approximately described by $\Upsilon \approx 1 + 2x{\rm Pr}_m(h/r)$, where $x \sim O(r_{\rm dead}/r_{\rm ms})$. The BZ luminosity is then enhanced by a factor of Υ^2.

It is interesting to explore the astrophysical consequences of the strong h/r dependence of the equilibrium hole-threading flux A_*. There is mounting empirical evidence that black hole systems produce jets only when a geometrically thick accretion disk is present. The best case can be made for the GBHBs, as discussed by Fender et al. (2004). In their X-ray low-hard (LH) state (aka the power-law state; McClintock & Remillard 2004), they display steady optically thick radio cores, which, in Cygnus X-1, can be spatially resolved into a jetlike structure by the Very Long Baseline Array (VLBA; Stirling et al. 2001). It is generally believed that the inner regions of the accretion flow in a LH-state GBHB system are radiatively inefficient, hot, and hence geometrically thick ($h/r \sim 0.5$). However, the radio jet is seen to shut off once the source has made a transition to the high-soft (HS) state (or thermal state; McClintock & Remillard 2004), which is believed to correspond to an inner accretion disk that is radiatively efficient, and hence significantly thinner. We postulate that the jet in the LH state is powered by the BZ effect, which is enhanced by the flux-trapping effect of the plunge region. Some time after a transition to a HS state, the system will possess a disk with a similar accretion rate but significantly reduced thickness. For a fixed accretion rate, the maximum pressure in a disk $p_{\rm max} \propto (h/r)^{-1}$. Using our parameterization for Υ, we expect the BZ luminosity $L_{\rm BZ} \propto f(h/r)$, provided $h/r \gtrsim 1/x{\rm Pr}_m$. Hence, due to the inability of a thin disk to trap flux on the black hole, the BZ luminosity of the HS state will be much reduced, leading to the suppression of the radio jet.

The actual LH → HS transition itself is particularly interesting. It is during this transition (when the source crosses the "jet line" on the X-ray flux/color diagram) that powerful relativistic outflows are produced, which, for example, produce the superluminal radio blobs seen from microquasars. It is likely that the transition is driven by the thermal collapse of the LH-state hot disk, producing a structure that eventually evolves into the HS-state cold disk. The nature of the intermediate structure is unclear, however. It has been suggested that the thermal collapse produces a magnetically dominated region (e.g., Meier 2005) in which MRI-driven turbulence is suppressed and accretion proceeds only through large-scale magnetic torques. If the precollapse disk is threaded by a large-scale magnetic field, this field could readily become dynamically important in the postcollapse disk (since rapid thermal collapse will proceed at constant surface density, producing a disk pressure $p_{\rm max} \propto h/r$). Subsequent magnetic braking of the disk would lead to rapid inflow, a rapid accretion of poloidal flux onto the black hole, and a rapid increase in the importance of the BZ effect. The powerful ejections seen from GBHBs as they undergo this transition might be the result of such a scenario. The ejections would terminate once the inner disk has ceased to be magnetically dominated (due to the accretion of matter from the outer disk), hence reestablishing a turbulent state with high effective magnetic diffusivity.

5. CONCLUSIONS

Black hole rotation is, in principle, a more than sufficient source of energy for energizing even the most powerful relativistic jets. The viability of magnetic extraction of black hole spin energy does, however, hinge on the strength of the horizon-threading poloidal magnetic field that can be established by the accretion flow. In this paper we have argued that the plunge region of the black hole accretion disk has an important role to play in enhancing the horizon-threading field well above the modest levels suggested by previous works. We support this hypothesis by constructing a toy model (that is nonrelativistic, assumes axisymmetry, and treats the fields away from the disk plane as potential) with which we can follow the dragging of an external magnetic field by the disk and its subsequent trapping by the plunge region. Our toy model suggests that the BZ effect can be enhanced above the canonical estimates of GA97 by a factor of $[1 + x\mathrm{Pr}_m(h/r)]^2$ where Pr_m is the effective magnetic Prandtl number of the disk and $x \sim O(r_{\rm dead}/r_{\rm ms})$. Even in cases in which the effective magnetic diffusivity is small due to the MHD turbulence (i.e., $\mathrm{Pr}_m \sim 1$), the BZ effect can be enhanced by 1 order of magnitude (or more) above the GA97 value if the disk is geometrically thick, $h/r \gtrsim r_{\rm ms}/r_{\rm dead}$. The h/r dependence of this effect has an appealing resonance with the empirical evidence from GBHBs, which points to a close connection between the existence of powerful black hole jets and the inferred properties of the accretion disk.

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