ASTR430 HW#4 (due 11/9/05)

1. Holy Shrinking Jupiters, Batman!

   (a) Jupiter has an estimated emissivity of 0.64. Compute the mean power in Watts emitted by Jupiter assuming the giant planet is only reemitting light absorbed from the Sun.

   (b) Jupiter actually glows like a blackbody of effective temperature 124 K. Compute its emitted power on that basis (recall, for a blackbody, the emitted *flux* is σT^4). Notice that the emitted power is larger that what you found in part (a)—Jupiter radiates more energy than it receives from the Sun!

   (c) The difference is attributable to the fact that Jupiter is still contracting and converting gravitational potential energy into heat. If Jupiter were to convert all its gravitational binding energy into heat, how long would it take for the giant planet to contract to a point, assuming it maintained the constant blackbody temperature of part (b)? (In reality, the contraction would be slowed to almost zero long before that—why?)

2. Everything You Didn't Want to Know about Moments of Inertia...

   The angular momentum $\mathbf{L}$ of a rigid body is related to its spin vector $\boldsymbol{\omega}$ by $\mathbf{L} = \mathbf{I}\boldsymbol{\omega}$, where $\mathbf{I}$ is a matrix called the *inertia tensor*. For continuous bodies, the elements of $\mathbf{I}$ are given by

$$I_{jk} = \int_V \rho(\mathbf{r})(r^2\delta_{jk} - x_j x_k)\, dV,$$

   where $\mathbf{r} = (x_1, x_2, x_3)$ is a point inside the volume, $\rho(\mathbf{r})$ is the mass density at that point, and δ_{jk} is the Kronecker delta function defined such that

$$\delta_{jk} = \left\{ \begin{array}{ll} 1, & j = k \\ 0, & j \neq k \end{array} \right. .$$

   The diagonal elements ($j = k$) of $\mathbf{I}$ are called the *coefficients of inertia* while the off-diagonal elements ($j \neq k$) are the *products of inertia.*

   (a) Compute the inertia tensor for a solid homogeneous sphere of radius R and mass M. *Hint:* compute I_{11} and I_{12} first, notice the pattern, and argue that the entire matrix reduces to the identity matrix times a single scalar of form kMR^2—find k. The quantity kMR^2 is the *moment of inertia* of a solid homogeneous sphere.

   (b) Compute the inertia tensor for a solid homogeneous spherical *shell* of inner radius r, outer radius R, and total mass M. *Hint:* all that's really changed from part (a) is the lower limit of the r integral—use this to your advantage to avoid a lot of work! Again, express your answer as kMR^2, the moment of inertia of a solid homogeneous spherical shell, and find k (which will be a function of r and R but will still be dimensionless). Verify the expected behaviour as $r \to 0$.

(c) *Bonus:* compute the inertia tensor of a *thin* shell of radius R and mass M. *Hint:* use your result from part (b)—you don't need to integrate!

(d) It can be shown that the free precession frequency of a symmetrical rigid body rotating at frequency ω_3 around its symmetry axis is given by

$$\Omega = \left(\frac{I_3 - I_1}{I_1}\right)\omega_3,$$

where we have set $I_1 \equiv I_{11}$ and $I_3 \equiv I_{33}$, and taken the z axis to be the symmetry axis (so that $I_2 = I_1$).[1] For the Earth, $(I_3 - I_1)/I_1 \doteq 0.00327$ (not surprisingly, this value is close to the Earth's *oblateness* $\epsilon \equiv (R_{\text{eq}} - R_{\text{polar}})/R_{\text{eq}} \doteq 0.00335$). Find the free precession *period* and compare it to the precession period of the equinoxes. (It turns out the free precession period is quite close and probably related to the so-called *Chandler wobble* period of the Earth—look it up with `google`!)

3. Term Project

(a) Provide the title and a 2–3-sentence description of your chosen term project (recall the essay is due Nov. 23 and the oral presentations will be Dec. 7 & 9).

[1] Strictly speaking, I_1, I_2, and I_3 are the *principal* moments of inertia obtained by diagonalizing the inertia tensor and solving for its eigenvalues. The eigenvectors are the principal axes of the body. In this case, we are just assuming the principal axes are aligned with the space axes for simplicity.