

Due October 2, 2007

Topics for this problem set include round-off error and linear algebra.

1. Write a program to generate N random single-precision floats between 0 and 1 (either inclusive or exclusive of the endpoints; it doesn't matter). Store them in an array.
 - (a) Write a simple function that takes the array and its size N as arguments and returns the sum of the array elements as a single-precision float. Just use brute-force summation for this function.
 - (b) Add a second function that repeats this exercise but using the "Kahan summation algorithm" (do a web search). Compare the results of these summations for $N = 100$ and $N = 10000$ by computing the fractional magnitude difference $|\Sigma_{\text{Kahan}} - \Sigma_{\text{brute}}|/\Sigma_{\text{Kahan}}$ in each case. Run each comparison a few times to get a feel for the statistical fluctuation. Comment.
 - (c) Generate the mean fractional magnitude difference for 1000 iterations for each of $N = 10, 100, 1000, 10000, 100000$. Plot the result and comment. Be sure to use the Kahan algorithm when computing the average!
 - (d) Repeat this entire exercise in double precision and comment.
2. Write a program to compute the instantaneous spin period of a rigid body made up of identical, discrete, point particles. Use the fact that the angular momentum is

$$\mathbf{L} = \sum_i m_i(\mathbf{r}_i \times \mathbf{v}_i) = \mathbf{I}\boldsymbol{\omega}, \quad (1)$$

where m_i is the mass of particle i , \mathbf{r}_i and \mathbf{v}_i are its position and velocity vectors with respect to the centre of mass, $\boldsymbol{\omega}$ is the spin vector, and \mathbf{I} is the inertia tensor

$$\mathbf{I} = \sum_i m_i(r_i^2 \mathbf{1} - \mathbf{r}_i \mathbf{r}_i),$$

where $\mathbf{1}$ is the unit matrix.¹ Write a program to solve Eq. (1) for $\boldsymbol{\omega}$ (feel free to use the routines in *NR*). The spin period is then $2\pi/|\boldsymbol{\omega}|$.

- (a) Test your code by reading the data file

<http://www.astro.umd.edu/~dcr/Courses/ASTR615/ps2.dat>

which is in the format $x \ y \ z \ v_x \ v_y \ v_z$ (i.e. 6 values to a line separated by white space). The units are mks (SI). What is the spin period in hours?

- (b) BONUS: Make a graphical representation of the body using your favorite graphing package. If you use 2-D projections, be sure to include enough viewing angles to get a complete picture.

¹For continuous bodies the summations are replaced by volume integrations and the particle masses become a mass density. In the present case the m_i 's can be omitted entirely since the particles are identical.