ASTR430 Handout 2:
The Two-body Cheat Sheet

General Equations

These general formulae govern all types of orbital motion in the gravitational two-body problem, including both bound and unbound orbits. More specialized formulae, valid only for certain types of orbits, can be derived from these.

Specific Energy	Specific Angular Momentum	Distance	Speed	Pericenter & Apocenter
$E_B = -\frac{GM}{2a}$	$h = \sqrt{GMa(1-e^2)}$	$r = \frac{a(1-e^2)}{1+e\cos f}$	$v = \sqrt{GM\left(\frac{2}{r} - \frac{1}{a}\right)}$	$q = a(1-e)$ $Q = a(1+e)$

(E_B & h are per unit reduced mass; $M = m_1 + m_2$; r & v are relative coordinates)

Equations for Specific Orbits

Energy determines whether an orbit is bound or not. Circles and ellipses are the only bound orbits; parabolae and hyperbolae are the only unbound ones. Note that $e = 1$ orbits (rectilinear or straight-line orbits) may be elliptical, parabolic, or hyperbolic.

	Bound Orbits ($E_B < 0$)		Unbound Orbits ($E_B \geq 0$)	
	Circle	Ellipse	Parabola	Hyperbola
Semimajor Axis:	$a = r$	$a > 0$	$a \to \pm\infty$	$a < 0$
Eccentricity:	$e = 0$	$0 < e \leq 1$	$e = 1$	$e \geq 1$
Distance:	$r = a$	$r \geq a(1-e)$ $r \leq a(1+e)$	$r \geq a(1-e)$ $r \to \infty$	$r \geq a(1-e)$ $r \to \infty$
Speed:	$v = \sqrt{\frac{GM}{a}}$	$v \leq \sqrt{\frac{GM(1+e)}{a(1-e)}}$ $v \geq \sqrt{\frac{GM(1-e)}{a(1+e)}}$	$v = \sqrt{\frac{2GM}{r}}$ $v_\infty = 0$	$v \leq \sqrt{\frac{GM(1+e)}{a(1-e)}}$ $v_\infty = \sqrt{-\frac{GM}{a}}$

($v_\infty = v$ in the limit $r \to \infty$, i.e., speed at "infinity")

Conservation Laws

In the absence of non-gravitational forces and external torques, total energy and angular momentum are conserved. Often this fact can be used to simplify a problem. The following formulae assume the origin is at the center of mass (so bulk motion can be ignored).

Total Energy:	$E = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 - \frac{Gm_1 m_2}{\lvert\mathbf{r}_2 - \mathbf{r}_1\rvert} = \frac{1}{2}\mu v^2 - \frac{Gm_1 m_2}{r} = \frac{1}{2}\mu v_\infty^2$
Angular Momentum:	$\mathbf{L} = m_1(\mathbf{r}_1 \times \mathbf{v}_1) + m_2(\mathbf{r}_2 \times \mathbf{v}_2) = \mu(\mathbf{r} \times \mathbf{v}) = \mu v_q q = \mu v_\infty b$

($\mu = m_1 m_2 / M$; b = impact parameter $= r\sin\phi$)