ASTR430 HW#3 (due 10/26/05)

1. Toomre's Q. Analogous to the collapse of a gas cloud, a thin rotating disk of particles may clump up if the random motions of the particles, plus the differential shear of the disk, is insufficient to prevent gravitational collapse. One measure of this is Toomre's Q parameter:

$$Q \equiv \frac{\kappa\sigma}{\pi G \Sigma},$$

where κ is the *epicyclic* (radial) frequency (which for Keplerian orbits is just the orbital frequency, ω), σ is the velocity *dispersion* (a measure of the randomness of motion), and Σ is the surface mass density of the disk (in kg/m^2). If $Q \lesssim 1$, the disk is unstable.

   (a) Compute Q as a function of Σ for a disk of identical 1 km radius planetesimals of mass density 2000 kg/m^3 orbiting the Sun at 1 AU, assuming the velocity dispersion is roughly equal to the escape speed from one of the planetesimals.

   (b) For what value of Σ is Q "critical" (i.e., equal to 1)?

   (c) Roughly speaking, the scale height of the disk is $H \sim \sigma/\nu$, where ν is the vertical frequency. Taking ν to be ω, estimate the *volume* mass density ρ of the critical disk, assuming it's uniformly distributed. Comment.

2. Textbook problems.

   (a) Problem #15 from Ch. 6 (p. 153).

   (b) Problem #14 from Ch. 7 (p. 189).

3. Hazard mitigation. Suppose a 10 km radius comet of bulk density 0.5 g/cc traveling at 20 km/s relative to Earth while still far away is on a direct (straight-line) collision course with our planet...

   (a) Using the solution strategy of Homework 1, Problem 3, compute the minimum required energy as a function of distance to deflect the comet so that it just grazes the Earth.

   (b) Using the result of Homework 2, Problem 2, compute the minimum required energy to fully disrupt the comet, assuming an explosive efficiency of only 0.1% (which is probably optimistic).

   (c) At what critical distance is it more energy efficient to just blow up the comet rather than deflect it? Express your answer in Earth radii.