ASTR430 HW#6 (due 12/12/05)

Your work should be handed to me (or placed under my door) by 5 pm Monday, December 12. I will have office hours as usual that day, so you could hand it in then (and discuss your oral presentation from the previous week if you like).

1. The Maxwell-Boltzmann distribution.

   From the kinetic theory of gases, the fraction of particles with speeds between v and $v + dv$ in an ideal gas at thermal equilibrium is given by the Maxwell-Boltzmann distribution function,

   $$p(v)\,dv = \left(\frac{2}{\pi}\right)^{1/2} \left(\frac{m}{kT}\right)^{3/2} v^2 e^{-mv^2/2kT}\,dv,$$

   where m is the molecular mass and T is the temperature (k is the Boltzmann constant).

   (a) Verify that this distribution is normalized (i.e., show that $\int_0^\infty p(v)\,dv = 1$). What are the units of $p(v)$?

   (b) Derive expressions for the most probable speed $v_{\rm max}$ (i.e., the speed corresponding to the peak in the distribution), the mean speed $v_{\rm avg} = \int_0^\infty v\,p(v)\,dv$, and the root-mean-square speed $v_{\rm rms} = [\int_0^\infty v^2\,p(v)\,dv]^{1/2}$. Show that $v_{\rm max} \le v_{\rm avg} \le v_{\rm rms}$. Evaluate $p(v)$ at each of these values.

   (c) Plot the distribution functions for hydrogen and oxygen (^{16}O) atoms at 900 K, for v between 0 and 25 km/s. For what $v > v_{\rm max}$ is $p(v)\,dv \lesssim 10^{-12}$ in each case? Express your answers in km/s, and also in terms of $v_{\rm max}$, $v_{\rm avg}$, and $v_{\rm rms}$ for both species.

2. Atmospheres.

   (a) Do problem #9 from Ch. 11 (p. 347).

   (b) Also do problem #10.

   (c) Infrared observations of 90377 Sedna show a peak at 88 μm. What is the temperature of Sedna? At that temperature, familiar gases like N_2 and CO_2 are frozen ices. Helium (He) and neon (Ne) would still be gaseous. Assuming Sedna has a radius of 800 km and a bulk density of 3000 kg/m^3, can this body retain an atmosphere of He? What about Ne? Comment.

3. Habitable zones. Do problem #9 from Ch. 12 (p. 369).